Computer Algebra
Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/151-5.3.5-u-a+b-arctan-c+d-x-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [70]. This is test number [151].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (70)	0.00 (0)
Maple	98.57 (69)	1.43 (1)
Mathematica	95.71 (67)	4.29 (3)
Maxima	52.86 (37)	47.14 (33)
Mupad	42.86 (30)	57.14 (40)
Fricas	40.00 (28)	60.00 (42)
Sympy	32.86 (23)	67.14 (47)
Giac	12.86 (9)	87.14 (61)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

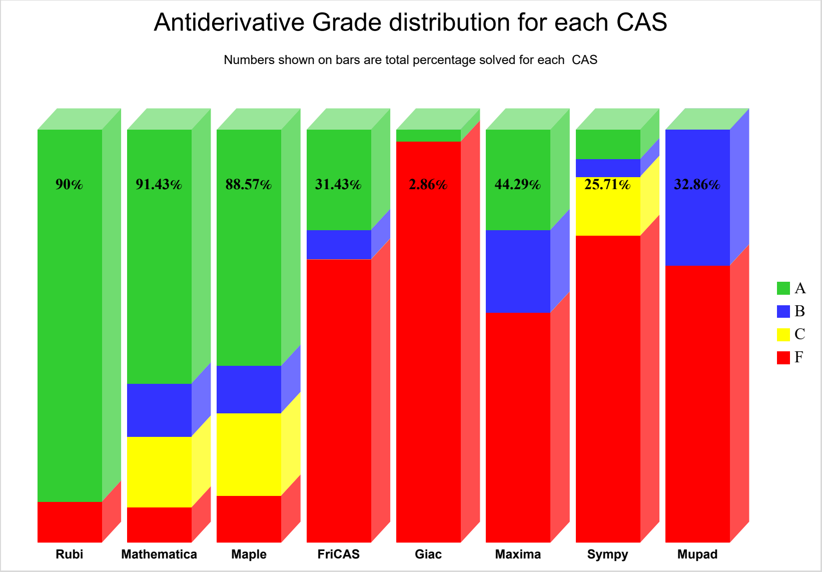
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

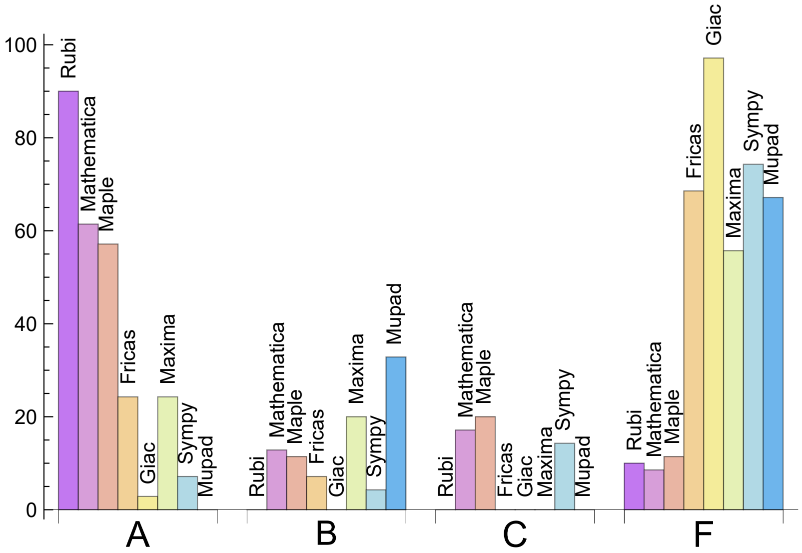
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.000	0.000	0.000	10.000
Mathematica	61.429	12.857	17.143	8.571
Maple	57.143	11.429	20.000	11.429
Fricas	24.286	7.143	0.000	68.571
Maxima	24.286	20.000	0.000	55.714
Sympy	7.143	4.286	14.286	74.286
Giac	2.857	0.000	0.000	97.143
Mupad	0.000	32.857	0.000	67.143

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	3	100.00	0.00	0.00
Rubi	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Maxima	33	93.94	0.00	6.06
Mupad	40	0.00	100.00	0.00
Fricas	42	97.62	0.00	2.38
Sympy	47	40.43	59.57	0.00
Giac	61	75.41	21.31	3.28

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.24
Fricas	0.31
Maxima	1.02
Mathematica	1.03
Mupad	1.47
Maple	1.50
Sympy	15.01
Giac	90.38

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	9.78	0.30	3.00	0.15
Fricas	140.75	1.45	81.00	1.25
Mupad	178.37	1.75	102.50	1.35
Rubi	230.81	1.00	159.50	1.00
Sympy	236.30	2.65	168.00	1.98
Mathematica	261.99	1.42	163.00	1.08
Maple	668.71	2.40	186.00	1.05
Maxima	1036.14	4.56	165.00	1.42

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

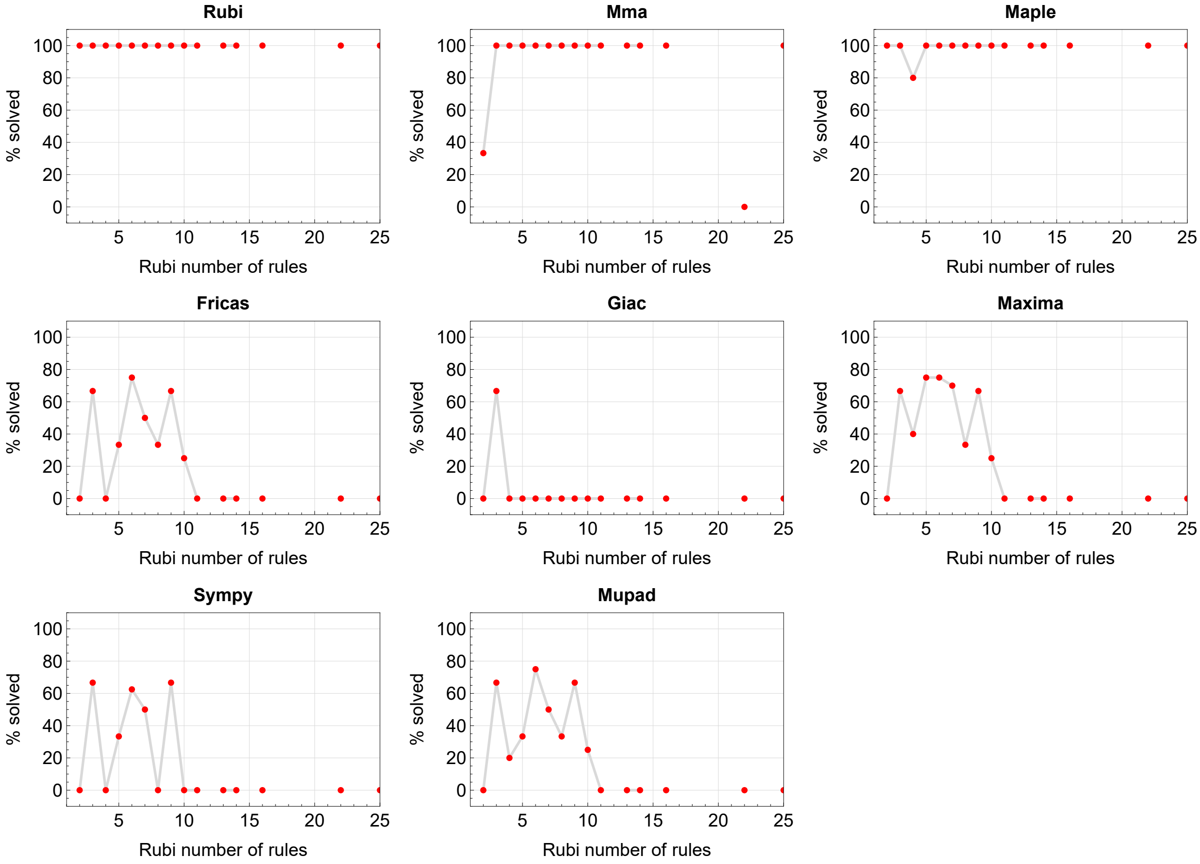


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

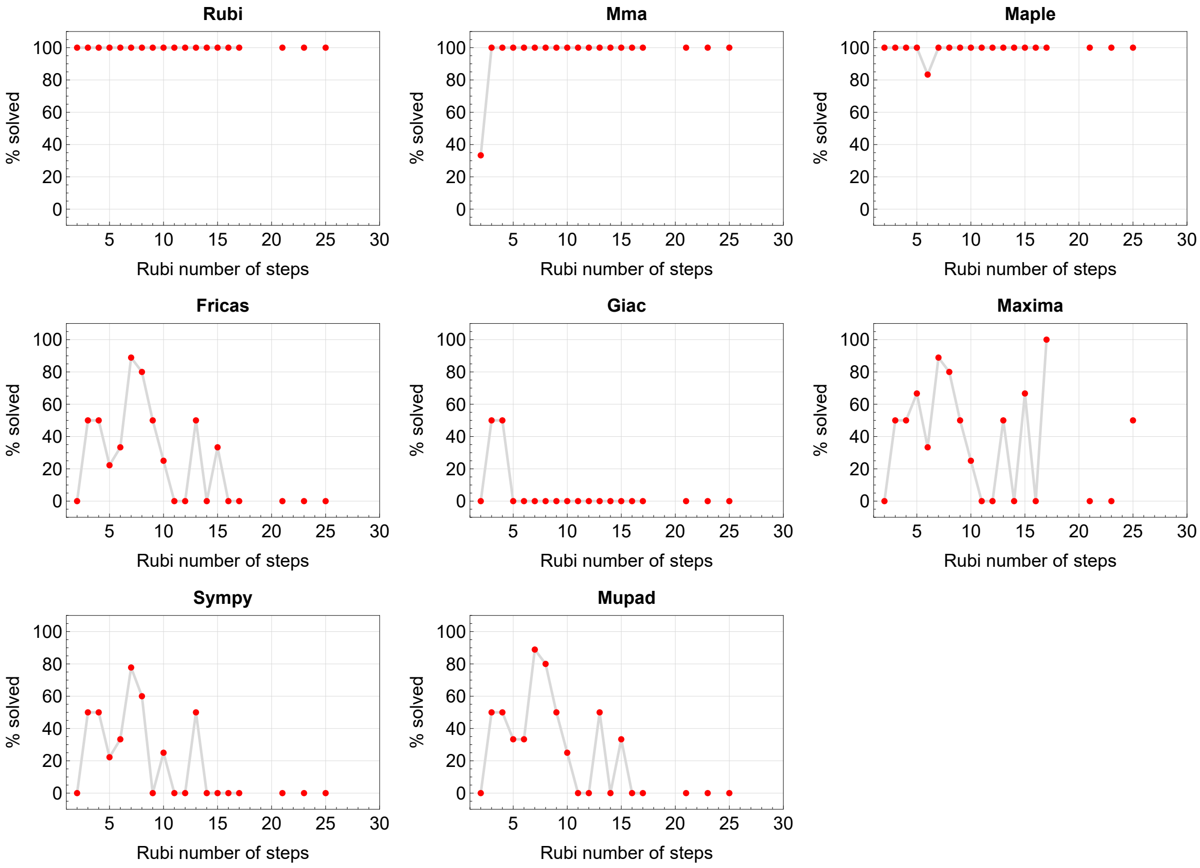


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

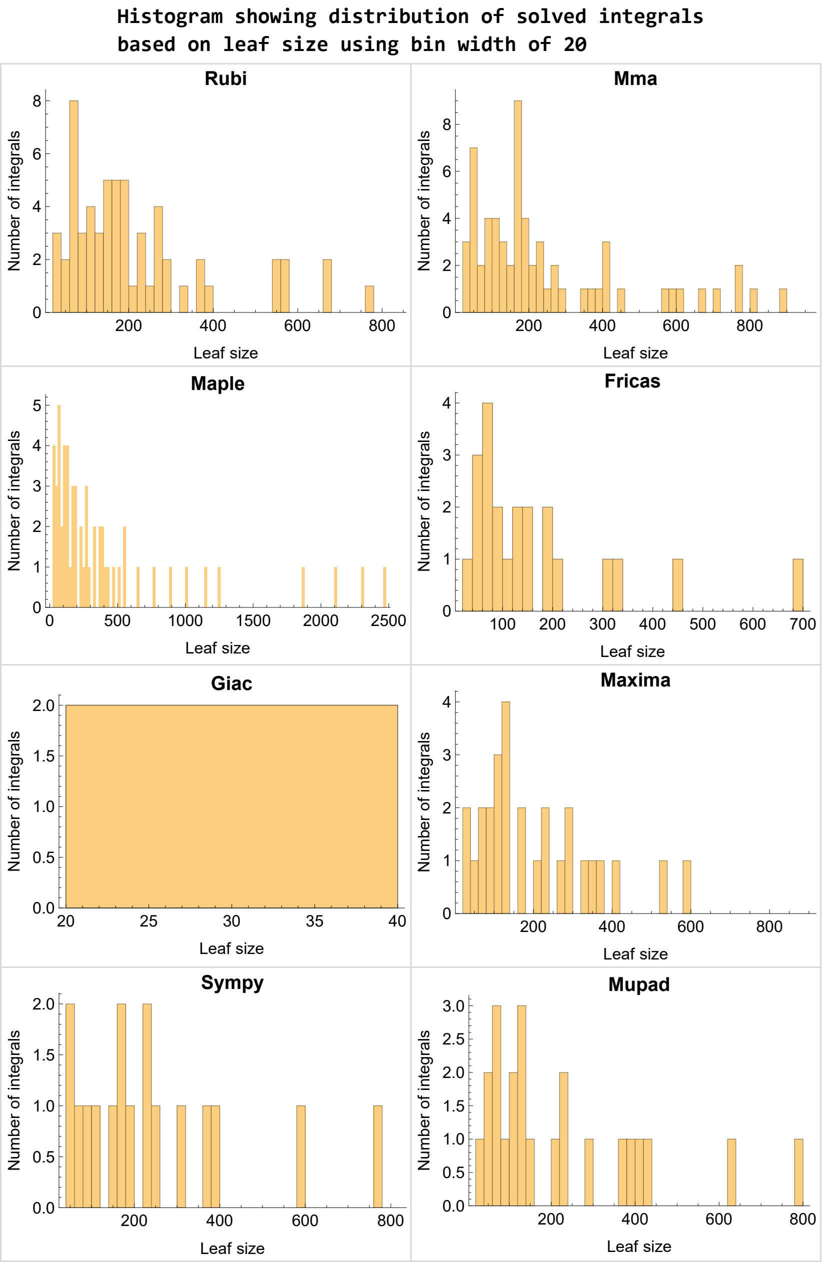


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

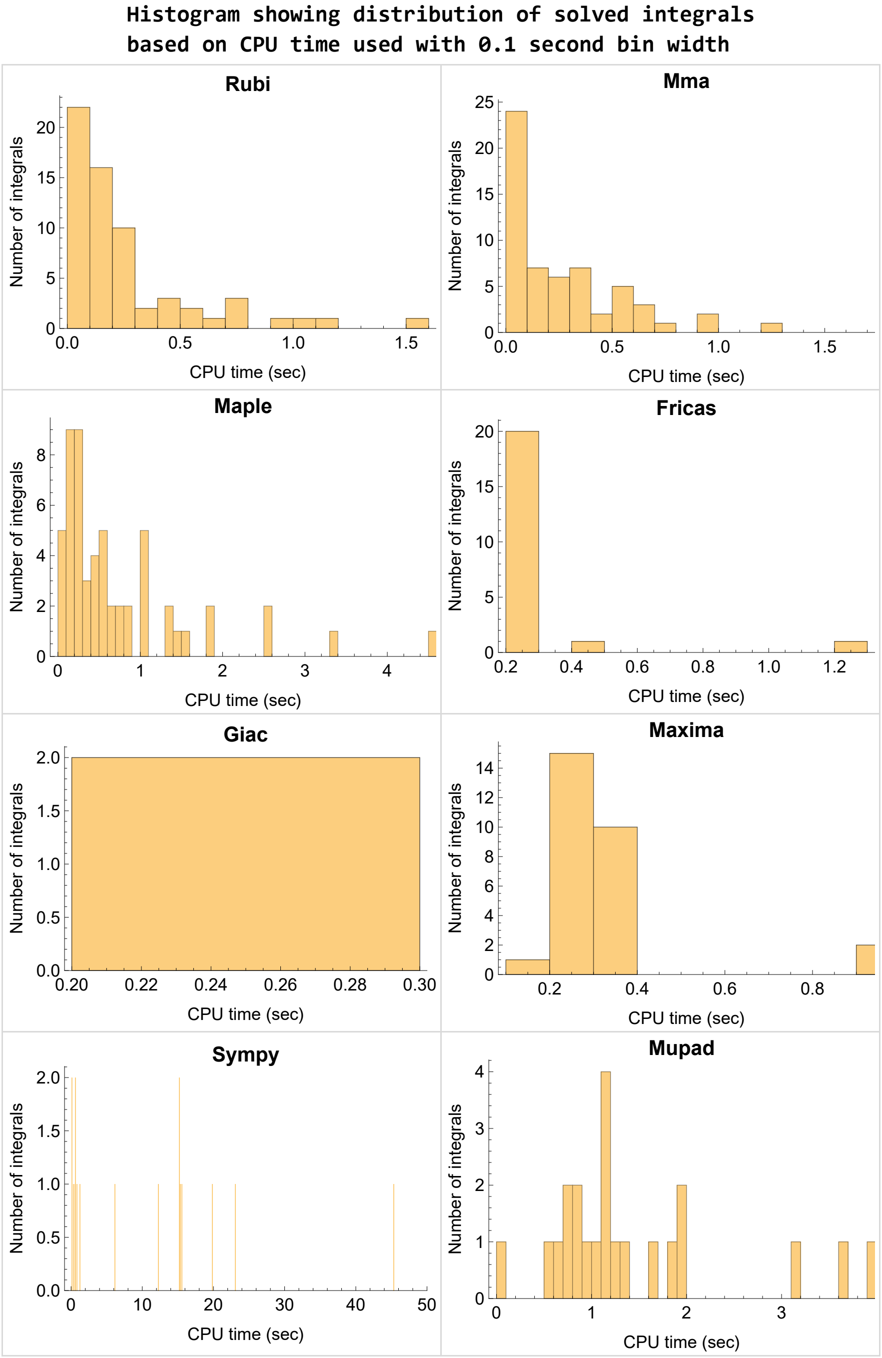


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

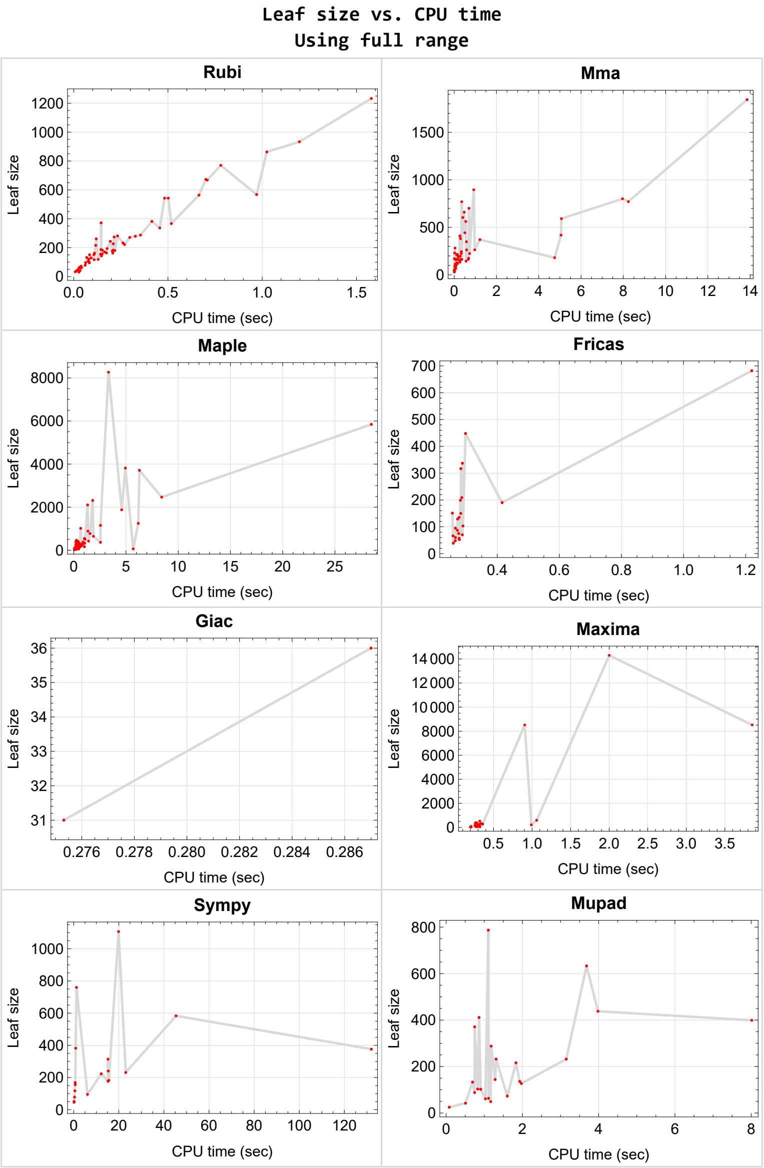


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{23, 42, 43, 65, 66, 69, 70\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{65, 66, 69, 70\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{36\}$

Maple $\{10, 15, 17, 18, 20, 34, 36, 37, 39, 40, 52, 57, 59\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
 # 1.7 is a fudge factor since it is low side from actual leaf count
 leafCount = round(1.7*count_ops(anti))

 except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

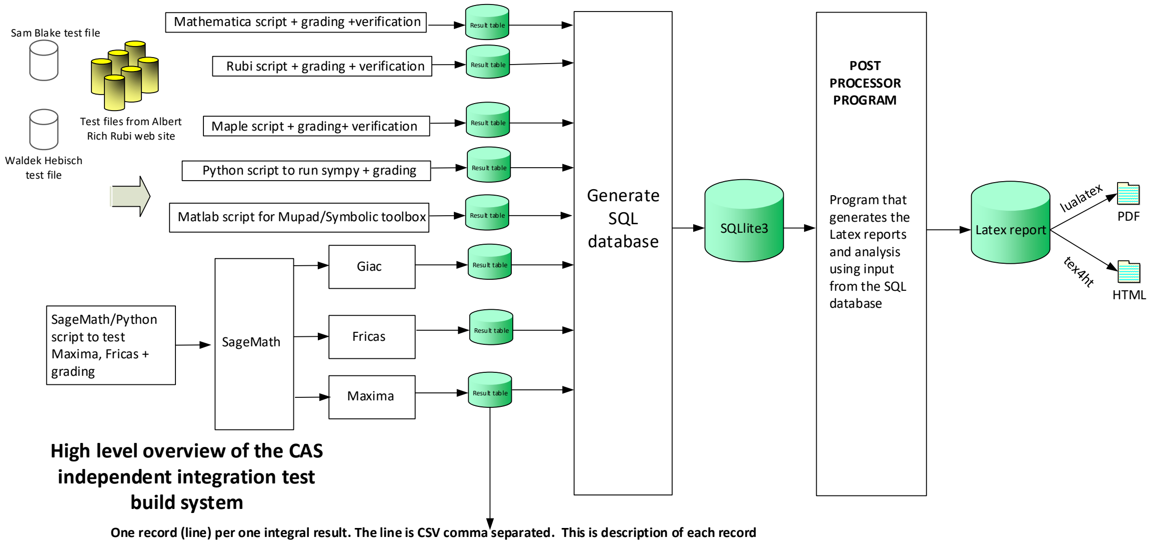
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 27, 28, 32, 33, 35, 37, 38, 41, 47, 48, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

B grade { 10, 17, 31, 36, 55, 65, 66, 69, 70 }

C grade { 6, 24, 25, 26, 29, 30, 44, 45, 46, 49, 50, 51 }

F normal fail { 34, 39, 40 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade $\{ 1,2,3,4,5,6,7,8,9,12,14,21,22,25,26,27,28,29,30,32,33,35,44,45,46,47,48,49,50,51,53,54,55,56,60,61,63,64,67,68 \}$

B grade $\{ 11,13,16,19,24,31,38,62 \}$

C grade $\{ 10,15,17,18,20,34,36,37,39,40,52,57,58,59 \}$

F normal fail $\{ 41 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ \}$

Fricas

A grade $\{ 3,5,6,9,12,24,25,26,27,29,44,45,46,47,49,50,51 \}$

B grade $\{ 1,2,7,14,30 \}$

C grade $\{ \}$

F normal fail $\{ 4,8,10,11,13,15,16,17,18,19,20,21,22,28,31,32,33,34,35,36,37,38,39,40,41,48,52,53,54,55,56,57,58,59,60,61,62,63,64,67,68 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ 23 \}$

Maxima

A grade $\{ 5,24,25,26,27,29,30,44,45,46,47,48,49,50,51,55,60 \}$

B grade $\{ 1,2,3,6,7,9,12,14,21,22,53,54,56,61 \}$

C grade $\{ \}$

F normal fail $\{ 4,8,10,11,13,15,16,17,18,19,20,28,31,32,33,34,35,36,37,38,39,40,41,52,57,58,59,63,64,67,68 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ 23,62 \}$

Giac

A grade { 27, 47 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 36, 37, 38, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68 }

F(-1) timedout fail { 10, 11, 12, 13, 14, 17, 18, 19, 20, 34, 35, 39, 40 }

F(-2) exception fail { 58, 59 }

Mupad

A grade { }

B grade { 1, 2, 3, 5, 6, 7, 9, 12, 14, 21, 24, 25, 26, 27, 29, 30, 44, 45, 46, 47, 49, 50, 51 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 22, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

F(-2) exception fail { }

Sympy

A grade { 27, 44, 45, 46, 47 }

B grade { 1, 3, 6 }

C grade { 2, 5, 7, 9, 12, 25, 26, 49, 50, 51 }

F normal fail { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 32, 33, 37, 38, 60, 63 }

F(-1) timedout fail { 14, 24, 28, 29, 30, 31, 34, 35, 36, 39, 40, 41, 42, 43, 48, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 64, 67, 68 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	64	370	151	231	0	371
N.S.	1	1.00	0.78	0.89	5.14	2.10	3.21	0.00	5.15
time (sec)	N/A	0.039	0.015	5.695	0.285	0.255	23.076	0.000	0.748

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	61	238	129	182	0	144
N.S.	1	1.00	0.81	0.91	3.55	1.93	2.72	0.00	2.15
time (sec)	N/A	0.039	0.018	0.230	0.312	0.272	15.567	0.000	1.285

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	51	120	60	95	0	73
N.S.	1	1.00	0.83	1.06	2.50	1.25	1.98	0.00	1.52
time (sec)	N/A	0.022	0.012	0.107	0.311	0.265	6.121	0.000	1.604

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	65	0	0	0	0	0
N.S.	1	1.00	0.83	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.016	0.348	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	52	58	92	75	223	0	88
N.S.	1	1.00	0.85	0.95	1.51	1.23	3.66	0.00	1.44
time (sec)	N/A	0.033	0.020	0.279	0.207	0.276	12.222	0.000	0.747

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	51	57	120	70	314	0	103
N.S.	1	1.00	0.81	0.90	1.90	1.11	4.98	0.00	1.63
time (sec)	N/A	0.031	0.016	0.469	0.265	0.287	15.213	0.000	0.828

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	216	152	597	337	583	0	633
N.S.	1	1.00	1.38	0.97	3.80	2.15	3.71	0.00	4.03
time (sec)	N/A	0.151	0.146	0.489	1.060	0.288	45.360	0.000	3.682

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	163	276	0	0	0	0	0
N.S.	1	1.00	0.89	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	0.362	0.845	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	113	218	150	240	0	216
N.S.	1	1.00	1.13	1.19	2.29	1.58	2.53	0.00	2.27
time (sec)	N/A	0.083	0.097	0.195	0.991	0.282	15.311	0.000	1.831

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	183	183	381	1154	0	0	0	0	0
N.S.	1	1.00	2.08	6.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.301	2.563	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	135	325	0	0	0	0	0
N.S.	1	1.00	1.13	2.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.277	1.053	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	194	129	268	209	1107	0	232
N.S.	1	1.00	1.66	1.10	2.29	1.79	9.46	0.00	1.98
time (sec)	N/A	0.108	0.202	0.537	0.320	0.286	19.884	0.000	3.150

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	163	368	0	0	0	0	0
N.S.	1	1.00	0.84	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.663	2.558	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	245	164	534	448	0	0	438
N.S.	1	1.00	1.44	0.96	3.14	2.64	0.00	0.00	2.58
time (sec)	N/A	0.164	0.339	1.024	0.322	0.298	0.000	0.000	3.981

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	271	271	349	1249	0	0	0	0	0
N.S.	1	1.00	1.29	4.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.583	6.181	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	196	321	0	0	0	0	0
N.S.	1	1.00	1.20	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	0.285	0.553	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	279	279	562	2313	0	0	0	0	0
N.S.	1	1.00	2.01	8.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	0.548	1.814	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	163	163	263	2104	0	0	0	0	0
N.S.	1	1.00	1.61	12.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.586	1.334	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	225	423	0	0	0	0	0
N.S.	1	1.00	1.25	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.334	1.419	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	287	287	371	2465	0	0	0	0	0
N.S.	1	1.00	1.29	8.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.354	1.217	8.420	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	44	0	0	0	25
N.S.	1	1.00	1.00	0.84	1.42	0.00	0.00	0.00	0.81
time (sec)	N/A	0.027	0.007	0.061	0.323	0.000	0.000	0.000	0.082

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	38	123	0	0	0	0
N.S.	1	1.00	0.83	0.93	3.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.008	0.184	0.309	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	0	0	17	3	18
N.S.	1	1.00	1.11	0.89	0.00	0.00	0.94	0.17	1.00
time (sec)	N/A	0.012	3.739	0.133	0.000	0.000	3.179	127.074	1.016

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	157	464	346	317	0	0	787
N.S.	1	1.00	0.67	1.99	1.48	1.36	0.00	0.00	3.38
time (sec)	N/A	0.261	0.226	0.250	0.263	0.283	0.000	0.000	1.106

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	118	265	220	199	376	0	411
N.S.	1	1.00	0.76	1.71	1.42	1.28	2.43	0.00	2.65
time (sec)	N/A	0.142	0.128	0.201	0.291	0.281	131.969	0.000	0.863

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	163	113	116	103	177	0	136
N.S.	1	1.00	1.68	1.16	1.20	1.06	1.82	0.00	1.40
time (sec)	N/A	0.083	0.053	0.121	0.285	0.289	15.212	0.000	1.929

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	49	35	36	48	51	36	49
N.S.	1	1.00	1.29	0.92	0.95	1.26	1.34	0.95	1.29
time (sec)	N/A	0.013	0.011	0.061	0.200	0.265	0.168	0.287	1.169

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	160	198	0	0	0	0	0
N.S.	1	1.00	0.99	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.093	0.275	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	121	160	177	190	0	0	127
N.S.	1	1.00	0.80	1.06	1.17	1.26	0.00	0.00	0.84
time (sec)	N/A	0.084	0.167	0.246	0.261	0.415	0.000	0.000	1.973

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	175	245	409	682	0	0	399
N.S.	1	1.00	0.77	1.08	1.80	3.00	0.00	0.00	1.76
time (sec)	N/A	0.210	0.684	0.598	0.271	1.220	0.000	0.000	8.015

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	801	1018	0	0	0	0	0
N.S.	1	1.00	2.10	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	7.963	0.666	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	264	413	0	0	0	0	0
N.S.	1	1.00	1.19	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.972	0.417	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	109	139	0	0	0	0	0
N.S.	1	1.00	1.07	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.092	0.286	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	261	261	0	1877	0	0	0	0	0
N.S.	1	1.00	0.00	7.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.000	4.594	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	568	568	419	776	0	0	0	0	0
N.S.	1	1.00	0.74	1.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.970	5.053	1.564	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	564	564	1844	5843	0	0	0	0	0
N.S.	1	1.00	3.27	10.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.665	13.855	28.488	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	337	337	592	8267	0	0	0	0	0
N.S.	1	1.00	1.76	24.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	5.071	3.333	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	212	279	0	0	0	0	0
N.S.	1	1.00	1.48	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.116	0.779	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	372	372	0	3817	0	0	0	0	0
N.S.	1	1.00	0.00	10.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.000	4.948	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	1233	1233	0	3708	0	0	0	0	0
N.S.	1	1.00	0.00	3.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.579	0.000	6.291	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	162	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.327	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	504	36	0	3	22
N.S.	1	1.00	1.10	1.00	25.20	1.80	0.00	0.15	1.10
time (sec)	N/A	0.041	4.368	0.187	8.769	0.267	0.000	111.038	0.542

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	659	52	0	3	22
N.S.	1	1.00	1.10	1.00	32.95	2.60	0.00	0.15	1.10
time (sec)	N/A	0.040	0.534	0.204	11.470	0.281	0.000	112.198	0.555

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	95	131	104	87	155	0	133
N.S.	1	1.00	0.90	1.24	0.98	0.82	1.46	0.00	1.25
time (sec)	N/A	0.077	0.058	0.126	0.269	0.271	0.688	0.000	0.693

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	114	102	85	66	117	0	102
N.S.	1	1.00	1.44	1.29	1.08	0.84	1.48	0.00	1.29
time (sec)	N/A	0.061	0.047	0.094	0.269	0.257	0.514	0.000	0.906

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	90	63	68	52	78	0	61
N.S.	1	1.00	1.50	1.05	1.13	0.87	1.30	0.00	1.02
time (sec)	N/A	0.041	0.026	0.084	0.291	0.278	0.370	0.000	1.030

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	39	30	31	39	46	31	42
N.S.	1	1.00	1.18	0.91	0.94	1.18	1.39	0.94	1.27
time (sec)	N/A	0.008	0.017	0.064	0.210	0.258	0.166	0.275	0.506

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	171	94	134	0	0	0	0
N.S.	1	1.00	1.42	0.78	1.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.011	0.170	0.318	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	67	61	77	57	168	0	63
N.S.	1	1.00	1.08	0.98	1.24	0.92	2.71	0.00	1.02
time (sec)	N/A	0.029	0.051	0.103	0.277	0.277	0.644	0.000	1.113

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	92	84	112	95	382	0	232
N.S.	1	1.00	0.96	0.88	1.17	0.99	3.98	0.00	2.42
time (sec)	N/A	0.063	0.082	0.138	0.293	0.265	0.872	0.000	1.309

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	128	115	165	135	760	0	288
N.S.	1	1.00	0.99	0.89	1.28	1.05	5.89	0.00	2.23
time (sec)	N/A	0.092	0.119	0.167	0.272	0.277	1.231	0.000	1.180

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	863	863	701	380	0	0	0	0	0
N.S.	1	1.00	0.81	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.024	0.698	0.885	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	409	542	8520	0	0	0	0
N.S.	1	1.00	0.75	1.00	15.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.284	1.041	3.854	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	231	186	284	0	0	0	0
N.S.	1	1.00	1.52	1.22	1.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.020	0.379	0.358	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	771	295	284	0	0	0	0
N.S.	1	1.00	3.16	1.21	1.16	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	8.240	0.345	0.359	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	668	668	660	647	8518	0	0	0	0
N.S.	1	1.00	0.99	0.97	12.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.708	0.475	1.885	0.905	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	933	933	896	511	0	0	0	0	0
N.S.	1	1.00	0.96	0.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.197	0.925	1.092	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	604	364	0	0	0	0	0
N.S.	1	1.00	0.90	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.701	0.407	0.238	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	770	770	770	388	0	0	0	0	0
N.S.	1	1.00	1.00	0.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.780	0.358	0.210	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	283	226	328	0	0	0	0
N.S.	1	1.00	1.03	0.82	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.041	0.779	0.340	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	409	542	14300	0	0	0	0
N.S.	1	1.00	0.75	1.00	26.34	0.00	0.00	0.00	0.00
time (sec)	N/A	0.501	0.267	1.011	2.001	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	443	890	0	0	0	0	0
N.S.	1	1.00	1.21	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.518	0.507	1.363	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	67	135	0	0	0	0	0
N.S.	1	1.00	0.51	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.072	0.427	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	95	176	0	0	0	0	0
N.S.	1	1.00	0.44	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.068	0.513	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	163	26	28	28	29	3	28
N.S.	1	1.00	5.82	0.93	1.00	1.00	1.04	0.11	1.00
time (sec)	N/A	0.030	0.311	0.246	0.259	0.262	0.882	56.779	0.574

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	165	31	33	33	31	3	33
N.S.	1	1.00	5.00	0.94	1.00	1.00	0.94	0.09	1.00
time (sec)	N/A	0.036	0.152	0.203	0.285	0.279	6.574	59.742	0.561

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	145	180	0	0	0	0	0
N.S.	1	1.00	0.78	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.559	0.677	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	189	222	0	0	0	0	0
N.S.	1	1.00	0.67	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.190	0.524	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	181	33	35	44	36	3	35
N.S.	1	1.00	5.17	0.94	1.00	1.26	1.03	0.09	1.00
time (sec)	N/A	0.089	4.753	0.230	0.357	0.289	3.725	172.488	0.594

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	225	38	40	49	37	3	40
N.S.	1	1.00	5.62	0.95	1.00	1.22	0.92	0.08	1.00
time (sec)	N/A	0.127	0.728	0.211	0.398	0.280	26.309	173.556	0.588

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [35] had the largest ratio of [1.25000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	21	0.238
2	A	6	5	1.00	21	0.238
3	A	5	5	1.00	19	0.263
4	A	5	4	1.00	21	0.190
5	A	7	7	1.00	21	0.333
6	A	5	5	1.00	21	0.238
7	A	13	9	1.00	23	0.391
8	A	11	10	1.00	23	0.435
9	A	8	7	1.00	21	0.333
10	A	8	7	1.00	23	0.304
11	A	6	6	1.00	23	0.261
12	A	10	9	1.00	23	0.391
13	A	10	9	1.00	23	0.391
14	A	15	10	1.00	23	0.435
15	A	14	11	1.00	23	0.478
16	A	10	10	1.00	21	0.476
17	A	10	8	1.00	23	0.348
18	A	7	8	1.00	23	0.348
19	A	9	8	1.00	23	0.348
20	A	16	13	1.00	23	0.565
21	A	5	4	1.00	12	0.333
22	A	5	4	1.00	19	0.210
23	N/A	0	0	1.00	18	0.000
24	A	7	6	1.00	18	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	7	6	1.00	18	0.333
26	A	7	6	1.00	16	0.375
27	A	4	3	1.00	10	0.300
28	A	5	5	1.00	18	0.278
29	A	8	8	1.00	18	0.444
30	A	9	8	1.00	18	0.444
31	A	16	13	1.00	20	0.650
32	A	13	10	1.00	18	0.556
33	A	6	6	1.00	12	0.500
34	A	2	2	1.00	20	0.100
35	A	25	25	1.00	20	1.250
36	A	21	14	1.00	20	0.700
37	A	15	11	1.00	18	0.611
38	A	6	7	1.00	12	0.583
39	A	2	2	1.00	20	0.100
40	A	35	22	1.00	20	1.100
41	A	6	4	1.00	18	0.222
42	N/A	0	0	1.00	20	0.000
43	N/A	0	0	1.00	20	0.000
44	A	7	6	1.00	10	0.600
45	A	7	6	1.00	10	0.600
46	A	7	6	1.00	8	0.750
47	A	3	3	1.00	6	0.500
48	A	5	5	1.00	10	0.500
49	A	7	7	1.00	10	0.700
50	A	8	7	1.00	10	0.700
51	A	8	7	1.00	10	0.700
52	A	23	5	1.00	16	0.312
53	A	17	5	1.00	16	0.312
54	A	5	5	1.00	14	0.357
55	A	15	7	1.00	16	0.438
56	A	25	7	1.00	16	0.438
57	A	31	7	1.00	16	0.438
58	A	31	13	1.00	18	0.722
59	A	37	16	1.00	18	0.889

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	17	5	1.00	14	0.357
61	A	17	5	1.00	16	0.312
62	A	12	8	1.00	19	0.421
63	A	2	2	1.00	28	0.071
64	A	3	3	1.00	33	0.091
65	N/A	0	0	1.00	28	0.000
66	N/A	0	0	1.00	33	0.000
67	A	4	4	1.00	35	0.114
68	A	5	5	1.00	40	0.125
69	N/A	0	0	1.00	35	0.000
70	N/A	0	0	1.00	40	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (ce+dex)^3(a+b\arctan(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx = \frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\arctan(c+dx)}{4d}+\frac{e^3(c+dx)^4(a+b\arctan(c+dx))}{4d}$$

```
[Out] 1/4*b*e^3*x-1/12*b*e^3*(d*x+c)^3/d-1/4*b*e^3*arctan(d*x+c)/d+1/4*e^3*(d*x+c
)^4*(a+b*arctan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5151, 12, 4946, 308, 209}

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx = \frac{e^3(c+dx)^4(a+b\arctan(c+dx))}{4d}-\frac{be^3\arctan(c+dx)}{4d}-\frac{be^3(c+dx)^3}{12d}+\frac{1}{4}be^3x$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (b*e^3*x)/4 - (b*e^3*(c + d*x)^3)/(12*d) - (b*e^3*ArcTan[c + d*x])/(4*d) +
(e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^3x^3(a+b\arctan(x))\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int x^3(a+b\arctan(x))\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3(c+dx)^4(a+b\arctan(c+dx))}{4d}-\frac{(be^3)\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx,x,c+dx\right)}{4d}\\
&= \frac{e^3(c+dx)^4(a+b\arctan(c+dx))}{4d}-\frac{(be^3)\,\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx,x,c+dx\right)}{4d}\\
&= \frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}+\frac{e^3(c+dx)^4(a+b\arctan(c+dx))}{4d}-\frac{(be^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{4d}\\
&= \frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\arctan(c+dx)}{4d}+\frac{e^3(c+dx)^4(a+b\arctan(c+dx))}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx$$
$$=\frac{e^3\left(-\frac{1}{4}b\left(-dx+\frac{1}{3}(c+dx)^3+\arctan(c+dx)\right)+\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))\right)}{d}$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e^3*(-1/4*(b*(-(d*x) + (c + d*x)^3/3 + ArcTan[c + d*x])) + ((c + d*x)^4*(a
 + b*ArcTan[c + d*x]))/4))/d
```

Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{e^3a(dx+c)^4}{4}+e^3b\left(\frac{(dx+c)^4\arctan(dx+c)}{4}-\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}-\frac{\arctan(dx+c)}{4}\right)}{d}$
default	$\frac{\frac{e^3a(dx+c)^4}{4}+e^3b\left(\frac{(dx+c)^4\arctan(dx+c)}{4}-\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}-\frac{\arctan(dx+c)}{4}\right)}{d}$
parts	$\frac{e^3a(dx+c)^4}{4d}+\frac{e^3b\left(\frac{(dx+c)^4\arctan(dx+c)}{4}-\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}-\frac{\arctan(dx+c)}{4}\right)}{d}$
parallelrisch	$3d^5e^3b\arctan(dx+c)x^4+3x^4a\,d^5e^3+12bc\,d^4e^3\arctan(dx+c)x^3+12x^3ac\,d^4e^3+18x^2\arctan(dx+c)b\,c^2d^3e^3-x^3b\,d^4e^3+18$
risch	$\frac{ie^3d^2bc\,x^3\ln(1-i(dx+c))}{2}+\frac{ie^3d^3b\,x^4\ln(1-i(dx+c))}{8}+\frac{ie^3b\,c^3x\ln(1-i(dx+c))}{2}+\frac{3ie^3db\,c^2x^2\ln(1-i(dx+c))}{4}+e^3$

```
[In] int((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*a*(d*x+c)^4+e^3*b*(1/4*(d*x+c)^4*arctan(d*x+c)-1/12*(d*x+c)^3+
1/4*d*x+1/4*c-1/4*arctan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.10

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx$$
$$=\frac{3\,ad^4e^3x^4+(12\,ac-b)d^3e^3x^3+3\,(6\,ac^2-bc)d^2e^3x^2+3\,(4\,ac^3-bc^2+b)de^3x+3\,(bd^4e^3x^4+4\,bcd^3e^3x^3+}{12\,d}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*e^3*x^4 + (12*a*c - b)*d^3*e^3*x^3 + 3*(6*a*c^2 - b*c)*d^2*e^
3*x^2 + 3*(4*a*c^3 - b*c^2 + b)*d*e^3*x + 3*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^3*
x^3 + 6*b*c^2*d^2*e^3*x^2 + 4*b*c^3*d*e^3*x + (b*c^4 - b)*e^3)*arctan(d*x +
 c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(61) = 122$.

Time = 23.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.21

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx$$
$$= \begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{atan}(c+dx)}{4d} + bc^3e^3x\operatorname{atan}(c+dx) + \frac{3bc^2de^3x^2\operatorname{atan}(c+dx)}{2} - bc \\ c^3e^3x(a+b\operatorname{atan}(c)) \end{cases}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*atan(c + d*x)/(4*d) + b*c**3*e**3*x*atan(c +
 d*x) + 3*b*c**2*d*e**3*x**2*atan(c + d*x)/2 - b*c**2*e**3*x/4 + b*c*d**2*e
**3*x**3*atan(c + d*x) - b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atan(c + d*x)
/4 - b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atan(c + d*x)/(4*d), Ne(d, 0
)), (c**3*e**3*x*(a + b*atan(c)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(64) = 128$.

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.14

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx = \frac{1}{4}\,ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}\,ac^2de^3x^2$$
$$+\frac{3}{2}\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)bc^2de^3$$
$$+\frac{1}{2}\left(2\,x^3\arctan(dx+c) - d\left(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log(d^2x^2+2\,cdx}{d^4}\right.\right.$$
$$+\frac{1}{12}\left(3\,x^4\arctan(dx+c) - d\left(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4} + \frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - 6\,(c\right.\right.$$
$$+ ac^3e^3x + \frac{\left(2\,(dx+c)\arctan(dx+c) - \log\left((dx+c)^2+1\right)\right)bc^3e^3}{2\,d}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/2*(x^2*arctan
(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)/d^3))*b*c^2*d*e^3 + 1/2*(2*x^3*arctan(d*x + c) - d*(
(d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 -
1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/12*(3*x^4*arctan(
d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2
+ 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)/d^5))*b*d^3*e^3 + a*c^3*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log
((d*x + c)^2 + 1))*b*c^3*e^3/d
```

Giac [F]

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx=\int (dex+ce)^3(b\arctan\left(dx+c\right)+a)\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.15

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx$$
$$\begin{aligned}&=\operatorname{atan}(c+d\,x)\,\left(b\,c^3\,e^3\,x+\frac{3\,b\,c^2\,d\,e^3\,x^2}{2}+b\,c\,d^2\,e^3\,x^3+\frac{b\,d^3\,e^3\,x^4}{4}\right)\\&\quad-x^3\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{12}+\frac{2\,a\,c\,d^2\,e^3}{3}\right)\\&\quad+x^2\left(\frac{c\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{d}+\frac{d\,e^3\,(10\,a\,c^2-b\,c+a)}{2}-\frac{a\,d\,e^3\,(4\,c^2+4)}{8}\right)\\&\quad+x\left(\frac{c\,e^3\,(20\,a\,c^2-3\,b\,c+6\,a)}{2}+\frac{(4\,c^2+4)\,\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{4\,d^2}\right.\\&\qquad\left.-\frac{2\,c\left(\frac{2\,c\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{d}+d\,e^3\,(10\,a\,c^2-b\,c+a)-\frac{a\,d\,e^3\,(4\,c^2+4)}{4}\right)}{d}\right)+\frac{a\,d^3\,e^3\,x^4}{4}\\&\quad-\frac{b\,e^3\,\operatorname{atan}\left(\frac{\frac{b\,c\,e^3\,\left(c^2+1\right)\,(c-1)\,(c+1)}{4}+\frac{b\,d\,e^3\,x\,\left(c^2+1\right)\,(c-1)\,(c+1)}{4}}{\frac{b\,e^3}{4}-\frac{b\,c^4\,e^3}{4}}\right)\,(c^2+1)\,(c-1)\,(c+1)}{4\,d}\end{aligned}$$

```
[In] int((c*e + d*e*x)^3*(a + b*atan(c + d*x)),x)
```

```
[Out] atan(c + d*x)*((b*d^3*e^3*x^4)/4 + b*c^3*e^3*x + (3*b*c^2*d*e^3*x^2)/2 + b*
c*d^2*e^3*x^3) - x^3*((d^2*e^3*(b - 20*a*c))/12 + (2*a*c*d^2*e^3)/3) + x^2*
((c*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/d + (d*e^3*(a - b*c + 10*a*
c^2))/2 - (a*d*e^3*(4*c^2 + 4))/8) + x*((c*e^3*(6*a - 3*b*c + 20*a*c^2))/2
+ ((4*c^2 + 4)*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/(4*d^2) - (2*c*(
(2*c*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/d + d*e^3*(a - b*c + 10*a*
c^2) - (a*d*e^3*(4*c^2 + 4))/4))/d) + (a*d^3*e^3*x^4)/4 - (b*e^3*atan(((b*c
*e^3*(c^2 + 1)*(c - 1)*(c + 1))/4 + (b*d*e^3*x*(c^2 + 1)*(c - 1)*(c + 1))/4
)/((b*e^3)/4 - (b*c^4*e^3)/4))*(c^2 + 1)*(c - 1)*(c + 1))/(4*d)
```

3.2 $\int (ce+dex)^2(a+b\arctan(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx=-\frac{be^2(c+dx)^2}{6d}+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))}{3d}+\frac{be^2\log\left(1+(c+dx)^2\right)}{6d}$$

```
[Out] -1/6*b*e^2*(d*x+c)^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*x+c))/d+1/6*b*e^2*ln
(1+(d*x+c)^2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5151, 12, 4946, 272, 45}

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx=\frac{e^2(c+dx)^3(a+b\arctan(c+dx))}{3d}-\frac{be^2(c+dx)^2}{6d}+\frac{be^2\log\left((c+dx)^2+1\right)}{6d}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -1/6*(b*e^2*(c + d*x)^2)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(3*d
) + (b*e^2*Log[1 + (c + d*x)^2])/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^2x^2(a+b\arctan(x))\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int x^2(a+b\arctan(x))\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2(c+dx)^3(a+b\arctan(c+dx))}{3d}-\frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{1+x^2}\,dx,x,c+dx\right)}{3d}\\
&= \frac{e^2(c+dx)^3(a+b\arctan(c+dx))}{3d}-\frac{(be^2)\,\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,(c+dx)^2\right)}{6d}\\
&= \frac{e^2(c+dx)^3(a+b\arctan(c+dx))}{3d}-\frac{(be^2)\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx,x,(c+dx)^2\right)}{6d}\\
&= -\frac{be^2(c+dx)^2}{6d}+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))}{3d}+\frac{be^2\log\left(1+(c+dx)^2\right)}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx$$
$$=\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))-\frac{1}{6}b\left((c+dx)^2-\log\left(1+(c+dx)^2\right)\right)\right)}{d}$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e^2*(((c + d*x)^3*(a + b*ArcTan[c + d*x]))/3 - (b*((c + d*x)^2 - Log[1 + (
c + d*x)^2]))/6))/d
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{e^2a(dx+c)^3}{3}+e^2b\left(\frac{(dx+c)^3\arctan(dx+c)}{3}-\frac{(dx+c)^2}{6}+\frac{\ln\left(1+(dx+c)^2\right)}{6}\right)}{d}$
default	$\frac{\frac{e^2a(dx+c)^3}{3}+e^2b\left(\frac{(dx+c)^3\arctan(dx+c)}{3}-\frac{(dx+c)^2}{6}+\frac{\ln\left(1+(dx+c)^2\right)}{6}\right)}{d}$
parts	$\frac{e^2a(dx+c)^3}{3d}+\frac{e^2b\left(\frac{(dx+c)^3\arctan(dx+c)}{3}-\frac{(dx+c)^2}{6}+\frac{\ln\left(1+(dx+c)^2\right)}{6}\right)}{d}$
parallelrisch	$\frac{2d^4e^2b\arctan(dx+c)x^3+2x^3a\,d^4e^2+6d^3e^2cb\arctan(dx+c)x^2+6x^2ac\,d^3e^2+6b\,c^2e^2\arctan(dx+c)x\,d^2-x^2b\,d^3e^2+6xa\,c^2d}{6d^2}$
risch	$-\frac{ie^2(dx+c)^3b\ln(1+i(dx+c))}{6d}+\frac{ie^2d^2b\,x^3\ln(1-i(dx+c))}{6}+\frac{ie^2dbc\,x^2\ln(1-i(dx+c))}{2}+\frac{ie^2b\,c^2x\ln(1-i(dx+c))}{2}+$

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*a*(d*x+c)^3+e^2*b*(1/3*(d*x+c)^3*arctan(d*x+c)-1/6*(d*x+c)^2+1
/6*ln(1+(d*x+c)^2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx$$
$$=\frac{2\,ad^3e^2x^3+(6\,ac-b)d^2e^2x^2+2\,(3\,ac^2-bc)de^2x+be^2\log\left(d^2x^2+2\,cdx+c^2+1\right)+2\,(bd^3e^2x^3+3\,bcd^2e}{6\,d}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*e^2*x^3 + (6*a*c - b)*d^2*e^2*x^2 + 2*(3*a*c^2 - b*c)*d*e^2*x
+ b*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b*d^3*e^2*x^3 + 3*b*c*d^2*e^2
*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*arctan(d*x + c))/d
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.57 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.72

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx$$
$$=\begin{cases} ac^2e^2x+acde^2x^2+\frac{ad^2e^2x^3}{3}+\frac{bc^3e^2\operatorname{atan}(c+dx)}{3d}+bc^2e^2x\operatorname{atan}(c+dx)+bcde^2x^2\operatorname{atan}(c+dx)-\frac{bce^2x}{3}+ \\ c^2e^2x(a+b\operatorname{atan}(c)) \end{cases}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*atan(c + d*x)/(3*d) + b*c**2*e**2*x*atan(c + d*x) + b*c*d*e**2*x**2*atan
(c + d*x) - b*c*e**2*x/3 + b*d**2*e**2*x**3*atan(c + d*x)/3 - b*d*e**2*x**2
/6 + b*e**2*log(c/d + x - I/d)/(3*d) - I*b*e**2*atan(c + d*x)/(3*d), Ne(d,
0)), (c**2*e**2*x*(a + b*atan(c)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.55

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx = \frac{1}{3}ad^2e^2x^3 + acde^2x^2$$

$$+\left(x^2\arctan\left(dx+c\right) - d\left(\frac{x}{d^2} + \frac{\left(c^2-1\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bcde^2$$

$$+\frac{1}{6}\left(2\,x^3\arctan\left(dx+c\right) - d\left(\frac{dx^2-4\,cx}{d^3} - \frac{2\left(c^3-3\,c\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{\left(3\,c^2-1\right)\log\left(d^2x^2+2\,cdx+\right.}{d^4}\right.\right.$$

$$+ac^2e^2x + \frac{\left(2\left(dx+c\right)\arctan\left(dx+c\right) - \log\left(\left(dx+c\right)^2+1\right)\right)bc^2e^2}{2\,d}$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2
- 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))
*b*c*d*e^2 + 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 -
 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - lo
g((d*x + c)^2 + 1))*b*c^2*e^2/d
```

Giac [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx = \int \left(dex+ce\right)^2\left(b\arctan\left(dx+c\right)+a\right)dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.15

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx = \frac{a\,d^2\,e^2\,x^3}{3} - \frac{b\,c\,e^2\,x}{3}$$

$$+\frac{b\,e^2\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{6\,d} + a\,c^2\,e^2\,x$$

$$-\frac{b\,d\,e^2\,x^2}{6} + b\,c^2\,e^2\,x\,\mathrm{atan}(c+d\,x)$$

$$+a\,c\,d\,e^2\,x^2 + \frac{b\,c^3\,e^2\,\mathrm{atan}(c+d\,x)}{3\,d}$$

$$+\frac{b\,d^2\,e^2\,x^3\,\mathrm{atan}(c+d\,x)}{3} + b\,c\,d\,e^2\,x^2\,\mathrm{atan}(c+d\,x)$$

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x)),x)
```

```
[Out] (a*d^2*e^2*x^3)/3 - (b*c*e^2*x)/3 + (b*e^2*log(c^2 + d^2*x^2 + 2*c*d*x + 1)
)/(6*d) + a*c^2*e^2*x - (b*d*e^2*x^2)/6 + b*c^2*e^2*x*atan(c + d*x) + a*c*d
*e^2*x^2 + (b*c^3*e^2*atan(c + d*x))/(3*d) + (b*d^2*e^2*x^3*atan(c + d*x))/
3 + b*c*d*e^2*x^2*atan(c + d*x)
```

3.3 $\int (ce+dex)(a+b\arctan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx=-\frac{1}{2}bex+\frac{be\arctan(c+dx)}{2d}+\frac{e(c+dx)^2(a+b\arctan(c+dx))}{2d}$$

```
[Out] -1/2*b*e*x+1/2*b*e*arctan(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arctan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5151, 12, 4946, 327, 209}

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx=\frac{e(c+dx)^2(a+b\arctan(c+dx))}{2d}+\frac{be\arctan(c+dx)}{2d}-\frac{bex}{2}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -1/2*(b*e*x) + (b*e*ArcTan[c + d*x])/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c
 + d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int ex(a+b\arctan(x))\,dx,x,c+dx\right)}{d}\\
&= \frac{e\text{Subst}\left(\int x(a+b\arctan(x))\,dx,x,c+dx\right)}{d}\\
&= \frac{e(c+dx)^2(a+b\arctan(c+dx))}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,c+dx\right)}{2d}\\
&= -\frac{1}{2}bex + \frac{e(c+dx)^2(a+b\arctan(c+dx))}{2d} + \frac{(be)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{2d}\\
&= -\frac{1}{2}bex + \frac{be\arctan(c+dx)}{2d} + \frac{e(c+dx)^2(a+b\arctan(c+dx))}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx \\ =\frac{e\left(b(-dx+\arctan(c+dx))+(c+dx)^2(a+b\arctan(c+dx))\right)}{2d}$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e*(b*(-(d*x) + ArcTan[c + d*x]) + (c + d*x)^2*(a + b*ArcTan[c + d*x])))/(2
*d)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{ea(dx+c)^2}{2}+be\left(\frac{(dx+c)^2\arctan(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}+\frac{\arctan(dx+c)}{2}\right)}{d}$
default	$\frac{\frac{ea(dx+c)^2}{2}+be\left(\frac{(dx+c)^2\arctan(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}+\frac{\arctan(dx+c)}{2}\right)}{d}$
parts	$ea\left(\frac{1}{2}d\,x^2+cx\right)+\frac{be\left(\frac{(dx+c)^2\arctan(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}+\frac{\arctan(dx+c)}{2}\right)}{d}$
parallelrisch	$\frac{d^3eb\arctan(dx+c)x^2+x^2a\,d^3e+2cbe\arctan(dx+c)x\,d^2+2xac\,d^2e+\arctan(dx+c)b\,c^2de-xb\,d^2e-5a\,c^2de+eb\arctan(dx+c}{2d^2}$
risch	$-\frac{ieb(d\,x^2+2cx)\ln(1+i(dx+c))}{4}+\frac{iedb\,x^2\ln(1-i(dx+c))}{4}+\frac{iebcx\ln(1-i(dx+c))}{2}+\frac{ade\,x^2}{2}+\frac{e\arctan(dx+c)b\,c^2}{2d}-$

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*a*(d*x+c)^2+b*e*(1/2*(d*x+c)^2*arctan(d*x+c)-1/2*d*x-1/2*c+1/2*a
rctan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx \\ =\frac{ad^2ex^2+(2\,ac-b)dex+(bd^2ex^2+2\,bcdex+(bc^2+b)e)\arctan\left(dx+c\right)}{2\,d}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*e*x^2 + (2*a*c - b)*d*e*x + (b*d^2*e*x^2 + 2*b*c*d*e*x + (b*c^2
+ b)*e)*arctan(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.

Time = 6.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int (ce + dex)(a + b\arctan(c + dx))\,dx = \begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\operatorname{atan}(c+dx)}{2d} + bcex\operatorname{atan}(c+dx) + \frac{bdex^2\operatorname{atan}(c+dx)}{2} - \frac{bex}{2} + \frac{be\operatorname{atan}(c+dx)}{2d} & \text{for}\ d \neq 0 \\ cex(a + b\operatorname{atan}(c)) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*atan(c + d*x)/(2*d) + b*c*e*x*
atan(c + d*x) + b*d*e*x**2*atan(c + d*x)/2 - b*e*x/2 + b*e*atan(c + d*x)/(2
*d), Ne(d, 0)), (c*e*x*(a + b*atan(c)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(42) = 84$.

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.50

$$\int (ce + dex)(a + b\arctan(c + dx))\,dx = \frac{1}{2}adex^2 + \frac{1}{2}\left(x^2\arctan(dx + c) - d\left(\frac{x}{d^2} + \frac{(c^2 - 1)\arctan\left(\frac{d^2x + cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2 + 2\,cdx + c^2 + 1)}{d^3}\right)\right)bde + acex + \frac{\left(2\,(dx + c)\arctan(dx + c) - \log\left((dx + c)^2 + 1\right)\right)bce}{2\,d}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*e*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2
*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*d*e + a*c*e*x
 + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*c*e/d
```

Giac [F]

$$\int (ce + dex)(a + b \arctan(c + dx)) \, dx = \int (dex + ce)(b \arctan(dx + c) + a) \, dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\begin{aligned} \int (ce + dex)(a + b \arctan(c + dx)) \, dx = {} & a\,c\,e\,x - \frac{b\,e\,x}{2} + \frac{b\,e\,\mathrm{atan}(c + d\,x)}{2\,d} \\ & + \frac{a\,d\,e\,x^2}{2} + \frac{b\,c^2\,e\,\mathrm{atan}(c + d\,x)}{2\,d} \\ & + b\,c\,e\,x\,\mathrm{atan}(c + d\,x) + \frac{b\,d\,e\,x^2\,\mathrm{atan}(c + d\,x)}{2} \end{aligned}$$

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x)),x)
```

```
[Out] a*c*e*x - (b*e*x)/2 + (b*e*atan(c + d*x))/(2*d) + (a*d*e*x^2)/2 + (b*c^2*e*
atan(c + d*x))/(2*d) + b*c*e*x*atan(c + d*x) + (b*d*e*x^2*atan(c + d*x))/2
```

3.4 $\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx = \frac{a\log(c+dx)}{de} + \frac{ib\,\mathrm{PolyLog}(2,-i(c+dx))}{2de} - \frac{ib\,\mathrm{PolyLog}(2,i(c+dx))}{2de}$$

```
[Out] a*ln(d*x+c)/d/e+1/2*I*b*polylog(2,-I*(d*x+c))/d/e-1/2*I*b*polylog(2,I*(d*x+
c))/d/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5151, 12, 4940, 2438}

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx = \frac{a\log(c+dx)}{de} + \frac{ib\,\mathrm{PolyLog}(2,-i(c+dx))}{2de} - \frac{ib\,\mathrm{PolyLog}(2,i(c+dx))}{2de}$$

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x])/(d*e) + ((I/2)*b*PolyLog[2, (-I)*(c + d*x)])/(d*e) - ((I/2
)*b*PolyLog[2, I*(c + d*x)])/(d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\arctan(x)}{ex}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\arctan(x)}{x}\,dx, x, c+dx\right)}{de}\\
&= \frac{a\log(c+dx)}{de}+\frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, c+dx\right)}{2de}-\frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, c+dx\right)}{2de}\\
&= \frac{a\log(c+dx)}{de}+\frac{ib\,\text{PolyLog}(2,-i(c+dx))}{2de}-\frac{ib\,\text{PolyLog}(2,i(c+dx))}{2de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx\\
&= \frac{a\log(c+dx)+\frac{1}{2}ib\,\text{PolyLog}(2,-i(c+dx))-\frac{1}{2}ib\,\text{PolyLog}(2,i(c+dx))}{de}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x] + (I/2)*b*PolyLog[2, (-I)*(c + d*x)] - (I/2)*b*PolyLog[2, I
*(c + d*x)])/(d*e)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{ib\,\mathrm{dilog}(-idx-ic+1)}{2ed}+\frac{a\ln(-idx-ic)}{ed}+\frac{ib\,\mathrm{dilog}(idx+ic+1)}{2ed}$
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\Big(\ln(dx+c)\arctan(dx+c)+\frac{i\ln(dx+c)\ln(1+i(dx+c))}{2}-\frac{i\ln(dx+c)\ln(1-i(dx+c))}{2}+\frac{i\,\mathrm{dilog}(1+i(dx+c))}{2}-\frac{i\,\mathrm{dilog}(1-i(dx+}{2}}{e}}{d}$
default	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\Big(\ln(dx+c)\arctan(dx+c)+\frac{i\ln(dx+c)\ln(1+i(dx+c))}{2}-\frac{i\ln(dx+c)\ln(1-i(dx+c))}{2}+\frac{i\,\mathrm{dilog}(1+i(dx+c))}{2}-\frac{i\,\mathrm{dilog}(1-i(dx+}{2}}{e}}{d}$
parts	$\frac{a\ln(dx+c)}{de}+\frac{b\Big(\ln(dx+c)\arctan(dx+c)+\frac{i\ln(dx+c)\ln(1+i(dx+c))}{2}-\frac{i\ln(dx+c)\ln(1-i(dx+c))}{2}+\frac{i\,\mathrm{dilog}(1+i(dx+c))}{2}-\frac{i\,\mathrm{dilog}(}{}}{ed}$

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/e/d*b*dilog(-I*d*x-I*c+1)+1/e/d*a*ln(-I*d*x-I*c)+1/2*I*b/e/d*dilog(I
*d*x+I*c+1)
```

Fricas [F]

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\int \frac{b\arctan(dx+c)+a}{dex+ce}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)/(d*e*x + c*e), x)
```

Sympy [F]

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\frac{\int \frac{a}{c+dx}\,dx+\int \frac{b\,\mathrm{atan}\,(c+dx)}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*atan(c + d*x)/(c + d*x), x))/e
```

Maxima [F]

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\int \frac{b\arctan\left(dx+c\right)+a}{dex+ce}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(d*x + c)/(d*e*x + c*e), x) + a*log(d*e*x + c*e)/(d
*e)
```

Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\int \frac{b\arctan\left(dx+c\right)+a}{dex+ce}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\int \frac{a+b\,\mathrm{atan}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atan(c + d*x))/(c*e + d*e*x), x)
```

3.5 $\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx=-\frac{a+b\arctan(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left(1+(c+dx)^2\right)}{2de^2}$$

```
[Out] (-a-b*arctan(d*x+c))/d/e^2/(d*x+c)+b*ln(d*x+c)/d/e^2-1/2*b*ln(1+(d*x+c)^2)/
d/e^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5151, 12, 4946, 272, 36, 29, 31}

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx=-\frac{a+b\arctan(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left((c+dx)^2+1\right)}{2de^2}$$

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcTan[c + d*x])/(d*e^2*(c + d*x))) + (b*Log[c + d*x])/(d*e^2) - (
b*Log[1 + (c + d*x)^2])/(2*d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\arctan(x)}{e^2x^2}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^2}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{a+b\arctan(c+dx)}{de^2(c+dx)} + \frac{b\text{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{a+b\arctan(c+dx)}{de^2(c+dx)} + \frac{b\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, (c+dx)^2\right)}{2de^2}
\end{aligned}
$$

$$= -\frac{a+b\arctan(c+dx)}{de^2(c+dx)} + \frac{b\text{Subst}\left(\int \frac{1}{x}\,dx, x, (c+dx)^2\right)}{2de^2} - \frac{b\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, (c+dx)^2\right)}{2de^2}$$

$$= -\frac{a+b\arctan(c+dx)}{de^2(c+dx)} + \frac{b\log(c+dx)}{de^2} - \frac{b\log\left(1+(c+dx)^2\right)}{2de^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx = \frac{\frac{-a-b\arctan(c+dx)}{c+dx} + b\left(\log(c+dx) - \frac{1}{2}\log\left(1+(c+dx)^2\right)\right)}{de^2}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] ((-a - b*ArcTan[c + d*x])/(c + d*x) + b*(Log[c + d*x] - Log[1 + (c + d*x)^2
]/2))/(d*e^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\arctan(dx+c)}{dx+c}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^2}}{d}$
default	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\arctan(dx+c)}{dx+c}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^2}}{d}$
parts	$-\frac{a}{d\,e^2(dx+c)}+\frac{b\left(-\frac{\arctan(dx+c)}{dx+c}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^2d}$
parallelrisch	$\frac{6\ln(dx+c)xbc\,d^2-3\ln\left(d^2x^2+2cdx+c^2+1\right)xbc\,d^2+6\ln(dx+c)b\,c^2d-3\ln\left(d^2x^2+2cdx+c^2+1\right)b\,c^2d+2xa\,d^2-6b\arctan(d}{6(dx+c)c\,d^2e^2}$
risch	$\frac{ib\ln(1+i(dx+c))}{2d\,e^2(dx+c)}-\frac{-2\ln(-dx-c)bdx+\ln\left(-d^2x^2-2cdx-c^2-1\right)bdx-2\ln(-dx-c)bc+\ln\left(-d^2x^2-2cdx-c^2-1\right)bc+ib}{2e^2(dx+c)d}$

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arctan(d*x+c)+ln(d*x+c)-1/2*ln(1+(d*x
+c)^2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx = -\frac{2\,b\arctan\left(dx+c\right)+\left(bdx+bc\right)\log\left(d^2x^2+2\,cdx+c^2+1\right)-2\left(bdx+bc\right)\log\left(dx+c\right)+2\,a}{2\left(d^2e^2x+cde^2\right)}$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*arctan(d*x + c) + (b*d*x + b*c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
- 2*(b*d*x + b*c)*log(d*x + c) + 2*a)/(d^2*e^2*x + c*d*e^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.66

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx = \begin{cases} -\frac{a}{cde^2+d^2e^2x}+\frac{bc\log\left(\frac{c}{d}+x\right)}{cde^2+d^2e^2x}-\frac{bc\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{cde^2+d^2e^2x}+\frac{ibc\operatorname{atan}\left(c+dx\right)}{cde^2+d^2e^2x}+\frac{bdx\log\left(\frac{c}{d}+x\right)}{cde^2+d^2e^2x}-\frac{bdx\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{cde^2+d^2e^2x}+\frac{ibdx\operatorname{atan}\left(c+dx\right)}{cde^2+d^2e^2x}- \\ \frac{x\left(a+b\operatorname{atan}\left(c\right)\right)}{c^2e^2} \end{cases}$$

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] Piecewise((-a/(c*d*e**2 + d**2*e**2*x) + b*c*log(c/d + x)/(c*d*e**2 + d**2*
e**2*x) - b*c*log(c/d + x - I/d)/(c*d*e**2 + d**2*e**2*x) + I*b*c*atan(c +
d*x)/(c*d*e**2 + d**2*e**2*x) + b*d*x*log(c/d + x)/(c*d*e**2 + d**2*e**2*x)
 - b*d*x*log(c/d + x - I/d)/(c*d*e**2 + d**2*e**2*x) + I*b*d*x*atan(c + d*x
)/(c*d*e**2 + d**2*e**2*x) - b*atan(c + d*x)/(c*d*e**2 + d**2*e**2*x), Ne(d
, 0)), (x*(a + b*atan(c))/(c**2*e**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.51

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx = -\frac{1}{2}\left(d\left(\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2}-\frac{2\,\log\left(dx+c\right)}{d^2e^2}\right)+\frac{2\,\arctan\left(dx+c\right)}{d^2e^2x+cde^2}\right)b - \frac{a}{d^2e^2x+cde^2}$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e
^2)) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*b - a/(d^2*e^2*x + c*d*e^2)
```

Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx=\int \frac{b\arctan\left(dx+c\right)+a}{\left(dex+ce\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx=\frac{b\,\ln\left(c+d\,x\right)}{d\,e^2}-\frac{b\,\mathrm{atan}(c+d\,x)}{x\,d^2\,e^2+c\,d\,e^2}-\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d\,e^2}-\frac{a}{x\,d^2\,e^2+c\,d\,e^2}$$

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] (b*log(c + d*x))/(d*e^2) - (b*atan(c + d*x))/(d^2*e^2*x + c*d*e^2) - (b*log
(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d*e^2) - a/(d^2*e^2*x + c*d*e^2)
```

3.6 $\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx=-\frac{b}{2de^3(c+dx)}-\frac{b\arctan(c+dx)}{2de^3}-\frac{a+b\arctan(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] -1/2*b/d/e^3/(d*x+c)-1/2*b*arctan(d*x+c)/d/e^3+1/2*(-a-b*arctan(d*x+c))/d/e
^3/(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5151, 12, 4946, 331, 209}

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx=-\frac{a+b\arctan(c+dx)}{2de^3(c+dx)^2}-\frac{b\arctan(c+dx)}{2de^3}-\frac{b}{2de^3(c+dx)}$$

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*b/(d*e^3*(c + d*x)) - (b*ArcTan[c + d*x])/(2*d*e^3) - (a + b*ArcTan[c
+ d*x])/(2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\arctan(x)}{e^3x^3}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^3}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{a+b\arctan(c+dx)}{2de^3(c+dx)^2}+\frac{b\text{Subst}\left(\int \frac{1}{x^2(1+x^2)}\,dx, x, c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{a+b\arctan(c+dx)}{2de^3(c+dx)^2}-\frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{b\arctan(c+dx)}{2de^3}-\frac{a+b\arctan(c+dx)}{2de^3(c+dx)^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx = -\frac{a+b\arctan(c+dx)+b(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-(c+dx)^2\right)}{2de^3(c+dx)^2}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a + b*ArcTan[c + d*x] + b*(c + d*x)*Hypergeometric2F1[-1/2, 1, 1/2, -
(c + d*x)^2])/(d*e^3*(c + d*x)^2)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx+c)}-\frac{\arctan(dx+c)}{2}\right)}{e^3}}{d}$
default	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx+c)}-\frac{\arctan(dx+c)}{2}\right)}{e^3}}{d}$
parts	$-\frac{a}{2e^3(dx+c)^2d}+\frac{b\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx+c)}-\frac{\arctan(dx+c)}{2}\right)}{e^3d}$
parallelrisch	$\frac{-4b\,d^4\arctan(dx+c)x^2c-8b\,c^2\arctan(dx+c)x\,d^3+b\,d^4x^2-4\arctan(dx+c)b\,c^3d^2-2xbc\,d^3-4b\arctan(dx+c)c\,d^2-3b\,c^2d^2}{8(dx+c)^2e^3c\,d^3}$
risch	$\frac{ib\ln(1+i(dx+c))}{4d\,e^3(dx+c)^2}-\frac{-i\ln(-dx-c+i)b\,d^2x^2+i\ln(-dx-c-i)b\,d^2x^2-2i\ln(-dx-c+i)bcdx+2i\ln(-dx-c-i)bcdx-i\ln(-}{4e^3(dx+c)^2d}$

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)-1/2/(d*x+c)-1
/2*arctan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx = -\frac{bdx+bc+\left(bd^2x^2+2\,bcdx+bc^2+b\right)\arctan\left(dx+c\right)+a}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*x + b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + b)*arctan(d*x + c) + a
)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(54) = 108$.

Time = 15.21 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.98

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx = \begin{cases} -\frac{a}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{bc^2\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2bcdx\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{bc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{bd^2x^2\,\mathrm{at}}{2c^2de^3+4cd^2} \\ \frac{x(a+b\operatorname{atan}(c))}{c^3e^3} \end{cases}$$

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*c**2
*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b*c
*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b
*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d**2*x**2*atan(
c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d*x/(2*c*
*2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*atan(c + d*x)/(2*c**2*d
*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*atan(c))/
(c**3*e**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(57) = 114$.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.90

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx = -\frac{1}{2}\left(d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)+\frac{\arctan\left(dx+c\right)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)b - \frac{a}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + a
rctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*b - 1/2*a/(d^3*e^
3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx = \int \frac{b\arctan\left(dx+c\right)+a}{\left(dex+ce\right)^3}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.63

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx = -\frac{\frac{a+b\,c}{d}+b\,x}{2\,c^2\,e^3+4\,c\,d\,e^3\,x+2\,d^2\,e^3\,x^2} - \frac{b\,\mathrm{atan}\left(\frac{b\,c+b\,d\,x}{b}\right)}{2\,d\,e^3} - \frac{b\,\mathrm{atan}(c+d\,x)}{2\,d^3\,e^3\,\left(x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}\right)}$$

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x)^3,x)
```

```
[Out] - ((a + b*c)/d + b*x)/(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x) - (b*atan((
b*c + b*d*x)/b))/(2*d*e^3) - (b*atan(c + d*x))/(2*d^3*e^3*(x^2 + c^2/d^2 +
(2*c*x)/d))
```

3.7 $\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\begin{aligned}\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx &= \frac{1}{2}abe^3x+\frac{b^2e^3(c+dx)^2}{12d}\\&\quad+\frac{b^2e^3(c+dx)\arctan(c+dx)}{2d}\\&\quad-\frac{be^3(c+dx)^3(a+b\arctan(c+dx))}{6d}\\&\quad-\frac{e^3(a+b\arctan(c+dx))^2}{4d}\\&\quad+\frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d}\\&\quad-\frac{b^2e^3\log\left(1+(c+dx)^2\right)}{3d}\end{aligned}$$

```
[Out] 1/2*a*b*e^3*x+1/12*b^2*e^3*(d*x+c)^2/d+1/2*b^2*e^3*(d*x+c)*arctan(d*x+c)/d-
1/6*b*e^3*(d*x+c)^3*(a+b*arctan(d*x+c))/d-1/4*e^3*(a+b*arctan(d*x+c))^2/d+1
/4*e^3*(d*x+c)^4*(a+b*arctan(d*x+c))^2/d-1/3*b^2*e^3*ln(1+(d*x+c)^2)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{5151, 12, 4946, 5036, 272, 45, 4930, 266, 5004\}$

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx = \frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d} - \frac{be^3(c+dx)^3(a+b\arctan(c+dx))}{6d} - \frac{e^3(a+b\arctan(c+dx))^2}{4d} + \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)\arctan(c+dx)}{2d} + \frac{b^2e^3(c+dx)^2}{12d} - \frac{b^2e^3\log\left((c+dx)^2+1\right)}{3d}$$

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (a*b*e^3*x)/2 + (b^2*e^3*(c + d*x)^2)/(12*d) + (b^2*e^3*(c + d*x)*ArcTan[c
+ d*x])/(2*d) - (b*e^3*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(6*d) - (e^3*(a
 + b*ArcTan[c + d*x])^2)/(4*d) + (e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x])^2
)/(4*d) - (b^2*e^3*Log[1 + (c + d*x)^2])/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^3x^3(a+b\arctan(x))^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3(a+b\arctan(x))^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\arctan(x))}{1+x^2}\,dx, x, c+dx\right)}{2d}$$

$$
\begin{aligned}
&= \frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d} \\
&\quad - \frac{(be^3)\,\mathrm{Subst}\left(\int x^2(a+b\arctan(x))\,dx, x, c+dx\right)}{2d} \\
&\quad + \frac{(be^3)\,\mathrm{Subst}\left(\int \frac{x^2(a+b\arctan(x))}{1+x^2}\,dx, x, c+dx\right)}{2d} \\
&= -\frac{be^3(c+dx)^3(a+b\arctan(c+dx))}{6d} + \frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d} \\
&\quad + \frac{(be^3)\,\mathrm{Subst}\left(\int (a+b\arctan(x))\,dx, x, c+dx\right)}{2d} \\
&\quad - \frac{(be^3)\,\mathrm{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx, x, c+dx\right)}{2d} + \frac{(b^2e^3)\,\mathrm{Subst}\left(\int \frac{x^3}{1+x^2}\,dx, x, c+dx\right)}{6d} \\
&= \frac{1}{2}abe^3x - \frac{be^3(c+dx)^3(a+b\arctan(c+dx))}{6d} \\
&\quad - \frac{e^3(a+b\arctan(c+dx))^2}{4d} + \frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d} \\
&\quad + \frac{(b^2e^3)\,\mathrm{Subst}\left(\int \frac{x}{1+x}\,dx, x, (c+dx)^2\right)}{12d} + \frac{(b^2e^3)\,\mathrm{Subst}\left(\int \arctan(x)\,dx, x, c+dx\right)}{2d} \\
&= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)\arctan(c+dx)}{2d} - \frac{be^3(c+dx)^3(a+b\arctan(c+dx))}{6d} \\
&\quad - \frac{e^3(a+b\arctan(c+dx))^2}{4d} + \frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d} \\
&\quad + \frac{(b^2e^3)\,\mathrm{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, (c+dx)^2\right)}{12d} - \frac{(b^2e^3)\,\mathrm{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{2d} \\
&= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)^2}{12d} + \frac{b^2e^3(c+dx)\arctan(c+dx)}{2d} - \frac{be^3(c+dx)^3(a+b\arctan(c+dx))}{6d} \\
&\quad - \frac{e^3(a+b\arctan(c+dx))^2}{4d} + \frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d} - \frac{b^2e^3\log\left(1+(c+dx)^2\right)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$
\begin{aligned}
&\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx \\
&= \frac{e^3((c+dx)\left(b^2(c+dx)+3a^2(c+dx)^3-2ab(-3+c^2+2cdx+d^2x^2)\right)+2b(-b(-3c+c^3-3dx+3c^2dx+}{}
\end{aligned}
$$

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e^3*((c + d*x)*(b^2*(c + d*x) + 3*a^2*(c + d*x)^3 - 2*a*b*(-3 + c^2 + 2*c*
d*x + d^2*x^2)) + 2*b*(-(b*(-3*c + c^3 - 3*d*x + 3*c^2*d*x + 3*c*d^2*x^2 +
```

```
d^3*x^3)) + 3*a*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x
^4))*ArcTan[c + d*x] + 3*b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^
3*x^3 + d^4*x^4)*ArcTan[c + d*x]^2 - 4*b^2*Log[1 + (c + d*x)^2]))/(12*d)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{e^{3} a^{2} (dx+c)^{4}}{4}+e^{3} b^{2} \Bigg(\frac{(dx+c)^{4} \arctan(dx+c)^{2}}{4}-\frac{(dx+c)^{3} \arctan(dx+c)}{6}+\frac{(dx+c) \arctan(dx+c)}{2}-\frac{\arctan(dx+c)^{2}}{4}+\frac{(dx+c)^{2}}{12}-\frac{\ln\Big(1+}{}}{d}$
default	$\frac{\frac{e^{3} a^{2} (dx+c)^{4}}{4}+e^{3} b^{2} \Bigg(\frac{(dx+c)^{4} \arctan(dx+c)^{2}}{4}-\frac{(dx+c)^{3} \arctan(dx+c)}{6}+\frac{(dx+c) \arctan(dx+c)}{2}-\frac{\arctan(dx+c)^{2}}{4}+\frac{(dx+c)^{2}}{12}-\frac{\ln\Big(1+}{}}{d}$
parts	$\frac{e^{3} a^{2} (dx+c)^{4}}{4d}+\frac{e^{3} b^{2} \Bigg(\frac{(dx+c)^{4} \arctan(dx+c)^{2}}{4}-\frac{(dx+c)^{3} \arctan(dx+c)}{6}+\frac{(dx+c) \arctan(dx+c)}{2}-\frac{\arctan(dx+c)^{2}}{4}+\frac{(dx+c)^{2}}{12}-\frac{\ln(}{}}{d}$
parallelrisch	$-\frac{18 e^{3} d c^{2} a^{2}+5 e^{3} d c^{2} b^{2}+e^{3} b^{2} d-36 x^{2} \arctan(dx+c) a b c^{2} d^{3} e^{3}-24 x^{3} \arctan(dx+c) a b c d^{4} e^{3}-24 x \arctan(dx+c) a b c^{3} d^{2} e}{}$
risch	$i e^{3} d^{2} a b c x^{3} \ln(1-i(dx+c))+\frac{3 i e^{3} d a b c^{2} x^{2} \ln(1-i(dx+c))}{2}-\frac{i e^{3} d b^{2} c x^{2} \ln(1-i(dx+c))}{4}+i e^{3} a b c^{3} x$

```
[In] int((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*a^2*(d*x+c)^4+e^3*b^2*(1/4*(d*x+c)^4*arctan(d*x+c)^2-1/6*(d*x+
c)^3*arctan(d*x+c)+1/2*(d*x+c)*arctan(d*x+c)-1/4*arctan(d*x+c)^2+1/12*(d*x+
c)^2-1/3*ln(1+(d*x+c)^2))+2*e^3*a*b*(1/4*(d*x+c)^4*arctan(d*x+c)-1/12*(d*x+
c)^3+1/4*d*x+1/4*c-1/4*arctan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(143) = 286.

Time = 0.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.15

$$\int (ce+dex)^{3}(a+b\arctan(c+dx))^{2}\,dx$$
$$=\frac{3\,a^{2}d^{4}e^{3}x^{4}+2\,(6\,a^{2}c-ab)d^{3}e^{3}x^{3}+(18\,a^{2}c^{2}-6\,abc+b^{2})d^{2}e^{3}x^{2}+2\,(6\,a^{2}c^{3}-3\,abc^{2}+b^{2}c+3\,ab)de^{3}x-}{}$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*d^4*e^3*x^4 + 2*(6*a^2*c - a*b)*d^3*e^3*x^3 + (18*a^2*c^2 - 6*a
*b*c + b^2)*d^2*e^3*x^2 + 2*(6*a^2*c^3 - 3*a*b*c^2 + b^2*c + 3*a*b)*d*e^3*x
 - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*e^3*x^4 + 4*b^2*
c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b^2*c^4 - b^2)
*e^3)*arctan(d*x + c)^2 + 2*(3*a*b*d^4*e^3*x^4 + (12*a*b*c - b^2)*d^3*e^3*x
```

```
^3 + 3*(6*a*b*c^2 - b^2*c)*d^2*e^3*x^2 + 3*(4*a*b*c^3 - b^2*c^2 + b^2)*d*e^
3*x + (3*a*b*c^4 - b^2*c^3 + 3*b^2*c - 3*a*b)*e^3)*arctan(d*x + c))/d
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 45.36 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.71

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx$$
$$=\begin{cases} a^2c^3e^3x+\frac{3a^2c^2de^3x^2}{2}+a^2cd^2e^3x^3+\frac{a^2d^3e^3x^4}{4}+\frac{abc^4e^3\operatorname{atan}(c+dx)}{2d}+2abc^3e^3x\operatorname{atan}(c+dx)+3abc^2de^3x^2\operatorname{atan} \\ c^3e^3x(a+b\operatorname{atan}(c))^2 \end{cases}$$

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*atan(c + d*x)/(2*d) + 2*a*b*c*
*3*e**3*x*atan(c + d*x) + 3*a*b*c**2*d*e**3*x**2*atan(c + d*x) - a*b*c**2*e
**3*x/2 + 2*a*b*c*d**2*e**3*x**3*atan(c + d*x) - a*b*c*d*e**3*x**2/2 + a*b*
d**3*e**3*x**4*atan(c + d*x)/2 - a*b*d**2*e**3*x**3/6 + a*b*e**3*x/2 - a*b*
e**3*atan(c + d*x)/(2*d) + b**2*c**4*e**3*atan(c + d*x)**2/(4*d) + b**2*c**
3*e**3*x*atan(c + d*x)**2 - b**2*c**3*e**3*atan(c + d*x)/(6*d) + 3*b**2*c**
2*d*e**3*x**2*atan(c + d*x)**2/2 - b**2*c**2*e**3*x*atan(c + d*x)/2 + b**2*
c*d**2*e**3*x**3*atan(c + d*x)**2 - b**2*c*d*e**3*x**2*atan(c + d*x)/2 + b*
*2*c*e**3*x/6 + b**2*c*e**3*atan(c + d*x)/(2*d) + b**2*d**3*e**3*x**4*atan(
c + d*x)**2/4 - b**2*d**2*e**3*x**3*atan(c + d*x)/6 + b**2*d*e**3*x**2/12 +
 b**2*e**3*x*atan(c + d*x)/2 - 2*b**2*e**3*log(c/d + x - I/d)/(3*d) - b**2*
e**3*atan(c + d*x)**2/(4*d) + 2*I*b**2*e**3*atan(c + d*x)/(3*d), Ne(d, 0)),
 (c**3*e**3*x*(a + b*atan(c))**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(143) = 286$.

Time = 1.06 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.80

$$
\begin{aligned}
\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx &= \frac{1}{4}\,a^2d^3e^3x^4 + a^2cd^2e^3x^3 + \frac{3}{2}\,a^2c^2de^3x^2 \\
&+3\Bigg(x^2\arctan\left(dx+c\right) - d\Bigg(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\Bigg)\Bigg)abc^2de^3 \\
&+\Bigg(2\,x^3\arctan\left(dx+c\right) - d\Bigg(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx+c\right.}{d^4} \\
&+\frac{1}{6}\Bigg(3\,x^4\arctan\left(dx+c\right) - d\Bigg(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4} + \frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - \frac{6\,(c^3}{} \\
&+a^2c^3e^3x + \frac{\left(2\,(dx+c)\arctan\left(dx+c\right) - \log\left((dx+c)^2+1\right)\right)abc^3e^3}{d} \\
&+\frac{b^2d^2e^3x^2 + 2\,b^2cde^3x - 4\,b^2e^3\log\left(d^2x^2+2\,cdx+c^2+1\right) + 3\,(b^2d^4e^3x^4 + 4\,b^2cd^3e^3x^3 + 6\,b^2c^2d^2e^3x^2 + 4}{}
\end{aligned}
$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3*(x^2*ar
ctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*c^2*d*e^3 + (2*x^3*arctan(d*x + c) - d
*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2
- 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*c*d^2*e^3 + 1/6*(3*x^4*arct
an(d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c
^2 + 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)/d^5))*a*b*d^3*e^3 + a^2*c^3*e^3*x + (2*(d*x + c)*arctan(d*x + c) -
log((d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/12*(b^2*d^2*e^3*x^2 + 2*b^2*c*d*e^3
*x - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*e^3*x^4 + 4*b^
2*c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b^2*c^4 - b^
2)*e^3)*arctan(d*x + c)^2 - 2*(b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*(b
^2*c^2 - b^2)*d*e^3*x + (b^2*c^3 - 3*b^2*c)*e^3)*arctan(d*x + c))/d
```

Giac [F]

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx=\int (dex+ce)^3(b\arctan\left(dx+c\right)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.03

$$\begin{aligned}
&\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx \\
&= x\left(\frac{c\,e^3\,\left(20\,a^2\,c^2+6\,a^2-6\,a\,b\,c+b^2\right)}{2}+\frac{\left(6\,c^2+6\right)\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,\left(b-10\,a\,c\right)}{2}\right)}{6\,d^2}\right. \\
&\qquad\left.-\frac{2\,c\,\left(\frac{2\,c\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,\left(b-10\,a\,c\right)}{2}\right)}{d}+\frac{d\,e^3\,\left(60\,a^2\,c^2+6\,a^2-12\,a\,b\,c+b^2\right)}{6}-\frac{a^2\,d\,e^3\,\left(6\,c^2+6\right)}{6}\right)}{d}\right) \\
&\quad+x^2\left(\frac{c\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,\left(b-10\,a\,c\right)}{2}\right)}{d}+\frac{d\,e^3\,\left(60\,a^2\,c^2+6\,a^2-12\,a\,b\,c+b^2\right)}{12}\right. \\
&\qquad\left.-\frac{a^2\,d\,e^3\,\left(6\,c^2+6\right)}{12}\right)-x^3\,\left(\frac{2\,a^2\,c\,d^2\,e^3}{3}+\frac{a\,d^2\,e^3\,\left(b-10\,a\,c\right)}{6}\right) \\
&\quad+\mathrm{atan}(c+d\,x)^2\,\left(b^2\,c^3\,e^3\,x-\frac{b^2\,e^3-b^2\,c^4\,e^3}{4\,d}+\frac{b^2\,d^3\,e^3\,x^4}{4}+\frac{3\,b^2\,c^2\,d\,e^3\,x^2}{2}+b^2\,c\,d^2\,e^3\,x^3\right) \\
&\quad-d^2\,\mathrm{atan}(c+d\,x)\,\left(x^3\,\left(\frac{b^2\,e^3}{6}-2\,a\,b\,c\,e^3\right)-\frac{x\,\left(-b^2\,c^2\,e^3+b^2\,e^3+4\,a\,b\,c^3\,e^3\right)}{2\,d^2}\right. \\
&\quad\left.+\frac{x^2\,\left(b^2\,c\,e^3-6\,a\,b\,c^2\,e^3\right)}{2\,d}-\frac{a\,b\,d\,e^3\,x^4}{2}\right)+\frac{a^2\,d^3\,e^3\,x^4}{4}-\frac{b^2\,e^3\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{3\,d} \\
&\quad+\frac{b\,e^3\,\mathrm{atan}\left(\frac{\frac{b\,c\,e^3\,\left(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a\right)}{6}+\frac{b\,d\,e^3\,x\,\left(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a\right)}{6}}{-\frac{b^2\,c^3\,e^3}{6}+\frac{b^2\,c\,e^3}{2}+\frac{a\,b\,c^4\,e^3}{2}-\frac{a\,b\,e^3}{2}}\right)\,\left(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a\right)}{6\,d}
\end{aligned}$$

```
[In] int((c*e + d*e*x)^3*(a + b*atan(c + d*x))^2,x)
```

```
[Out] x*((c*e^3*(6*a^2 + b^2 + 20*a^2*c^2 - 6*a*b*c))/2 + ((6*c^2 + 6)*(2*a^2*c*d
^2*e^3 + (a*d^2*e^3*(b - 10*a*c))/2))/(6*d^2) - (2*c*((2*c*(2*a^2*c*d^2*e^3
 + (a*d^2*e^3*(b - 10*a*c))/2))/d + (d*e^3*(6*a^2 + b^2 + 60*a^2*c^2 - 12*a
*b*c))/6 - (a^2*d*e^3*(6*c^2 + 6))/6))/d) + x^2*((c*(2*a^2*c*d^2*e^3 + (a*d
^2*e^3*(b - 10*a*c))/2))/d + (d*e^3*(6*a^2 + b^2 + 60*a^2*c^2 - 12*a*b*c))/
12 - (a^2*d*e^3*(6*c^2 + 6))/12) - x^3*((2*a^2*c*d^2*e^3)/3 + (a*d^2*e^3*(b
 - 10*a*c))/6) + atan(c + d*x)^2*(b^2*c^3*e^3*x - (b^2*e^3 - b^2*c^4*e^3)/(
4*d) + (b^2*d^3*e^3*x^4)/4 + (3*b^2*c^2*d*e^3*x^2)/2 + b^2*c*d^2*e^3*x^3) -
 d^2*atan(c + d*x)*(x^3*((b^2*e^3)/6 - 2*a*b*c*e^3) - (x*(b^2*e^3 - b^2*c^2
*e^3 + 4*a*b*c^3*e^3))/(2*d^2) + (x^2*(b^2*c*e^3 - 6*a*b*c^2*e^3))/(2*d) -
(a*b*d*e^3*x^4)/2) + (a^2*d^3*e^3*x^4)/4 - (b^2*e^3*log(c^2 + d^2*x^2 + 2*c
*d*x + 1))/(3*d) + (b*e^3*atan(((b*c*e^3*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/6
 + (b*d*e^3*x*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/6)/((b^2*c*e^3)/2 - (b^2*c^3
*e^3)/6 - (a*b*e^3)/2 + (a*b*c^4*e^3)/2))*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/
(6*d)
```

3.8 $\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\begin{aligned}\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx = {} & \frac{1}{3}b^2e^2x-\frac{b^2e^2\arctan(c+dx)}{3d} \\ & -\frac{be^2(c+dx)^2(a+b\arctan(c+dx))}{3d} \\ & -\frac{ie^2(a+b\arctan(c+dx))^2}{3d} \\ & +\frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d} \\ & -\frac{2be^2(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d} \\ & -\frac{ib^2e^2\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d}\end{aligned}$$

```
[Out] 1/3*b^2*e^2*x-1/3*b^2*e^2*arctan(d*x+c)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arctan(d
*x+c))/d-1/3*I*e^2*(a+b*arctan(d*x+c))^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*
x+c))^2/d-2/3*b*e^2*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d-1/3*I*b^2*e^2
*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5151, 12, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352}

$$\begin{aligned}\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx = &\frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d}\\ &-\frac{be^2(c+dx)^2(a+b\arctan(c+dx))}{3d}\\ &-\frac{ie^2(a+b\arctan(c+dx))^2}{3d}\\ &-\frac{2be^2\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))}{3d}\\ &-\frac{b^2e^2\arctan(c+dx)}{3d}\\ &-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{3d}+\frac{1}{3}b^2e^2x\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*e^2*x)/3 - (b^2*e^2*ArcTan[c + d*x])/(3*d) - (b*e^2*(c + d*x)^2*(a + b
*ArcTan[c + d*x]))/(3*d) - ((I/3)*e^2*(a + b*ArcTan[c + d*x])^2)/d + (e^2*(
c + d*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*d) - (2*b*e^2*(a + b*ArcTan[c + d*
x])*Log[2/(1 + I*(c + d*x))])/(3*d) - ((I/3)*b^2*e^2*PolyLog[2, 1 - 2/(1 +
I*(c + d*x))])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^2x^2(a+b\arctan(x))^2\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int x^2(a+b\arctan(x))^2\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d} - \frac{(2be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{3d}\\
&= \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d}\\
&\quad - \frac{(2be^2)\,\text{Subst}\left(\int x(a+b\arctan(x))\,dx,x,c+dx\right)}{3d}\\
&\quad + \frac{(2be^2)\,\text{Subst}\left(\int \frac{x(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{3d}\\
&= -\frac{be^2(c+dx)^2(a+b\arctan(c+dx))}{3d} - \frac{ie^2(a+b\arctan(c+dx))^2}{3d}\\
&\quad + \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d}\\
&\quad - \frac{(2be^2)\,\text{Subst}\left(\int \frac{a+b\arctan(x)}{i-x}\,dx,x,c+dx\right)}{3d} + \frac{(b^2e^2)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,c+dx\right)}{3d}\\
&= \frac{1}{3}b^2e^2x - \frac{be^2(c+dx)^2(a+b\arctan(c+dx))}{3d} - \frac{ie^2(a+b\arctan(c+dx))^2}{3d}\\
&\quad + \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d} - \frac{2be^2(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d}\\
&\quad - \frac{(b^2e^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{3d} + \frac{(2b^2e^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{3d}\\
&= \frac{1}{3}b^2e^2x - \frac{b^2e^2\arctan(c+dx)}{3d} - \frac{be^2(c+dx)^2(a+b\arctan(c+dx))}{3d}\\
&\quad - \frac{ie^2(a+b\arctan(c+dx))^2}{3d} + \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d}\\
&\quad - \frac{2be^2(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d}\\
&\quad - \frac{(2ib^2e^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{3d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{3}b^2e^2x - \frac{b^2e^2\arctan(c+dx)}{3d} - \frac{be^2(c+dx)^2(a+b\arctan(c+dx))}{3d} \\
&\quad - \frac{ie^2(a+b\arctan(c+dx))^2}{3d} + \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d} \\
&\quad - \frac{2be^2(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d} - \frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx$$

$$= e^2\big(a^2(c+dx)^3 + ab(-(c+dx)^2 + 2(c+dx)^3\arctan(c+dx) + \log\left(1+(c+dx)^2\right)) + b^2(c+dx-\arctan(c$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e^2*(a^2*(c + d*x)^3 + a*b*(-(c + d*x)^2 + 2*(c + d*x)^3*ArcTan[c + d*x] +
 Log[1 + (c + d*x)^2]) + b^2*(c + d*x - ArcTan[c + d*x] - (c + d*x)^2*ArcTa
n[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan[c + d*x]^2 - 2*ArcTan
[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*PolyLog[2, -E^((2*I)*ArcTa
n[c + d*x])])))/(3*d)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{e^2a^2(dx+c)^3}{3}+b^2e^2\Big(\frac{(dx+c)^3\arctan(dx+c)^2}{3}-\frac{(dx+c)^2\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{dx}{3}+\frac{c}{3}-\frac{\arctan(dx+c)}{3}+$
default	$\frac{e^2a^2(dx+c)^3}{3}+b^2e^2\Big(\frac{(dx+c)^3\arctan(dx+c)^2}{3}-\frac{(dx+c)^2\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{dx}{3}+\frac{c}{3}-\frac{\arctan(dx+c)}{3}+$
parts	$\frac{e^2a^2(dx+c)^3}{3d}+\frac{b^2e^2\Big(\frac{(dx+c)^3\arctan(dx+c)^2}{3}-\frac{(dx+c)^2\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{dx}{3}+\frac{c}{3}-\frac{\arctan(dx+c)}{3}+}{}$
risch	Expression too large to display

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*a^2*(d*x+c)^3+b^2*e^2*(1/3*(d*x+c)^3*arctan(d*x+c)^2-1/3*(d*x+
c)^2*arctan(d*x+c)+1/3*arctan(d*x+c)*ln(1+(d*x+c)^2)+1/3*d*x+1/3*c-1/3*arct
```

```
an(d*x+c)+1/6*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I
*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))-1/6*I*(ln(d*x+c+I)*ln(1+(d*x+
c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-
I))))+2*e^2*a*b*(1/3*(d*x+c)^3*arctan(d*x+c)-1/6*(d*x+c)^2+1/6*ln(1+(d*x+c)
^2)))
```

Fricas [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx=\int(dex+ce)^2(b\arctan(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^2
 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arctan(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2 +
2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arctan(d*x + c), x)
```

Sympy [F]

$$\begin{aligned}\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx\\ =e^2\left(\int a^2c^2\,dx+\int a^2d^2x^2\,dx+\int b^2c^2\operatorname{atan}^2(c+dx)\,dx+\int 2abc^2\operatorname{atan}(c+dx)\,dx\right.\\ +\int 2a^2cdx\,dx+\int b^2d^2x^2\operatorname{atan}^2(c+dx)\,dx+\int 2abd^2x^2\operatorname{atan}(c+dx)\,dx\\ \left.+\int 2b^2cdx\operatorname{atan}^2(c+dx)\,dx+\int 4abcdx\operatorname{atan}(c+dx)\,dx\right)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2*
c**2*atan(c + d*x)**2, x) + Integral(2*a*b*c**2*atan(c + d*x), x) + Integra
l(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*atan(c + d*x)**2, x) + Integra
l(2*a*b*d**2*x**2*atan(c + d*x), x) + Integral(2*b**2*c*d*x*atan(c + d*x)**
2, x) + Integral(4*a*b*c*d*x*atan(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx=\int (dex+ce)^2\left(b\arctan\left(dx+c\right)+a\right)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^4*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan
(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^
4*e^2 + 1/3*a^2*d^2*e^2*x^3 + 36*b^2*d^4*e^2*integrate(1/48*x^4*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^4*e^2*integrate(1/48*x^4
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144
*b^2*c*d^3*e^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 4*b^2*d^4*e^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d^3*e^2*integrate(1/48*x^3
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 216
*b^2*c^2*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 16*b^2*c*d^3*e^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^2*c^2*d^2*e^2*integrate(1
/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 144*b^2*c^3*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 24*b^2*c^2*d^2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c^3*d*e^2*integrat
e(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 12*b^2*c^3*d*e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^4*e^2*integrate(1/48*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^2*c*d*e^2*x^2 +
 3/4*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 8*b^2*d^3*e^
2*integrate(1/48*x^3*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24
*b^2*c*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) - 24*b^2*c^2*d*e^2*integrate(1/48*x*arctan(d*x + c)/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d -
arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*e^2 + 2*(x^2*arctan(d*x + c) - d*(x/d^
2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/d^3))*a*b*c*d*e^2 + 1/3*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3
 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + 36*b^2*d^2*e^2*integra
te(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2
*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + 72*b^2*c*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*e^2*integrate(1/48*x*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*e^2*int
egrate(1/48*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1
), x) + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c^2*e^2/d
```

```
+ 1/12*(b^2*d^2*e^2*x^3 + 3*b^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*arctan(d*x +
 c)^2 - 1/48*(b^2*d^2*e^2*x^3 + 3*b^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Giac [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx=\int (dex+ce)^2(b\arctan(dx+c)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{atan}(c+d\,x))^2\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^2, x)
```

3.9 $\int (ce+dex)(a+b\arctan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx = -abex-\frac{b^2e(c+dx)\arctan(c+dx)}{d}+\frac{e(a+b\arctan(c+dx))^2}{2d}+\frac{e(c+dx)^2(a+b\arctan(c+dx))^2}{2d}+\frac{b^2e\log\left(1+(c+dx)^2\right)}{2d}$$

```
[Out] -a*b*e*x-b^2*e*(d*x+c)*arctan(d*x+c)/d+1/2*e*(a+b*arctan(d*x+c))^2/d+1/2*e*
(d*x+c)^2*(a+b*arctan(d*x+c))^2/d+1/2*b^2*e*ln(1+(d*x+c)^2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5151, 12, 4946, 5036, 4930, 266, 5004}

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx = \frac{e(c+dx)^2(a+b\arctan(c+dx))^2}{2d}+\frac{e(a+b\arctan(c+dx))^2}{2d}-abex-\frac{b^2e(c+dx)\arctan(c+dx)}{d}+\frac{b^2e\log\left((c+dx)^2+1\right)}{2d}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] -(a*b*e*x) - (b^2*e*(c + d*x)*ArcTan[c + d*x])/d + (e*(a + b*ArcTan[c + d*x
])^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) + (b^2*e*Log[
1 + (c + d*x)^2])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int ex(a+b\arctan(x))^2\,dx,x,c+dx\right)}{d}\\
&= \frac{e\text{Subst}\left(\int x(a+b\arctan(x))^2\,dx,x,c+dx\right)}{d}\\
&= \frac{e(c+dx)^2(a+b\arctan(c+dx))^2}{2d}-\frac{(be)\text{Subst}\left(\int \frac{x^2(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{e(c+dx)^2(a+b\arctan(c+dx))^2}{2d}-\frac{(be)\text{Subst}\left(\int (a+b\arctan(x))\,dx,x,c+dx\right)}{d}\\
&\quad+\frac{(be)\text{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= -abex+\frac{e(a+b\arctan(c+dx))^2}{2d}+\frac{e(c+dx)^2(a+b\arctan(c+dx))^2}{2d}\\
&\quad-\frac{(b^2e)\,\text{Subst}\left(\int \arctan(x)\,dx,x,c+dx\right)}{d}\\
&= -abex-\frac{b^2e(c+dx)\arctan(c+dx)}{d}+\frac{e(a+b\arctan(c+dx))^2}{2d}\\
&\quad+\frac{e(c+dx)^2(a+b\arctan(c+dx))^2}{2d}+\frac{(b^2e)\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= -abex-\frac{b^2e(c+dx)\arctan(c+dx)}{d}+\frac{e(a+b\arctan(c+dx))^2}{2d}\\
&\quad+\frac{e(c+dx)^2(a+b\arctan(c+dx))^2}{2d}+\frac{b^2e\log\left(1+(c+dx)^2\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$
\int (ce+dex)(a+b\arctan(c+dx))^2\,dx
$$

$$
=\frac{e(a(c+dx)(-2b+ac+adx)+2b(-b(c+dx)+a(1+c^2+2cdx+d^2x^2))\arctan(c+dx)+b^2(1+c^2+2c}{2d}
$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e*(a*(c + d*x)*(-2*b + a*c + a*d*x) + 2*b*(-(b*(c + d*x)) + a*(1 + c^2 + 2
*c*d*x + d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcT
an[c + d*x]^2 + b^2*Log[1 + (c + d*x)^2]))/(2*d)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{e\,a^2(dx+c)^2}{2}+e\,b^2\left(\frac{(dx+c)^2\arctan(dx+c)^2}{2}+\frac{\arctan(dx+c)^2}{2}-(dx+c)\arctan(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)+2eab\Bigg(}{d}$
default	$\frac{\frac{e\,a^2(dx+c)^2}{2}+e\,b^2\left(\frac{(dx+c)^2\arctan(dx+c)^2}{2}+\frac{\arctan(dx+c)^2}{2}-(dx+c)\arctan(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)+2eab\Bigg(}{d}$
parts	$e\,a^2\left(\frac{1}{2}d\,x^2+cx\right)+\frac{e\,b^2\left(\frac{(dx+c)^2\arctan(dx+c)^2}{2}+\frac{\arctan(dx+c)^2}{2}-(dx+c)\arctan(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}+$
parallelrisch	$d^3e\,b^2\arctan(dx+c)^2x^2+2x^2\arctan(dx+c)ab\,d^3e+2ce\,b^2\arctan(dx+c)^2x\,d^2+x^2a^2d^3e+4x\arctan(dx+c)abc\,d^2e+$
risch	$-\frac{e\,b^2\left(d^2x^2+2cdx+c^2+1\right)\ln(1+i(dx+c))^2}{8d}+\frac{be\left(-2ia\,d^2x^2+b\,d^2x^2\ln(1-i(dx+c))-4iacdx+2bcdx\ln(1-i(dx+c))+\right.}{4d}$

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*a^2*(d*x+c)^2+e*b^2*(1/2*(d*x+c)^2*arctan(d*x+c)^2+1/2*arctan(d*
x+c)^2-(d*x+c)*arctan(d*x+c)+1/2*ln(1+(d*x+c)^2))+2*e*a*b*(1/2*(d*x+c)^2*ar
ctan(d*x+c)-1/2*d*x-1/2*c+1/2*arctan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.58

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx$$
$$=\frac{a^2d^2ex^2+2\left(a^2c-ab\right)dex+b^2e\log\left(d^2x^2+2\,cdx+c^2+1\right)+\left(b^2d^2ex^2+2\,b^2cdex+\left(b^2c^2+b^2\right)e\right)\arctan}{2\,d}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*e*x^2 + 2*(a^2*c - a*b)*d*e*x + b^2*e*log(d^2*x^2 + 2*c*d*x +
c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (b^2*c^2 + b^2)*e)*arctan(d*x +
 c)^2 + 2*(a*b*d^2*e*x^2 + (2*a*b*c - b^2)*d*e*x + (a*b*c^2 - b^2*c + a*b)*
e)*arctan(d*x + c))/d
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.53

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx$$

$$= \begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\,\mathrm{atan}\,(c+dx)}{d} + 2abcex\,\mathrm{atan}\,(c+dx) + abdex^2\,\mathrm{atan}\,(c+dx) - abex + \frac{abe\,\mathrm{atan}\,(c+dx)}{d} + \\ cex(a+b\,\mathrm{atan}\,(c))^2 \end{cases}$$

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atan(c + d*x)/d + 2*a*
b*c*e*x*atan(c + d*x) + a*b*d*e*x**2*atan(c + d*x) - a*b*e*x + a*b*e*atan(c
 + d*x)/d + b**2*c**2*e*atan(c + d*x)**2/(2*d) + b**2*c*e*x*atan(c + d*x)**
2 - b**2*c*e*atan(c + d*x)/d + b**2*d*e*x**2*atan(c + d*x)**2/2 - b**2*e*x*
atan(c + d*x) + b**2*e*log(c/d + x - I/d)/d + b**2*e*atan(c + d*x)**2/(2*d)
 - I*b**2*e*atan(c + d*x)/d, Ne(d, 0)), (c*e*x*(a + b*atan(c))**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(89) = 178.

Time = 0.99 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.29

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx = \frac{1}{2}\,a^2dex^2$$

$$+ \left(x^2\arctan\left(dx+c\right) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)abde$$

$$+ a^2cex + \frac{\left(2\left(dx+c\right)\arctan\left(dx+c\right) - \log\left(\left(dx+c\right)^2+1\right)\right)abce}{d}$$

$$+ \frac{b^2e\log\left(d^2x^2+2\,cdx+c^2+1\right) + \left(b^2d^2ex^2+2\,b^2cdex+(b^2c^2+b^2)e\right)\arctan\left(dx+c\right)^2 - 2\left(b^2dex+b^2ce\right)}{2\,d}$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*e*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*d*e + a^2*c*e
*x + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c*e/d + 1/2*(
b^2*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (
b^2*c^2 + b^2)*e)*arctan(d*x + c)^2 - 2*(b^2*d*e*x + b^2*c*e)*arctan(d*x +
c))/d
```

Giac [F]

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx=\int (dex+ce)(b\arctan\left(dx+c\right)+a)^2\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.27

$$\begin{aligned}&\int (ce+dex)(a+b\arctan(c+dx))^2\,dx\\&=\mathrm{atan}(c+d\,x)^2\left(\frac{e\,b^2\,c^2+e\,b^2}{2\,d}+b^2\,c\,e\,x+\frac{b^2\,d\,e\,x^2}{2}\right)-x\,\left(a\,e\,(b-3\,a\,c)+2\,a^2\,c\,e\right)\\&\quad-d^2\,\mathrm{atan}(c+d\,x)\,\left(\frac{x\,\left(b^2\,e-2\,a\,b\,c\,e\right)}{d^2}-\frac{a\,b\,e\,x^2}{d}\right)+\frac{b^2\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}\\&\quad+\frac{a^2\,d\,e\,x^2}{2}+\frac{b\,e\,\mathrm{atan}\left(\frac{b\,c\,e\,\left(a\,c^2-b\,c+a\right)+b\,d\,e\,x\,\left(a\,c^2-b\,c+a\right)}{-e\,b^2\,c+a\,e\,b\,c^2+a\,e\,b}\right)\,\left(a\,c^2-b\,c+a\right)}{d}\end{aligned}$$

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x))^2,x)
```

```
[Out] atan(c + d*x)^2*((b^2*e + b^2*c^2*e)/(2*d) + b^2*c*e*x + (b^2*d*e*x^2)/2) -
 x*(a*e*(b - 3*a*c) + 2*a^2*c*e) - d^2*atan(c + d*x)*((x*(b^2*e - 2*a*b*c*e
))/d^2 - (a*b*e*x^2)/d) + (b^2*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) +
(a^2*d*e*x^2)/2 + (b*e*atan((b*c*e*(a - b*c + a*c^2) + b*d*e*x*(a - b*c + a
*c^2))/(a*b*e - b^2*c*e + a*b*c^2*e))*(a - b*c + a*c^2))/d
```

3.10 $\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx = {} & \frac{2(a+b\arctan(c+dx))^2\operatorname{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} \\ & -\frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{de} \\ & +\frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)}{de} \\ & -\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2de} \\ & +\frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)}{2de}\end{aligned}$$

```
[Out] -2*(a+b*arctan(d*x+c))^2*arctanh(-1+2/(1+I*(d*x+c)))/d/e-I*b*(a+b*arctan(d*
x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d/e+I*b*(a+b*arctan(d*x+c))*polylog(2,-1
+2/(1+I*(d*x+c)))/d/e-1/2*b^2*polylog(3,1-2/(1+I*(d*x+c)))/d/e+1/2*b^2*poly
log(3,-1+2/(1+I*(d*x+c)))/d/e
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5151, 12, 4942, 5108, 5004, 5114, 6745}

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx = & \frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2}{de} \\ & - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))}{de} \\ & + \frac{ib\,\text{PolyLog}\left(2,\frac{2}{i(c+dx)+1}-1\right)(a+b\arctan(c+dx))}{de} \\ & - \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)}{2de} \\ & + \frac{b^2\,\text{PolyLog}\left(3,\frac{2}{i(c+dx)+1}-1\right)}{2de}\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTan[c + d*x])^2*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*e) + (I*b*
(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 + I*(c + d*x))])/(d*e) - (b^2*
PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*d*e) + (b^2*PolyLog[3, -1 + 2/(1 +
I*(c + d*x))])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{ex}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x}\,dx, x, c+dx\right)}{de}\\
&= \frac{2(a+b\arctan(c+dx))^2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de}\\
&\quad - \frac{(4b)\text{Subst}\left(\int \frac{(a+b\arctan(x))\text{arctanh}\left(1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2(a+b\arctan(c+dx))^2\operatorname{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad - \frac{(2b)\text{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(2-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de} \\
&\quad + \frac{(2b)\text{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(a+b\arctan(c+dx))^2\operatorname{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad - \frac{ib(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad + \frac{ib(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad + \frac{(ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de} \\
&\quad - \frac{(ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-1+\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(a+b\arctan(c+dx))^2\operatorname{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad - \frac{ib(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad + \frac{ib(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad - \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2de} + \frac{b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)}{2de}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 381 vs. $2(183) = 366$.

Time = 0.30 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.08

$$
\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx
$$

$$
= \frac{-6iab\pi^2 - ib^2\pi^3 + 24iab\pi\arctan(c+dx) - 48iab\arctan(c+dx)^2 + 16ib^2\arctan(c+dx)^3 - ab\pi\log(167}{}
$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] ((-6*I)*a*b*Pi^2 - I*b^2*Pi^3 + (24*I)*a*b*Pi*ArcTan[c + d*x] - (48*I)*a*b*
ArcTan[c + d*x]^2 + (16*I)*b^2*ArcTan[c + d*x]^3 - a*b*Pi*Log[16777216] + 2
4*b^2*ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] + 24*a*b*Pi*Log
[1 + E^((-2*I)*ArcTan[c + d*x])] - 48*a*b*ArcTan[c + d*x]*Log[1 + E^((-2*I)
*ArcTan[c + d*x])] + 48*a*b*ArcTan[c + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x
])] - 24*b^2*ArcTan[c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 24*a^2*
Log[c + d*x] + 12*a*b*Pi*Log[1 + c^2 + 2*c*d*x + d^2*x^2] - (24*I)*a*b*Poly
Log[2, -E^((-2*I)*ArcTan[c + d*x])] + (24*I)*b^2*ArcTan[c + d*x]*PolyLog[2,
 E^((-2*I)*ArcTan[c + d*x])] + (24*I)*b^2*ArcTan[c + d*x]*PolyLog[2, -E^((2
*I)*ArcTan[c + d*x])] - (24*I)*a*b*PolyLog[2, E^((2*I)*ArcTan[c + d*x])] +
12*b^2*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])] - 12*b^2*PolyLog[3, -E^((2*I)
*ArcTan[c + d*x])])/(24*d*e)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.56 (sec) , antiderivative size = 1154, normalized size of antiderivative = 6.31

method	result	size
derivativedivides	Expression too large to display	1154
default	Expression too large to display	1154
parts	Expression too large to display	1159

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/e*ln(d*x+c)+b^2/e*(ln(d*x+c)*arctan(d*x+c)^2+I*arctan(d*x+c)*polyl
og(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-1/2*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x
+c)^2))-arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+arctan(d*x+c)^2
*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-2*I*arctan(d*x+c)*polylog(2,-(1+I*
(d*x+c))/(1+(d*x+c)^2)^(1/2))+2*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2
))+arctan(d*x+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-2*I*arctan(d*x+c
)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+2*polylog(3,(1+I*(d*x+c))/(1
+(d*x+c)^2)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1
+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+
I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I
*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csg
n(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2
)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^
2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+
c)^2)))^2-csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2
/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn(I*((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3-csgn(I*((1+I*(d*x
+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+
```

```
c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn(((1+I*(d*x
+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3+1)*arctan(d*x+
c)^2)+2*a*b/e*(ln(d*x+c)*arctan(d*x+c)+1/2*I*ln(d*x+c)*ln(1+I*(d*x+c))-1/2*
I*ln(d*x+c)*ln(1-I*(d*x+c))+1/2*I*dilog(1+I*(d*x+c))-1/2*I*dilog(1-I*(d*x+c
))))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx=\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{dex+ce}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d*e*x + c*e
), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx=\frac{\int \frac{a^2}{c+dx}\,dx+\int \frac{b^2\,\mathrm{atan}^2\,(c+dx)}{c+dx}\,dx+\int \frac{2ab\,\mathrm{atan}\,(c+dx)}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*atan(c + d*x)**2/(c + d*x), x)
 + Integral(2*a*b*atan(c + d*x)/(c + d*x), x))/e
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx=\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{dex+ce}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*log(d*e*x + c*e)/(d*e) + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^2
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(d*e*x + c*e)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^2}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x), x)
```

3.11 $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = -\frac{i(a+b\arctan(c+dx))^2}{de^2} - \frac{(a+b\arctan(c+dx))^2}{de^2(c+dx)} + \frac{2b(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2}$$

```
[Out] -I*(a+b*arctan(d*x+c))^2/d/e^2-(a+b*arctan(d*x+c))^2/d/e^2/(d*x+c)+2*b*(a+b
*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^2-I*b^2*polylog(2,-1+2/(1-I*(d*x+
c)))/d/e^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5151, 12, 4946, 5044, 4988, 2497}

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = -\frac{(a+b\arctan(c+dx))^2}{de^2(c+dx)} - \frac{i(a+b\arctan(c+dx))^2}{de^2} + \frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{de^2} - \frac{ib^2\,\mathrm{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)}{de^2}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^2)/(d*e^2) - (a + b*ArcTan[c + d*x])^2/(d*e^2
*(c + d*x)) + (2*b*(a + b*ArcTan[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(d
*e^2) - (I*b^2*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{e^2x^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x^2}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{(a+b\arctan(c+dx))^2}{de^2(c+dx)} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\arctan(x)}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{i(a+b\arctan(c+dx))^2}{de^2} - \frac{(a+b\arctan(c+dx))^2}{de^2(c+dx)} \\
&\quad + \frac{(2ib)\text{Subst}\left(\int \frac{a+b\arctan(x)}{x(i+x)}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{i(a+b\arctan(c+dx))^2}{de^2} - \frac{(a+b\arctan(c+dx))^2}{de^2(c+dx)} \\
&\quad + \frac{2b(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} \\
&\quad - \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{i(a+b\arctan(c+dx))^2}{de^2} - \frac{(a+b\arctan(c+dx))^2}{de^2(c+dx)} \\
&\quad + \frac{2b(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{ib^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13

$$
\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx
$$

$$
= \frac{-ib^2(-i+c+dx)\arctan(c+dx)^2 + 2b\arctan(c+dx)\left(-a+b(c+dx)\log\left(1-e^{2i\arctan(c+dx)}\right)\right) + a\Big(-a}{de^2(c+dx)}
$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(-a + b*(c
 + d*x)*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(c
+ d*x)/Sqrt[1 + (c + d*x)^2]]) - I*b^2*(c + d*x)*PolyLog[2, E^((2*I)*ArcTan
[c + d*x])])/(d*e^2*(c + d*x))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(115) = 230.

Time = 1.05 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.73

method	result
derivativedivides	$\frac{-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{dx+c}+2\ln(dx+c)\arctan(dx+c)-\arctan(dx+c)\ln\left(1+(dx+c)^2\right)-\frac{i\left(\ln(dx+c-i)\ln\left(1+(dx+c)^2\right)-\frac{\ln(dx+}{2}\right.}{}\right.}{}}{}$
default	$\frac{-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{dx+c}+2\ln(dx+c)\arctan(dx+c)-\arctan(dx+c)\ln\left(1+(dx+c)^2\right)-\frac{i\left(\ln(dx+c-i)\ln\left(1+(dx+c)^2\right)-\frac{\ln(dx+}{2}\right.}{}\right.}{}}{}$
parts	$-\frac{a^2}{e^2(dx+c)d}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{dx+c}+2\ln(dx+c)\arctan(dx+c)-\arctan(dx+c)\ln\left(1+(dx+c)^2\right)-\frac{i\left(\ln(dx+c-i)\ln\left(1+(dx-\right.\right.}{}\right.}{}$

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/e^2/(d*x+c)+b^2/e^2*(-1/(d*x+c)*arctan(d*x+c)^2+2*ln(d*x+c)*arcta
n(d*x+c)-arctan(d*x+c)*ln(1+(d*x+c)^2)-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1
/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+
1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))
-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I)))+I*ln(d*x+c)*ln(1+I*(d*x+c))-I*ln(d*x+c)*l
n(1-I*(d*x+c))+I*dilog(1+I*(d*x+c))-I*dilog(1-I*(d*x+c)))+2*a*b/e^2*(-1/(d*
x+c)*arctan(d*x+c)+ln(d*x+c)-1/2*ln(1+(d*x+c)^2)))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx=\int \frac{(b\arctan(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^2*e^2*x^2
 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\,\mathrm{atan}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\,\mathrm{atan}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atan(c + d*
x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atan(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{\left(dex+ce\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e^2))
 + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*a*b - 1/16*(4*arctan(d*x + c)^2
 - 16*(d^2*e^2*x + c*d*e^2)*integrate(1/16*(12*(d^2*x^2 + 2*c*d*x + c^2 + 1
)*arctan(d*x + c)^2 + (d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^2 + 8*(d*x + c)*arctan(d*x + c) - 4*(d^2*x^2 + 2*c*d*x + c^2)*log
(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + (6*c^2 + 1)
*d^2*e^2*x^2 + 2*(2*c^3 + c)*d*e^2*x + (c^4 + c^2)*e^2), x) - log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2)*b^2/(d^2*e^2*x + c*d*e^2) - a^2/(d^2*e^2*x + c*d*e^2
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}(c+d\,x)\right)^2}{\left(c\,e+d\,e\,x\right)^2}\,dx$$

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^2, x)
```

3.12 $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx = -\frac{b(a+b\arctan(c+dx))}{de^3(c+dx)}-\frac{(a+b\arctan(c+dx))^2}{2de^3} -\frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2} +\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left(1+(c+dx)^2\right)}{2de^3}$$

```
[Out] -b*(a+b*arctan(d*x+c))/d/e^3/(d*x+c)-1/2*(a+b*arctan(d*x+c))^2/d/e^3-1/2*(a
+b*arctan(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-1/2*b^2*ln(1+(d*x+c
)^2)/d/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5151, 12, 4946, 5038, 272, 36, 29, 31, 5004}

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx = -\frac{b(a+b\arctan(c+dx))}{de^3(c+dx)}-\frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2} -\frac{(a+b\arctan(c+dx))^2}{2de^3} +\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left((c+dx)^2+1\right)}{2de^3}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*(a + b*ArcTan[c + d*x]))/(d*e^3*(c + d*x))) - (a + b*ArcTan[c + d*x])^
2/(2*d*e^3) - (a + b*ArcTan[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c
+ d*x])/(d*e^3) - (b^2*Log[1 + (c + d*x)^2])/(2*d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
```

```
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{e^3x^3}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x^3}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2} + \frac{b\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^2(1+x^2)}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2} + \frac{b\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^2}\,dx, x, c+dx\right)}{de^3}\\
&\quad - \frac{b\text{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{b(a+b\arctan(c+dx))}{de^3(c+dx)} - \frac{(a+b\arctan(c+dx))^2}{2de^3}\\
&\quad - \frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{b(a+b\arctan(c+dx))}{de^3(c+dx)} - \frac{(a+b\arctan(c+dx))^2}{2de^3}\\
&\quad - \frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, (c+dx)^2\right)}{2de^3}\\
&= -\frac{b(a+b\arctan(c+dx))}{de^3(c+dx)} - \frac{(a+b\arctan(c+dx))^2}{2de^3} - \frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2}\\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{1}{x}\,dx, x, (c+dx)^2\right)}{2de^3} - \frac{b^2\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, (c+dx)^2\right)}{2de^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b(a+b\arctan(c+dx))}{de^3(c+dx)}-\frac{(a+b\arctan(c+dx))^2}{2de^3} \\
&\quad -\frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left(1+(c+dx)^2\right)}{2de^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.66

$$
\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx = \\
-\frac{a^2+2abc+2abdx+2b(b(c+dx)+a(1+c^2+2cdx+d^2x^2))\arctan(c+dx)+b^2(1+c^2+2cdx+d^2x^2}{}
$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a^2 + 2*a*b*c + 2*a*b*d*x + 2*b*(b*(c + d*x) + a*(1 + c^2 + 2*c*d*x +
 d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d
*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x] + b^2*c^2*Log[1 + c^2 + 2*c*d*x + d^
2*x^2] + 2*b^2*c*d*x*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + b^2*d^2*x^2*Log[1 +
 c^2 + 2*c*d*x + d^2*x^2])/(d*e^3*(c + d*x)^2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{2(dx+c)^2}-\frac{\arctan(dx+c)}{dx+c}-\frac{\arctan(dx+c)^2}{2}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^3}+\frac{2ab\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx+c}\right.}{e^3}}{d}$
default	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{2(dx+c)^2}-\frac{\arctan(dx+c)}{dx+c}-\frac{\arctan(dx+c)^2}{2}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^3}+\frac{2ab\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx+c}\right.}{e^3}}{d}$
parts	$-\frac{a^2}{2e^3(dx+c)^2d}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{2(dx+c)^2}-\frac{\arctan(dx+c)}{dx+c}-\frac{\arctan(dx+c)^2}{2}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^3d}+\frac{2ab\left(-\frac{\arctan(dx+}{2(dx+c)^2}\right.}{}$
parallelrisch	$\frac{-2a^2cd^2-8x\arctan(dx+c)abc^2d^3-4x^2\arctan(dx+c)abcd^4+abd^4x^2-2b^2\arctan(dx+c)^2cd^2-4\arctan(dx+c)b^2c^2d^2-2}{}$
risch	$\frac{b^2\left(d^2x^2+2cdx+c^2+1\right)\ln(1+i(dx+c))^2}{8e^3(dx+c)^2d}-\frac{b\left(bd^2x^2\ln(1-i(dx+c))+2bcdx\ln(1-i(dx+c))-2ibdx+\ln(1-i(dx+c))bc^2-2\right.}{4e^3(dx+c)^2d}$

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2/e^3/(d*x+c)^2+b^2/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)^2-1/(d*x+
c)*arctan(d*x+c)-1/2*arctan(d*x+c)^2+ln(d*x+c)-1/2*ln(1+(d*x+c)^2))+2*a*b/e
```

```
^3*(-1/2/(d*x+c)^2*arctan(d*x+c)-1/2/(d*x+c)-1/2*arctan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx =$$
$$-\frac{2\,abdx+2\,abc+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2+b^2\right)\arctan\left(dx+c\right)^2+a^2+2\left(abd^2x^2+abc^2+b^2c+(2\,abc\right.}{2\,(}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*d*x + 2*a*b*c + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + b^2)*arc
tan(d*x + c)^2 + a^2 + 2*(a*b*d^2*x^2 + a*b*c^2 + b^2*c + (2*a*b*c + b^2)*d
*x + a*b)*arctan(d*x + c) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x +
 c))/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.88 (sec) , antiderivative size = 1107, normalized size of antiderivative = 9.46

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx$$
$$=\begin{cases} -\frac{a^2}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{2abc^2\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{4abcdx\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{2abc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{2abd^2x^2\,\mathrm{a}}{2c^2de^3+4cd^2} \\ \frac{x(a+b\operatorname{atan}(c))^2}{c^3e^3} \end{cases}$$

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a
*b*c**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
- 4*a*b*c*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*
x**2) - 2*a*b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*
b*d**2*x**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x*
*2) - 2*a*b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*
b*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b*
*2*c**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) -
 2*b**2*c**2*log(c/d + x - I/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e
**3*x**2) - b**2*c**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2
```

```
*d**3*e**3*x**2) + 2*I*b**2*c**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e*
*3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d
**2*e**3*x + 2*d**3*e**3*x**2) - 4*b**2*c*d*x*log(c/d + x - I/d)/(2*c**2*d*
e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c*d*x*atan(c + d*x)**2/
(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*I*b**2*c*d*x*atan(
c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c*at
an(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*d
**2*x**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
 - 2*b**2*d**2*x**2*log(c/d + x - I/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*
d**3*e**3*x**2) - b**2*d**2*x**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2
*e**3*x + 2*d**3*e**3*x**2) + 2*I*b**2*d**2*x**2*atan(c + d*x)/(2*c**2*d*e*
*3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*d*x*atan(c + d*x)/(2*c**2
*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b**2*atan(c + d*x)**2/(2*c*
*2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*atan(
c))**2/(c**3*e**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(111) = 222.

Time = 0.32 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.29

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx$$

$$= -\left(d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)+\frac{\arctan\left(dx+c\right)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)ab$$

$$-\frac{1}{2}\left(2\,d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)\arctan\left(dx+c\right)-\frac{\arctan\left(dx+c\right)^2-\log\left(d^2x^2+2\,cdx+c^2\right.}{de^3}\right.$$

$$-\frac{b^2\arctan\left(dx+c\right)^2}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}-\frac{a^2}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + arcta
n(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a*b - 1/2*(2*d*(1/(d^
3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3))*arctan(d*x + c) -
 (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x + c))/(d
*e^3))*b^2 - 1/2*b^2*arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d
*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.98

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx=\frac{b^2\,\ln\left(c+d\,x\right)}{d\,e^3}-\frac{\frac{a^2+2\,b\,c\,a}{2\,d}+a\,b\,x}{c^2\,e^3+2\,c\,d\,e^3\,x+d^2\,e^3\,x^2}-\frac{\mathrm{atan}(c+d\,x)\,\left(\frac{b^2\,c}{d^3\,e^3}+\frac{b^2\,x}{d^2\,e^3}+\frac{a\,b}{d^3\,e^3}\right)}{x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}}-\mathrm{atan}(c+d\,x)^2\,\left(\frac{b^2}{2\,d\,e^3}+\frac{b^2}{2\,d^3\,e^3\,\left(x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}\right)}\right)+\frac{\ln\left(c+d\,x-\mathrm{i}\right)\,\left(-\frac{b^2}{2}+\frac{a\,b\,1\mathrm{i}}{2}\right)}{d\,e^3}-\frac{\ln\left(c+d\,x+1\mathrm{i}\right)\,\left(\frac{b^2}{2}+\frac{1\mathrm{i}\,a\,b}{2}\right)}{d\,e^3}$$

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/(d*e^3) - ((a^2 + 2*a*b*c)/(2*d) + a*b*x)/(c^2*e^3 + d^2
*e^3*x^2 + 2*c*d*e^3*x) - (atan(c + d*x)*((b^2*c)/(d^3*e^3) + (b^2*x)/(d^2*
e^3) + (a*b)/(d^3*e^3)))/(x^2 + c^2/d^2 + (2*c*x)/d) - atan(c + d*x)^2*(b^2
/(2*d*e^3) + b^2/(2*d^3*e^3*(x^2 + c^2/d^2 + (2*c*x)/d))) + (log(c + d*x -
1i)*((a*b*1i)/2 - b^2/2))/(d*e^3) - (log(c + d*x + 1i)*((a*b*1i)/2 + b^2/2)
)/(d*e^3)
```

3.13 $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx =& -\frac{b^2}{3de^4(c+dx)}-\frac{b^2\arctan(c+dx)}{3de^4}\\ &-\frac{b(a+b\arctan(c+dx))}{3de^4(c+dx)^2}\\ &+\frac{i(a+b\arctan(c+dx))^2}{3de^4}-\frac{(a+b\arctan(c+dx))^2}{3de^4(c+dx)^3}\\ &-\frac{2b(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{3de^4}\\ &+\frac{ib^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{3de^4}\end{aligned}$$

```
[Out] -1/3*b^2/d/e^4/(d*x+c)-1/3*b^2*arctan(d*x+c)/d/e^4-1/3*b*(a+b*arctan(d*x+c)
)/d/e^4/(d*x+c)^2+1/3*I*(a+b*arctan(d*x+c))^2/d/e^4-1/3*(a+b*arctan(d*x+c))
^2/d/e^4/(d*x+c)^3-2/3*b*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^4+1/
3*I*b^2*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5151, 12, 4946, 5038, 331, 209, 5044, 4988, 2497}

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx = &-\frac{b(a+b\arctan(c+dx))}{3de^4(c+dx)^2}-\frac{(a+b\arctan(c+dx))^2}{3de^4(c+dx)^3}\\ &+\frac{i(a+b\arctan(c+dx))^2}{3de^4}\\ &-\frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{3de^4}\\ &-\frac{b^2\arctan(c+dx)}{3de^4}\\ &+\frac{ib^2\operatorname{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)}{3de^4}-\frac{b^2}{3de^4(c+dx)}\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*b^2/(d*e^4*(c + d*x)) - (b^2*ArcTan[c + d*x])/(3*d*e^4) - (b*(a + b*Ar
cTan[c + d*x]))/(3*d*e^4*(c + d*x)^2) + ((I/3)*(a + b*ArcTan[c + d*x])^2)/(
d*e^4) - (a + b*ArcTan[c + d*x])^2/(3*d*e^4*(c + d*x)^3) - (2*b*(a + b*ArcT
an[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(3*d*e^4) + ((I/3)*b^2*PolyLog[2
, -1 + 2/(1 - I*(c + d*x))])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{(a+b\arctan(c+dx))^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^3(1+x^2)}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{(a+b\arctan(c+dx))^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^3}\,dx, x, c+dx\right)}{3de^4} - \frac{(2b)\text{Subst}\left(\int \frac{a+b\arctan(x)}{x(1+x^2)}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{b(a+b\arctan(c+dx))}{3de^4(c+dx)^2} + \frac{i(a+b\arctan(c+dx))^2}{3de^4} - \frac{(a+b\arctan(c+dx))^2}{3de^4(c+dx)^3} - \frac{(2ib)\text{Subst}\left(\int \frac{a+b\arctan(x)}{x(i+x)}\,dx, x, c+dx\right)}{3de^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{x^2(1+x^2)}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b(a+b\arctan(c+dx))}{3de^4(c+dx)^2} + \frac{i(a+b\arctan(c+dx))^2}{3de^4} - \frac{(a+b\arctan(c+dx))^2}{3de^4(c+dx)^3} - \frac{2b(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{3de^4} - \frac{b^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{3de^4} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b^2\arctan(c+dx)}{3de^4} - \frac{b(a+b\arctan(c+dx))}{3de^4(c+dx)^2} + \frac{i(a+b\arctan(c+dx))^2}{3de^4} - \frac{(a+b\arctan(c+dx))^2}{3de^4(c+dx)^3} - \frac{2b(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{3de^4} + \frac{ib^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{3de^4}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx =$$

$$-\frac{ab+\frac{a^2}{(c+dx)^3}+\frac{ab}{(c+dx)^2}+\frac{b^2}{c+dx}+b^2\left(-i+\frac{1}{(c+dx)^3}\right)\arctan(c+dx)^2+b\arctan(c+dx)\left(b+\frac{2a}{(c+dx)^3}+\frac{b}{(c+}\right.}{3de^4}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*(a*b + a^2/(c + d*x)^3 + (a*b)/(c + d*x)^2 + b^2/(c + d*x) + b^2*(-I +
 (c + d*x)^(-3))*ArcTan[c + d*x]^2 + b*ArcTan[c + d*x]*(b + (2*a)/(c + d*x)
^3 + b/(c + d*x)^2 + 2*b*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + 2*a*b*Log[(c
 + d*x)/Sqrt[1 + (c + d*x)^2]] - I*b^2*PolyLog[2, E^((2*I)*ArcTan[c + d*x])
])/(d*e^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(176) = 352$.

Time = 2.56 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.90

method	result
derivativedivides	$-\frac{a^{2}}{3e^{4}(dx+c)^{3}}+\frac{b^{2}\Bigg(-\frac{\arctan(dx+c)^{2}}{3(dx+c)^{3}}-\frac{\arctan(dx+c)}{3(dx+c)^{2}}-\frac{2\ln(dx+c)\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\big(1+(dx+c)^{2}\big)}{3}+\frac{i\Big(\ln(dx+c-i)\ln\Big(1+}{}}{}$
default	$-\frac{a^{2}}{3e^{4}(dx+c)^{3}}+\frac{b^{2}\Bigg(-\frac{\arctan(dx+c)^{2}}{3(dx+c)^{3}}-\frac{\arctan(dx+c)}{3(dx+c)^{2}}-\frac{2\ln(dx+c)\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\big(1+(dx+c)^{2}\big)}{3}+\frac{i\Big(\ln(dx+c-i)\ln\Big(1+}{}}{}$
parts	$-\frac{a^{2}}{3e^{4}(dx+c)^{3}d}+\frac{b^{2}\Bigg(-\frac{\arctan(dx+c)^{2}}{3(dx+c)^{3}}-\frac{\arctan(dx+c)}{3(dx+c)^{2}}-\frac{2\ln(dx+c)\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\big(1+(dx+c)^{2}\big)}{3}+\frac{i\Big(\ln(dx+c-}{}}{}$

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/e^4/(d*x+c)^3+b^2/e^4*(-1/3/(d*x+c)^3*arctan(d*x+c)^2-1/3/(d*
x+c)^2*arctan(d*x+c)-2/3*ln(d*x+c)*arctan(d*x+c)+1/3*arctan(d*x+c)*ln(1+(d*
x+c)^2)+1/6*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(
d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))-1/6*I*(ln(d*x+c+I)*ln(1+(d*x+c)
^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I)
))-1/3/(d*x+c)-1/3*arctan(d*x+c)-1/3*I*ln(d*x+c)*ln(1+I*(d*x+c))+1/3*I*ln(d
*x+c)*ln(1-I*(d*x+c))-1/3*I*dilog(1+I*(d*x+c))+1/3*I*dilog(1-I*(d*x+c)))+2*
a*b/e^4*(-1/3/(d*x+c)^3*arctan(d*x+c)-1/6/(d*x+c)^2-1/3*ln(d*x+c)+1/6*ln(1+
(d*x+c)^2)))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{\left(dex+ce\right)^4}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^4*e^4*x^4
 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx = \frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^2\operatorname{atan}^2\left(c+dx\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2ab\operatorname{atan}\left(c+dx\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*atan(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**
2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*atan(c + d*x)/(c**
4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{\left(dex+ce\right)^4}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(d*(1/(d^4*e^4*x^2 + 2*c*d^3*e^4*x + c^2*d^2*e^4) - log(d^2*x^2 + 2*c*
d*x + c^2 + 1)/(d^2*e^4) + 2*log(d*x + c)/(d^2*e^4)) + 2*arctan(d*x + c)/(d
^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4))*a*b - 1/48*(4*
arctan(d*x + c)^2 - 48*(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c
^3*d*e^4)*integrate(1/48*(36*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^
2 + 3*(d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 8*
(d*x + c)*arctan(d*x + c) - 4*(d^2*x^2 + 2*c*d*x + c^2)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1))/(d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + (15*c^2 + 1)*d^4*e^4*x^4 +
4*(5*c^3 + c)*d^3*e^4*x^3 + 3*(5*c^4 + 2*c^2)*d^2*e^4*x^2 + 2*(3*c^5 + 2*c^
3)*d*e^4*x + (c^6 + c^4)*e^4), x) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)*b^2
/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^2/(d
^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^4}\,dx$$

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^4,x)
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^4, x)
```

3.14 $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx = &-\frac{b^2}{12de^5(c+dx)^2}-\frac{b(a+b\arctan(c+dx))}{6de^5(c+dx)^3}\\&+\frac{b(a+b\arctan(c+dx))}{2de^5(c+dx)}\\&+\frac{(a+b\arctan(c+dx))^2}{4de^5}-\frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4}\\&-\frac{2b^2\log(c+dx)}{3de^5}+\frac{b^2\log\left(1+(c+dx)^2\right)}{3de^5}\end{aligned}$$

```
[Out] -1/12*b^2/d/e^5/(d*x+c)^2-1/6*b*(a+b*arctan(d*x+c))/d/e^5/(d*x+c)^3+1/2*b*(
a+b*arctan(d*x+c))/d/e^5/(d*x+c)+1/4*(a+b*arctan(d*x+c))^2/d/e^5-1/4*(a+b*a
rctan(d*x+c))^2/d/e^5/(d*x+c)^4-2/3*b^2*ln(d*x+c)/d/e^5+1/3*b^2*ln(1+(d*x+c
)^2)/d/e^5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5151, 12, 4946, 5038, 272, 46, 36, 29, 31, 5004}

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx = &\frac{b(a+b\arctan(c+dx))}{2de^5(c+dx)}-\frac{b(a+b\arctan(c+dx))}{6de^5(c+dx)^3}\\&-\frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4}+\frac{(a+b\arctan(c+dx))^2}{4de^5}\\&-\frac{b^2}{12de^5(c+dx)^2}-\frac{2b^2\log(c+dx)}{3de^5}+\frac{b^2\log\left((c+dx)^2+1\right)}{3de^5}\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*b^2/(d*e^5*(c + d*x)^2) - (b*(a + b*ArcTan[c + d*x]))/(6*d*e^5*(c + d
*x)^3) + (b*(a + b*ArcTan[c + d*x]))/(2*d*e^5*(c + d*x)) + (a + b*ArcTan[c
+ d*x])^2/(4*d*e^5) - (a + b*ArcTan[c + d*x])^2/(4*d*e^5*(c + d*x)^4) - (2*
b^2*Log[c + d*x])/(3*d*e^5) + (b^2*Log[1 + (c + d*x)^2])/(3*d*e^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{e^5x^5}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x^5}\,dx, x, c+dx\right)}{de^5}\\
&= -\frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^4(1+x^2)}\,dx, x, c+dx\right)}{2de^5}\\
&= -\frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^4}\,dx, x, c+dx\right)}{2de^5}\\
&\quad - \frac{b\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^2(1+x^2)}\,dx, x, c+dx\right)}{2de^5}\\
&= -\frac{b(a+b\arctan(c+dx))}{6de^5(c+dx)^3} - \frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4}\\
&\quad - \frac{b\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^2}\,dx, x, c+dx\right)}{2de^5} + \frac{b\text{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx, x, c+dx\right)}{2de^5}\\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{1}{x^3(1+x^2)}\,dx, x, c+dx\right)}{6de^5}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b(a+b\arctan(c+dx))}{6de^5(c+dx)^3} + \frac{b(a+b\arctan(c+dx))}{2de^5(c+dx)} \\
&\quad + \frac{(a+b\arctan(c+dx))^2}{4de^5} - \frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4} \\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, (c+dx)^2\right)}{12de^5} - \frac{b^2\text{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx, x, c+dx\right)}{2de^5} \\
&= -\frac{b(a+b\arctan(c+dx))}{6de^5(c+dx)^3} + \frac{b(a+b\arctan(c+dx))}{2de^5(c+dx)} \\
&\quad + \frac{(a+b\arctan(c+dx))^2}{4de^5} - \frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4} \\
&\quad + \frac{b^2\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)\,dx, x, (c+dx)^2\right)}{12de^5} - \frac{b^2\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, (c+dx)^2\right)}{4de^5} \\
&= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b(a+b\arctan(c+dx))}{6de^5(c+dx)^3} \\
&\quad + \frac{b(a+b\arctan(c+dx))}{2de^5(c+dx)} + \frac{(a+b\arctan(c+dx))^2}{4de^5} \\
&\quad - \frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4} - \frac{b^2\log(c+dx)}{6de^5} + \frac{b^2\log\left(1+(c+dx)^2\right)}{12de^5} \\
&\quad - \frac{b^2\text{Subst}\left(\int \frac{1}{x}\,dx, x, (c+dx)^2\right)}{4de^5} + \frac{b^2\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, (c+dx)^2\right)}{4de^5} \\
&= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b(a+b\arctan(c+dx))}{6de^5(c+dx)^3} \\
&\quad + \frac{b(a+b\arctan(c+dx))}{2de^5(c+dx)} + \frac{(a+b\arctan(c+dx))^2}{4de^5} \\
&\quad - \frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4} - \frac{2b^2\log(c+dx)}{3de^5} + \frac{b^2\log\left(1+(c+dx)^2\right)}{3de^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.44

$$
\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx = \\
-\frac{3a^2+2ab(c+dx)+b^2(c+dx)^2-6ab(c+dx)^3-2b(b(-c+3c^3-dx+9c^2dx+9cd^2x^2+3d^3x^3)+3a(}{}
$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*(3*a^2 + 2*a*b*(c + d*x) + b^2*(c + d*x)^2 - 6*a*b*(c + d*x)^3 - 2*b*
(b*(-c + 3*c^3 - d*x + 9*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3) + 3*a*(-1 + c^4
```

```
 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4))*ArcTan[c + d*x] - 3*
b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)*ArcTan[c
 + d*x]^2 + 8*b^2*(c + d*x)^4*Log[c + d*x] - 4*b^2*(c + d*x)^4*Log[1 + c^2
+ 2*c*d*x + d^2*x^2])/(d*e^5*(c + d*x)^4)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{a^2}{4e^5(dx+c)^4}+\frac{b^2\Bigg(-\frac{\arctan(dx+c)^2}{4(dx+c)^4}-\frac{\arctan(dx+c)}{6(dx+c)^3}+\frac{\arctan(dx+c)}{2dx+2c}+\frac{\arctan(dx+c)^2}{4}-\frac{1}{12(dx+c)^2}-\frac{2\ln(dx+c)}{3}+\frac{\ln\left(1+(dx+c)^2\right)}{3}\Bigg)}{e^5}}{d}$
default	$\frac{-\frac{a^2}{4e^5(dx+c)^4}+\frac{b^2\Bigg(-\frac{\arctan(dx+c)^2}{4(dx+c)^4}-\frac{\arctan(dx+c)}{6(dx+c)^3}+\frac{\arctan(dx+c)}{2dx+2c}+\frac{\arctan(dx+c)^2}{4}-\frac{1}{12(dx+c)^2}-\frac{2\ln(dx+c)}{3}+\frac{\ln\left(1+(dx+c)^2\right)}{3}\Bigg)}{e^5}}{d}$
parts	$-\frac{a^2}{4e^5(dx+c)^4d}+\frac{b^2\Bigg(-\frac{\arctan(dx+c)^2}{4(dx+c)^4}-\frac{\arctan(dx+c)}{6(dx+c)^3}+\frac{\arctan(dx+c)}{2dx+2c}+\frac{\arctan(dx+c)^2}{4}-\frac{1}{12(dx+c)^2}-\frac{2\ln(dx+c)}{3}+\frac{\ln\Big(1+(dx}{3}}{e^5d}$
parallelrisch	$\frac{6ab\,c^3d^5-x^2b^2d^7-2xab\,d^6+6x^3ab\,d^8-2x\arctan(dx+c)b^2d^6+6x^3\arctan(dx+c)b^2d^8+3\arctan(dx+c)^2b^2c^4d^5+6\arctan}{}$
risch	Expression too large to display

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a^2/e^5/(d*x+c)^4+b^2/e^5*(-1/4/(d*x+c)^4*arctan(d*x+c)^2-1/6/(d*
x+c)^3*arctan(d*x+c)+1/2/(d*x+c)*arctan(d*x+c)+1/4*arctan(d*x+c)^2-1/12/(d*
x+c)^2-2/3*ln(d*x+c)+1/3*ln(1+(d*x+c)^2))+2*a*b/e^5*(-1/4/(d*x+c)^4*arctan(
d*x+c)-1/12/(d*x+c)^3+1/4/(d*x+c)+1/4*arctan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(156) = 312$.

Time = 0.30 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.64

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx$$
$$= \frac{6\,abd^3x^3+6\,abc^3+(18\,abc-b^2)d^2x^2-b^2c^2-2\,abc+2\,(9\,abc^2-b^2c-ab)dx+3\,(b^2d^4x^4+4\,b^2cd^3x^3+6}{}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*b*d^3*x^3 + 6*a*b*c^3 + (18*a*b*c - b^2)*d^2*x^2 - b^2*c^2 - 2*a*
b*c + 2*(9*a*b*c^2 - b^2*c - a*b)*d*x + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 +
6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*arctan(d*x + c)^2 - 3*a^
```

```
2 + 2*(3*a*b*d^4*x^4 + 3*(4*a*b*c + b^2)*d^3*x^3 + 3*a*b*c^4 + 3*b^2*c^3 +
9*(2*a*b*c^2 + b^2*c)*d^2*x^2 - b^2*c + (12*a*b*c^3 + 9*b^2*c^2 - b^2)*d*x
- 3*a*b)*arctan(d*x + c) + 4*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2
*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 8*(b^2*d
^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log
(d*x + c))/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e
^5*x + c^4*d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(156) = 312$.

Time = 0.32 (sec) , antiderivative size = 534, normalized size of antiderivative = 3.14

$$\begin{aligned}
&\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx \\
&= \frac{1}{6}\left(d\left(\frac{3\,d^2x^2+6\,cdx+3\,c^2-1}{d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^5}\right)-\frac{3\,\arctan\left(dx+\right.}{d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2}\right. \\
&\quad+\frac{1}{12}\left(2\,d\left(\frac{3\,d^2x^2+6\,cdx+3\,c^2-1}{d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^5}\right)\arctan\left(dx+c\right)-\frac{\left(3\left(d^2x^2+2\right.\right.}{}\right. \\
&\quad-\frac{b^2\arctan\left(dx+c\right)^2}{4\left(d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3d^2e^5x+c^4de^5\right)} \\
&\quad-\frac{a^2}{4\left(d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3d^2e^5x+c^4de^5\right)}
\end{aligned}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 +
3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*arctan((d^2*x + c*d)/d)/(d^2*e^5)) - 3*a
rctan(d*x + c)/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d
^2*e^5*x + c^4*d*e^5))*a*b + 1/12*(2*d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(
d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*arctan((
```

```
d^2*x + c*d)/d)/(d^2*e^5))*arctan(d*x + c) - (3*(d^2*x^2 + 2*c*d*x + c^2)*a
rctan(d*x + c)^2 - 4*(d^2*x^2 + 2*c*d*x + c^2)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1) + 8*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c) + 1)*d^2/(d^5*e^5*x^2 + 2*c
*d^4*e^5*x + c^2*d^3*e^5))*b^2 - 1/4*b^2*arctan(d*x + c)^2/(d^5*e^5*x^4 + 4
*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5) - 1/4*a^2
/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4
*d*e^5)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.58

$$\begin{aligned}
&\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx \\
&= \mathrm{atan}(c+d\,x)^2\left(\frac{b^2}{4\,d\,e^5}-\frac{b^2}{4\,d^3\,e^5\,\left(\frac{c^4}{d^2}+6\,c^2\,x^2+d^2\,x^4+\frac{4\,c^3\,x}{d}+4\,c\,d\,x^3\right)}\right) \\
&\quad -\frac{x^2\left(\frac{b^2\,d}{2}-9\,a\,b\,c\,d\right)+x\,\left(b^2\,c-9\,a\,b\,c^2+a\,b\right)+\frac{3\,a^2-6\,a\,b\,c^3+2\,a\,b\,c+b^2\,c^2}{2\,d}-3\,a\,b\,d^2\,x^3}{6\,c^4\,e^5+24\,c^3\,d\,e^5\,x+36\,c^2\,d^2\,e^5\,x^2+24\,c\,d^3\,e^5\,x^3+6\,d^4\,e^5\,x^4} \\
&\quad +\frac{\mathrm{atan}(c+d\,x)\,\left(\frac{b^2\,x^3}{2\,e^5}-\frac{a\,b}{2\,d^3\,e^5}+\frac{b^2\,c\,\left(\frac{c^2-1}{3\,d^2}+\frac{2\,c^2}{3\,d^2}\right)}{2\,d\,e^5}+\frac{b^2\,x\,\left(d\,\left(\frac{c^2-1}{3\,d^2}+\frac{2\,c^2}{3\,d^2}\right)+\frac{2\,c^2}{d}\right)}{2\,d\,e^5}+\frac{3\,b^2\,c\,x^2}{2\,d\,e^5}\right)}{\frac{c^4}{d^2}+6\,c^2\,x^2+d^2\,x^4+\frac{4\,c^3\,x}{d}+4\,c\,d\,x^3} \\
&\quad -\frac{2\,b^2\,\ln\left(c+d\,x\right)}{3\,d\,e^5}-\frac{\ln\left(c+d\,x-\mathrm{i}\right)\,\left(-\frac{b^2}{3}+\frac{a\,b\,1\mathrm{i}}{4}\right)}{d\,e^5}+\frac{\ln\left(c+d\,x+1\mathrm{i}\right)\,\left(\frac{b^2}{3}+\frac{1\mathrm{i}\,a\,b}{4}\right)}{d\,e^5}
\end{aligned}$$

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^5,x)
```

```
[Out] atan(c + d*x)^2*(b^2/(4*d*e^5) - b^2/(4*d^3*e^5*(c^4/d^2 + 6*c^2*x^2 + d^2*
x^4 + (4*c^3*x)/d + 4*c*d*x^3))) - (x^2*((b^2*d)/2 - 9*a*b*c*d) + x*(a*b +
b^2*c - 9*a*b*c^2) + (3*a^2 + b^2*c^2 + 2*a*b*c - 6*a*b*c^3)/(2*d) - 3*a*b*
d^2*x^3)/(6*c^4*e^5 + 6*d^4*e^5*x^4 + 24*c*d^3*e^5*x^3 + 36*c^2*d^2*e^5*x^2
 + 24*c^3*d*e^5*x) + (atan(c + d*x)*((b^2*x^3)/(2*e^5) - (a*b)/(2*d^3*e^5)
+ (b^2*c*((c^2 - 1)/(3*d^2) + (2*c^2)/(3*d^2)))/(2*d*e^5) + (b^2*x*(d*((c^2
```

```
 - 1)/(3*d^2) + (2*c^2)/(3*d^2)) + (2*c^2)/d))/(2*d*e^5) + (3*b^2*c*x^2)/(2
*d*e^5)))/(c^4/d^2 + 6*c^2*x^2 + d^2*x^4 + (4*c^3*x)/d + 4*c*d*x^3) - (2*b^
2*log(c + d*x))/(3*d*e^5) - (log(c + d*x - 1i)*((a*b*1i)/4 - b^2/3))/(d*e^5
) + (log(c + d*x + 1i)*((a*b*1i)/4 + b^2/3))/(d*e^5)
```

3.15 $\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 271

$$\begin{aligned}
&\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx \\
&= ab^2e^2x+\frac{b^3e^2(c+dx)\arctan(c+dx)}{d}-\frac{be^2(a+b\arctan(c+dx))^2}{2d} \\
&\quad-\frac{be^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d}-\frac{ie^2(a+b\arctan(c+dx))^3}{3d} \\
&\quad+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d}-\frac{be^2(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad-\frac{b^3e^2\log\left(1+(c+dx)^2\right)}{2d}-\frac{ib^2e^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad-\frac{b^3e^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}
\end{aligned}$$

```
[Out] a*b^2*e^2*x+b^3*e^2*(d*x+c)*arctan(d*x+c)/d-1/2*b*e^2*(a+b*arctan(d*x+c))^2
/d-1/2*b*e^2*(d*x+c)^2*(a+b*arctan(d*x+c))^2/d-1/3*I*e^2*(a+b*arctan(d*x+c)
)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*x+c))^3/d-b*e^2*(a+b*arctan(d*x+c))^2
*ln(2/(1+I*(d*x+c)))/d-1/2*b^3*e^2*ln(1+(d*x+c)^2)/d-I*b^2*e^2*(a+b*arctan(
d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d-1/2*b^3*e^2*polylog(3,1-2/(1+I*(d*x+
c)))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5151, 12, 4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$\begin{aligned}
&\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx\\
&=-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))}{d}\\
&\quad-\frac{be^2(a+b\arctan(c+dx))^2}{2d}-\frac{be^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d}\\
&\quad+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d}-\frac{ie^2(a+b\arctan(c+dx))^3}{3d}\\
&\quad-\frac{be^2\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2}{d}+ab^2e^2x+\frac{b^3e^2(c+dx)\arctan(c+dx)}{d}\\
&\quad-\frac{b^3e^2\operatorname{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)}{2d}-\frac{b^3e^2\log\left((c+dx)^2+1\right)}{2d}
\end{aligned}$$

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] a*b^2*e^2*x + (b^3*e^2*(c + d*x)*ArcTan[c + d*x])/d - (b*e^2*(a + b*ArcTan[
c + d*x])^2)/(2*d) - (b*e^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) -
((I/3)*e^2*(a + b*ArcTan[c + d*x])^3)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c
+ d*x])^3)/(3*d) - (b*e^2*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))
])/d - (b^3*e^2*Log[1 + (c + d*x)^2])/(2*d) - (I*b^2*e^2*(a + b*ArcTan[c +
d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1
 + I*(c + d*x))])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
```

```
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^2x^2(a+b\arctan(x))^3\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int x^2(a+b\arctan(x))^3\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d}-\frac{(be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\arctan(x))^2}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d}\\
&\quad-\frac{(be^2)\,\text{Subst}\left(\int x(a+b\arctan(x))^2\,dx,x,c+dx\right)}{d}\\
&\quad+\frac{(be^2)\,\text{Subst}\left(\int \frac{x(a+b\arctan(x))^2}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= -\frac{be^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d}-\frac{ie^2(a+b\arctan(c+dx))^3}{3d}\\
&\quad+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d}-\frac{(be^2)\,\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{i-x}\,dx,x,c+dx\right)}{d}\\
&\quad+\frac{(b^2e^2)\,\text{Subst}\left(\int \frac{x^2(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= -\frac{be^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d}-\frac{ie^2(a+b\arctan(c+dx))^3}{3d}\\
&\quad+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d}-\frac{be^2(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d}\\
&\quad+\frac{(b^2e^2)\,\text{Subst}\left(\int (a+b\arctan(x))\,dx,x,c+dx\right)}{d}\\
&\quad-\frac{(b^2e^2)\,\text{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&\quad+\frac{(2b^2e^2)\,\text{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= ab^2e^2x - \frac{be^2(a+b\arctan(c+dx))^2}{2d} - \frac{be^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d} \\
&\quad - \frac{ie^2(a+b\arctan(c+dx))^3}{3d} + \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d} \\
&\quad - \frac{be^2(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad - \frac{ib^2e^2(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad + \frac{(ib^3e^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d} \\
&\quad + \frac{(b^3e^2)\,\text{Subst}(\int \arctan(x)\,dx, x, c+dx)}{d} \\
&= ab^2e^2x + \frac{b^3e^2(c+dx)\arctan(c+dx)}{d} - \frac{be^2(a+b\arctan(c+dx))^2}{2d} \\
&\quad - \frac{be^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d} - \frac{ie^2(a+b\arctan(c+dx))^3}{3d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d} - \frac{be^2(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad - \frac{ib^2e^2(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad - \frac{b^3e^2\,\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d} - \frac{(b^3e^2)\,\text{Subst}(\int \frac{x}{1+x^2}\,dx, x, c+dx)}{d} \\
&= ab^2e^2x + \frac{b^3e^2(c+dx)\arctan(c+dx)}{d} - \frac{be^2(a+b\arctan(c+dx))^2}{2d} \\
&\quad - \frac{be^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d} - \frac{ie^2(a+b\arctan(c+dx))^3}{3d} \\
&\quad + \frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d} - \frac{be^2(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad - \frac{b^3e^2\log(1+(c+dx)^2)}{2d} - \frac{ib^2e^2(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad - \frac{b^3e^2\,\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.29

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx$$
$$=\frac{e^2\Big(-3a^2b(c+dx)^2+2a^3(c+dx)^3+6a^2b(c+dx)^3\arctan(c+dx)+3a^2b\log\left(1+(c+dx)^2\right)+6ab^2(c+d}{}$$

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (e^2*(-3*a^2*b*(c + d*x)^2 + 2*a^3*(c + d*x)^3 + 6*a^2*b*(c + d*x)^3*ArcTan
[c + d*x] + 3*a^2*b*Log[1 + (c + d*x)^2] + 6*a*b^2*(c + d*x - ArcTan[c + d*
x] - (c + d*x)^2*ArcTan[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan
[c + d*x]^2 - 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*Poly
Log[2, -E^((2*I)*ArcTan[c + d*x])]) + b^3*(6*(c + d*x)*ArcTan[c + d*x] - 3*
(1 + (c + d*x)^2)*ArcTan[c + d*x]^2 + (2*I)*ArcTan[c + d*x]^3 - 2*(c + d*x)
*ArcTan[c + d*x]^3 + 2*(c + d*x)*(1 + (c + d*x)^2)*ArcTan[c + d*x]^3 - 6*Ar
cTan[c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*Log[1/Sqrt[1 + (c +
d*x)^2]] + (6*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] - 3
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])))/(6*d)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.18 (sec) , antiderivative size = 1249, normalized size of antiderivative = 4.61

method	result	size
derivativedivides	Expression too large to display	1249
default	Expression too large to display	1249
parts	Expression too large to display	1257

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*a^3*(d*x+c)^3+e^2*b^3*(1/3*(d*x+c)^3*arctan(d*x+c)^3-1/2*(d*x+
c)^2*arctan(d*x+c)^2+1/2*arctan(d*x+c)^2*ln(1+(d*x+c)^2)-arctan(d*x+c)^2*ln
((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+I*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c)
)^2/(1+(d*x+c)^2))-1/2*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/12*I*arc
tan(d*x+c)*(3*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+
c))^2/(1+(d*x+c)^2))*Pi*arctan(d*x+c)-6*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^
(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2*Pi*arctan(d*x+c)+3*csgn(I*(1
+I*(d*x+c))^2/(1+(d*x+c)^2))^3*Pi*arctan(d*x+c)-3*csgn(I*(1+I*(d*x+c))^2/(1
+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*
x+c)^2))^2)^2*Pi*arctan(d*x+c)+3*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn
```

```
(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(
I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*Pi*arctan(d*x+c)-3*csgn(I*(1+(1+I*(d
*x+c))^2/(1+(d*x+c)^2)))^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*Pi*a
rctan(d*x+c)+6*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(1+(1+I*(d*
x+c))^2/(1+(d*x+c)^2))^2)^2*Pi*arctan(d*x+c)-3*csgn(I*(1+(1+I*(d*x+c))^2/(1
+(d*x+c)^2))^2)^3*Pi*arctan(d*x+c)+3*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(
1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3*Pi*arctan(d*x+c)-3*csgn(I*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2*csgn(I/(1+(1+I*(d
*x+c))^2/(1+(d*x+c)^2))^2)*Pi*arctan(d*x+c)+4*arctan(d*x+c)^2+12*I*ln(2)*ar
ctan(d*x+c)+6*I*arctan(d*x+c)-12-12*I*(d*x+c))+ln(1+(1+I*(d*x+c))^2/(1+(d*x
+c)^2)))+3*e^2*a*b^2*(1/3*(d*x+c)^3*arctan(d*x+c)^2-1/3*(d*x+c)^2*arctan(d*
x+c)+1/3*arctan(d*x+c)*ln(1+(d*x+c)^2)+1/3*d*x+1/3*c-1/3*arctan(d*x+c)+1/6*
I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln
(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))-1/6*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d
*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))+3*e^2*a^
2*b*(1/3*(d*x+c)^3*arctan(d*x+c)-1/6*(d*x+c)^2+1/6*ln(1+(d*x+c)^2)))
```

Fricas [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)^2(b\arctan(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^2
 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arctan(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x^2
+ 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arctan(d*x + c)^2 + 3*(a^2*b*d^2*e^2*x
^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arctan(d*x + c), x)
```

Sympy [F]

$$\begin{aligned}\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx\\ =e^2\bigg(\int a^3c^2\,dx+\int a^3d^2x^2\,dx+\int b^3c^2\operatorname{atan}^3(c+dx)\,dx+\int 3ab^2c^2\operatorname{atan}^2(c+dx)\,dx\\ +\int 3a^2bc^2\operatorname{atan}(c+dx)\,dx+\int 2a^3cdx\,dx+\int b^3d^2x^2\operatorname{atan}^3(c+dx)\,dx\\ +\int 3ab^2d^2x^2\operatorname{atan}^2(c+dx)\,dx+\int 3a^2bd^2x^2\operatorname{atan}(c+dx)\,dx\\ +\int 2b^3cdx\operatorname{atan}^3(c+dx)\,dx+\int 6ab^2cdx\operatorname{atan}^2(c+dx)\,dx\\ +\int 6a^2bcdx\operatorname{atan}(c+dx)\,dx\bigg)\end{aligned}$$

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3*
c**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*c**2*atan(c + d*x)**2, x) + I
ntegral(3*a**2*b*c**2*atan(c + d*x), x) + Integral(2*a**3*c*d*x, x) + Integ
ral(b**3*d**2*x**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x**2*atan(
c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*atan(c + d*x), x) + Integral(
2*b**3*c*d*x*atan(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*atan(c + d*x)**
2, x) + Integral(6*a**2*b*c*d*x*atan(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)^2(b\arctan(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^4*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^4*e
^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan(
(d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^4*e^2 - 7/32*(6
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((
d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^4*e^2 + 1/3*a^3*d^
2*e^2*x^3 + 7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 2
8*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + 3*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^4*e^2*i
ntegrate(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112
*b^3*c*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 4*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d^3*e^2*in
tegrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d^3*e^2*integrate(1/32*x^3*arctan(
d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 168*b^3*c^2*d^2*e^2*integrat
e(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*b^3*c*d
^3*e^2*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^3*c^2*d^2*e^2*integrate(1/32*x^2*a
rctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 576*a*b^2*c^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*
x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c^2*d^2*e^2*integrate(1
/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 12*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c
^3*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
```

```
x) + 12*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^4*e^2*integrate(1/
32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + a^3*c*d*e^2*x^2 + 3*a*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((
d^2*x + c*d)/d)/d - 4*b^3*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d^3*e^2*integrate(1/32*x^3*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*c*d^2*e^2
*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3
*b^3*c*d^2*e^2*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*c^2*d*e^2*integrate(1/32*x*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^2*d*e^2*integrate(1/32*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*
arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*
a*b^2*c^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*a
rctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b
^3*c^2*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x +
c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*c*d*e^2 + 1/2*
(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2
*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b
*d^2*e^2 + a^3*c^2*e^2*x + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*ar
ctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 96*a*b^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^3/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*e^2*integrate(1/32*x*arctan(d
*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)
*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/24*(b^3*d^2*e^
2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*arctan(d*x + c)^3 - 1/32*(b^3*
d^2*e^2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2
```

Giac [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)^2\left(b\arctan\left(dx+c\right)+a\right)^3\,dx$$

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx=\int \left(c\,e+d\,e\,x\right)^2\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^3\,dx$$

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^3, x)
```

3.16 $\int (ce+dex)(a+b\arctan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 164

$$\begin{aligned}\int (ce+dex)(a+b\arctan(c+dx))^3\,dx = &-\frac{3ibe(a+b\arctan(c+dx))^2}{2d}\\&-\frac{3be(c+dx)(a+b\arctan(c+dx))^2}{2d}\\&+\frac{e(a+b\arctan(c+dx))^3}{2d}\\&+\frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d}\\&-\frac{3b^2e(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}\\&-\frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d}\end{aligned}$$

```
[Out] -3/2*I*b*e*(a+b*arctan(d*x+c))^2/d-3/2*b*e*(d*x+c)*(a+b*arctan(d*x+c))^2/d+
1/2*e*(a+b*arctan(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arctan(d*x+c))^3/d-3*b^2
*e*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d-3/2*I*b^3*e*polylog(2,1-2/(1+I
*(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5151, 12, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$\begin{aligned}\int (ce+dex)(a+b\arctan(c+dx))^3\,dx = &-\frac{3b^2e\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))}{d}\\&-\frac{3ibe(a+b\arctan(c+dx))^2}{2d}\\&-\frac{3be(c+dx)(a+b\arctan(c+dx))^2}{2d}\\&+\frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d}\\&+\frac{e(a+b\arctan(c+dx))^3}{2d}\\&-\frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{2d}\end{aligned}$$

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c + d*x])^2)/d - (3*b*e*(c + d*x)*(a + b*ArcT
an[c + d*x])^2)/(2*d) + (e*(a + b*ArcTan[c + d*x])^3)/(2*d) + (e*(c + d*x)^
2*(a + b*ArcTan[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*ArcTan[c + d*x])*Log[2
/(1 + I*(c + d*x))])/d - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int ex(a+b\arctan(x))^3\,dx,x,c+dx\right)}{d}\\
&= \frac{e\text{Subst}\left(\int x(a+b\arctan(x))^3\,dx,x,c+dx\right)}{d}\\
&= \frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d} - \frac{(3be)\text{Subst}\left(\int \frac{x^2(a+b\arctan(x))^2}{1+x^2}\,dx,x,c+dx\right)}{2d}\\
&= \frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d} - \frac{(3be)\text{Subst}\left(\int (a+b\arctan(x))^2\,dx,x,c+dx\right)}{2d}\\
&\quad + \frac{(3be)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{1+x^2}\,dx,x,c+dx\right)}{2d}\\
&= -\frac{3be(c+dx)(a+b\arctan(c+dx))^2}{2d} + \frac{e(a+b\arctan(c+dx))^3}{2d}\\
&\quad + \frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d} + \frac{(3b^2e)\,\text{Subst}\left(\int \frac{x(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= -\frac{3ibe(a+b\arctan(c+dx))^2}{2d} - \frac{3be(c+dx)(a+b\arctan(c+dx))^2}{2d}\\
&\quad + \frac{e(a+b\arctan(c+dx))^3}{2d} + \frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d}\\
&\quad - \frac{(3b^2e)\,\text{Subst}\left(\int \frac{a+b\arctan(x)}{i-x}\,dx,x,c+dx\right)}{d}\\
&= -\frac{3ibe(a+b\arctan(c+dx))^2}{2d} - \frac{3be(c+dx)(a+b\arctan(c+dx))^2}{2d}\\
&\quad + \frac{e(a+b\arctan(c+dx))^3}{2d} + \frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d}\\
&\quad - \frac{3b^2e(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}\\
&\quad + \frac{(3b^3e)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= -\frac{3ibe(a+b\arctan(c+dx))^2}{2d} - \frac{3be(c+dx)(a+b\arctan(c+dx))^2}{2d}\\
&\quad + \frac{e(a+b\arctan(c+dx))^3}{2d} + \frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d}\\
&\quad - \frac{3b^2e(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}\\
&\quad - \frac{(3ib^3e)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3ibe(a+b\arctan(c+dx))^2}{2d} - \frac{3be(c+dx)(a+b\arctan(c+dx))^2}{2d} \\
&\quad + \frac{e(a+b\arctan(c+dx))^3}{2d} + \frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d} \\
&\quad - \frac{3b^2e(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d} - \frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$
\int (ce+dex)(a+b\arctan(c+dx))^3\,dx = \frac{e\Big(3b^2(-i+c+dx)(-b+a(i+c+dx))\arctan(c+dx)^2 + b^3(1+c^2+2cdx+d^2x^2)\arctan(c+dx)^3 + 3}{}
$$

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (e*(3*b^2*(-I + c + d*x)*(-b + a*(I + c + d*x))*ArcTan[c + d*x]^2 + b^3*(1
+ c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*b*ArcTan[c + d*x]*(a*(-2*b
*(c + d*x) + a*(1 + c^2 + 2*c*d*x + d^2*x^2)) - 2*b^2*Log[1 + E^((2*I)*ArcT
an[c + d*x])]) + a*(a*(c + d*x)*(-3*b + a*c + a*d*x) - 6*b^2*Log[1/Sqrt[1 +
 (c + d*x)^2]]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/(2*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(150) = 300.

Time = 0.55 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.96

method	result
derivativedivides	$\frac{e a^3 (dx+c)^2}{2} + e b^3 \Bigg(\frac{(dx+c)^2 \arctan(dx+c)^3}{2} + \frac{\arctan(dx+c)^3}{2} - \frac{3(dx+c)\arctan(dx+c)^2}{2} + \frac{3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2} + 3i\Big(\ln(d$
default	$\frac{e a^3 (dx+c)^2}{2} + e b^3 \Bigg(\frac{(dx+c)^2 \arctan(dx+c)^3}{2} + \frac{\arctan(dx+c)^3}{2} - \frac{3(dx+c)\arctan(dx+c)^2}{2} + \frac{3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2} + 3i\Big(\ln(d$
parts	$e a^3 \left(\frac{1}{2} d x^2 + cx\right) + e b^3 \Bigg(\frac{(dx+c)^2 \arctan(dx+c)^3}{2} + \frac{\arctan(dx+c)^3}{2} - \frac{3(dx+c)\arctan(dx+c)^2}{2} + \frac{3\arctan(dx+c)\ln\big(1+(dx+$
risch	Expression too large to display

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*a^3*(d*x+c)^2+e*b^3*(1/2*(d*x+c)^2*arctan(d*x+c)^3+1/2*arctan(d*
x+c)^3-3/2*(d*x+c)*arctan(d*x+c)^2+3/2*arctan(d*x+c)*ln(1+(d*x+c)^2)+3/4*I*
(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d
*x+c-I)*ln(-1/2*I*(d*x+c+I)))-3/4*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x
+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))+3*e*a*b^2*
(1/2*(d*x+c)^2*arctan(d*x+c)^2+1/2*arctan(d*x+c)^2-(d*x+c)*arctan(d*x+c)+1/
2*ln(1+(d*x+c)^2))+3*e*a^2*b*(1/2*(d*x+c)^2*arctan(d*x+c)-1/2*d*x-1/2*c+1/2
*arctan(d*x+c)))
```

Fricas [F]

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)(b\arctan{(dx+c)}+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arctan(d*x + c)^3 + 3*
(a*b^2*d*e*x + a*b^2*c*e)*arctan(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e)*a
rctan(d*x + c), x)
```

Sympy [F]

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx=e\left(\int a^3c\,dx+\int a^3dx\,dx+\int b^3c\,\operatorname{atan}^3{(c+dx)}\,dx+\int 3ab^2c\,\operatorname{atan}^2{(c+dx)}\,dx+\int 3a^2bc\,\operatorname{atan}{(c+dx)}\,dx+\int b^3dx\,\operatorname{atan}^3{(c+dx)}\,dx+\int 3ab^2dx\,\operatorname{atan}^2{(c+dx)}\,dx+\int 3a^2bdx\,\operatorname{atan}{(c+dx)}\,dx\right)$$

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**3,x)
```

```
[Out] e*(Integral(a**3*c, x) + Integral(a**3*d*x, x) + Integral(b**3*c*atan(c + d
*x)**3, x) + Integral(3*a*b**2*c*atan(c + d*x)**2, x) + Integral(3*a**2*b*c
*atan(c + d*x), x) + Integral(b**3*d*x*atan(c + d*x)**3, x) + Integral(3*a*
b**2*d*x*atan(c + d*x)**2, x) + Integral(3*a**2*b*d*x*atan(c + d*x), x))
```

Maxima [F]

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)(b\arctan\left(dx+c\right)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*e*x^2 + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d
^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*d*e + a
^3*c*e*x + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c
*e/d + 1/32*(8*(b^3*d^2*e*x^2 + 2*b^3*c*d*e*x + (b^3*c^2 + b^3)*e)*arctan(d
*x + c)^3 + 12*(a*b^2*d^2*e*x^2 + (2*a*b^2*c - b^3)*d*e*x)*arctan(d*x + c)^
2 - 3*(a*b^2*d^2*e*x^2 + (2*a*b^2*c - b^3)*d*e*x)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2 + 4*(4*b^3*c^3*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 18
*a*b^2*c^3*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 6*(3*arctan(d*x
+ c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^3*e
 - (6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arc
tan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^3*e - 3*b^3*c
^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d + 4*b^3*c*e*arctan(d*x + c
)^3*arctan((d^2*x + c*d)/d)/d + 128*b^3*d^3*e*integrate(1/32*x^3*arctan(d*x
 + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d^3*e*integrate(1/32*
x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c*d^2*e*i
ntegrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48*
a*b^2*d^3*e*integrate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c*d^2*e*integrate(1/32*x^2*arctan(d*x
 + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c^2*d*e*integrate(1/32*
x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^3*e*inte
grate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 144*a*b^2*c*d^2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c^2*d*e*integrate(1/32
*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 288*a*b^2*c*d^2*e*
integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 144*a*b^2*c^2*d*e*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*d*e*integrate(1/3
2*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48
*a*b^2*c^3*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - ar
ctan((d^2*x + c*d)/d)^3/d)*b^3*c^2*e + 18*a*b^2*c*e*arctan(d*x + c)^2*arcta
n((d^2*x + c*d)/d)/d - 96*b^3*d^2*e*integrate(1/32*x^2*arctan(d*x + c)^2/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^3*d^2*e*integrate(1/32*x^2*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 192*a*b^2*d^2
*e*integrate(1/32*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 1
92*b^3*c*d*e*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) - 96*b^3*d^2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(
```

```
d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 48*b^3*c*d*e*integrate(1/32*x*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 384*a*b^2*c*d*
e*integrate(1/32*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 96*b
^3*c*d*e*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) - 24*b^3*c^2*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 6*(3*arctan(d*x + c)*arctan((d^
2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c*e - (6*arctan(d*x
+ c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)
/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c*e - 3*b^3*e*arctan(d*x + c)^2*
arctan((d^2*x + c*d)/d)/d + 128*b^3*d*e*integrate(1/32*x*arctan(d*x + c)^3/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d*e*integrate(1/32*x*arctan(d
*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48*a*b^2*d*e*integrate(1/32*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192
*b^3*d*e*integrate(1/32*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 48*a*b^2*c*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d -
 arctan((d^2*x + c*d)/d)^3/d)*b^3*e - 24*b^3*e*integrate(1/32*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x))*d)/d
```

Giac [F]

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)(b\arctan(dx+c)+a)^3\,dx$$

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^3\,dx$$

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*atan(c + d*x))^3, x)
```

3.17 $\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 279

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx &= \frac{2(a+b\arctan(c+dx))^3\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de}\\ &-\frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2de}\\ &+\frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)}{2de}\\ &-\frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2de}\\ &+\frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)}{2de}\\ &+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+i(c+dx)}\right)}{4de}\\ &-\frac{3ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+i(c+dx)}\right)}{4de}\end{aligned}$$

```
[Out] -2*(a+b*arctan(d*x+c))^3*arctanh(-1+2/(1+I*(d*x+c)))/d/e-3/2*I*b*(a+b*arcta
n(d*x+c))^2*polylog(2,1-2/(1+I*(d*x+c)))/d/e+3/2*I*b*(a+b*arctan(d*x+c))^2*
polylog(2,-1+2/(1+I*(d*x+c)))/d/e-3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2
/(1+I*(d*x+c)))/d/e+3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,-1+2/(1+I*(d*x+c)
))/d/e+3/4*I*b^3*polylog(4,1-2/(1+I*(d*x+c)))/d/e-3/4*I*b^3*polylog(4,-1+2/
(1+I*(d*x+c)))/d/e
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5151, 12, 4942, 5108, 5004, 5114, 5118, 6745}

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = {} & \frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^3}{de} \\ & -\frac{3b^2\,\text{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))}{2de} \\ & +\frac{3b^2\,\text{PolyLog}\left(3,\frac{2}{i(c+dx)+1}-1\right)(a+b\arctan(c+dx))}{2de} \\ & -\frac{3ib\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))^2}{2de} \\ & +\frac{3ib\,\text{PolyLog}\left(2,\frac{2}{i(c+dx)+1}-1\right)(a+b\arctan(c+dx))^2}{2de} \\ & +\frac{3ib^3\,\text{PolyLog}\left(4,1-\frac{2}{i(c+dx)+1}\right)}{4de} \\ & -\frac{3ib^3\,\text{PolyLog}\left(4,\frac{2}{i(c+dx)+1}-1\right)}{4de}\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTan[c + d*x])^3*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (((3
*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*
e) + (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, -1 + 2/(1 + I*(c + d
*x))])/(d*e) - (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, 1 - 2/(1 + I*(c +
d*x))])/(2*d*e) + (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, -1 + 2/(1 + I*(
c + d*x))])/(2*d*e) + (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*(c + d*x))])/(
d*e) - (((3*I)/4)*b^3*PolyLog[4, -1 + 2/(1 + I*(c + d*x))])/(d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2(a+b\arctan(c+dx))^3\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(6b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2\text{arctanh}\left(1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2(a+b\arctan(c+dx))^3\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2\log\left(2-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2(a+b\arctan(c+dx))^3\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib(a+b\arctan(c+dx))^2\,\text{PolyLog}\left(2, 1-\frac{2}{1+i(c+dx)}\right)}{2de} + \frac{3ib(a+b\arctan(c+dx))^2\,\text{PolyLog}\left(2, -1+\frac{2}{1+i(c+dx)}\right)}{2de} + \frac{(3ib^2)\,\text{Subst}\left(\int \frac{(a+b\arctan(x))\,\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de} - \frac{(3ib^2)\,\text{Subst}\left(\int \frac{(a+b\arctan(x))\,\text{PolyLog}\left(2,-1+\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de}$$

$$
\begin{aligned}
&= \frac{2(a+b\arctan(c+dx))^3\operatorname{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad - \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2de} \\
&\quad + \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)}{2de} \\
&\quad - \frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2de} \\
&\quad + \frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)}{2de} \\
&\quad + \frac{(3b^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{2de} \\
&\quad - \frac{(3b^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{2de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(a+b\arctan(c+dx))^3\operatorname{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} \\
&\quad - \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2de} \\
&\quad + \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)}{2de} \\
&\quad - \frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2de} \\
&\quad + \frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)}{2de} \\
&\quad + \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+i(c+dx)}\right)}{4de} - \frac{3ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+i(c+dx)}\right)}{4de}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 562 vs. 2(279) = 558.

Time = 0.55 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.01

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \frac{64a^3\log(c+dx)-24ia^2b\left(\pi^2-4\pi\arctan(c+dx)+8\arctan(c+dx)^2-i\pi\log(16)+4i\pi\log\left(1+e^{-2i\,\mathrm{arcta}}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (64*a^3*Log[c + d*x] - (24*I)*a^2*b*(Pi^2 - 4*Pi*ArcTan[c + d*x] + 8*ArcTan
[c + d*x]^2 - I*Pi*Log[16] + (4*I)*Pi*Log[1 + E^((-2*I)*ArcTan[c + d*x])] -
 (8*I)*ArcTan[c + d*x]*Log[1 + E^((-2*I)*ArcTan[c + d*x])] + (8*I)*ArcTan[c
 + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x])] + (2*I)*Pi*Log[1 + c^2 + 2*c*d*x
 + d^2*x^2] + 4*PolyLog[2, -E^((-2*I)*ArcTan[c + d*x])] + 4*PolyLog[2, E^((
2*I)*ArcTan[c + d*x])]) + 8*a*b^2*((-I)*Pi^3 + (16*I)*ArcTan[c + d*x]^3 + 2
4*ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] - 24*ArcTan[c + d*x
]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + (24*I)*ArcTan[c + d*x]*PolyLog[2,
E^((-2*I)*ArcTan[c + d*x])] + (24*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*A
rcTan[c + d*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])] - 12*PolyLog[3
, -E^((2*I)*ArcTan[c + d*x])]) - I*b^3*(Pi^4 - 32*ArcTan[c + d*x]^4 + (64*I
)*ArcTan[c + d*x]^3*Log[1 - E^((-2*I)*ArcTan[c + d*x])] - (64*I)*ArcTan[c +
 d*x]^3*Log[1 + E^((2*I)*ArcTan[c + d*x])] - 96*ArcTan[c + d*x]^2*PolyLog[2
, E^((-2*I)*ArcTan[c + d*x])] - 96*ArcTan[c + d*x]^2*PolyLog[2, -E^((2*I)*A
rcTan[c + d*x])] + (96*I)*ArcTan[c + d*x]*PolyLog[3, E^((-2*I)*ArcTan[c + d
*x])] - (96*I)*ArcTan[c + d*x]*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])] + 48*
PolyLog[4, E^((-2*I)*ArcTan[c + d*x])] + 48*PolyLog[4, -E^((2*I)*ArcTan[c +
 d*x])]))/(64*d*e)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.81 (sec) , antiderivative size = 2313, normalized size of antiderivative = 8.29

method	result	size
derivativedivides	Expression too large to display	2313
default	Expression too large to display	2313
parts	Expression too large to display	2321

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3/e*ln(d*x+c)+b^3/e*(ln(d*x+c)*arctan(d*x+c)^3-arctan(d*x+c)^3*ln((1
+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+arctan(d*x+c)^3*ln(1-(1+I*(d*x+c))/(1+(d*x+c
)^2)^(1/2))-3*I*arctan(d*x+c)^2*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2)
)+6*arctan(d*x+c)*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6*I*polylog(
4,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+arctan(d*x+c)^3*ln(1+(1+I*(d*x+c))/(1+
(d*x+c)^2)^(1/2))-3*I*arctan(d*x+c)^2*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2
)^(1/2))+6*arctan(d*x+c)*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6*I*
polylog(4,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*(d*x+c
))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(((1+I*(d*x+c))
^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn(I*((1+I*(d*x+
c))^2/(1+(d*x+c)^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((
1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(I*(
(1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/
(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2-csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2
)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^
2)))^2+csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+
c)^2)))^3-csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d
*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*
x+c)^2)))^2+csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d
*x+c)^2)))^3+1)*arctan(d*x+c)^3+3/2*I*arctan(d*x+c)^2*polylog(2,-(1+I*(d*x+
c))^2/(1+(d*x+c)^2))-3/2*arctan(d*x+c)*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c
)^2))-3/4*I*polylog(4,-(1+I*(d*x+c))^2/(1+(d*x+c)^2)))+3*a*b^2/e*(ln(d*x+c)
*arctan(d*x+c)^2+I*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-
1/2*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-arctan(d*x+c)^2*ln((1+I*(d*x+
c))^2/(1+(d*x+c)^2)-1)+arctan(d*x+c)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/
2))-2*I*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+2*polyl
og(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+arctan(d*x+c)^2*ln(1-(1+I*(d*x+c))
/(1+(d*x+c)^2)^(1/2))-2*I*arctan(d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^
2)^(1/2))+2*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/2*I*Pi*(csgn(I*(
(1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((
1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(((1
+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn(I*
((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))
)*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)
))-csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*
x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2-csgn(I/(1+(1+I*(d*x+c))^2/(
1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/
(1+(d*x+c)^2)))^2+csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))
^2/(1+(d*x+c)^2)))^3-csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+
c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c
))^2/(1+(d*x+c)^2)))^2+csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+
c))^2/(1+(d*x+c)^2)))^3+1)*arctan(d*x+c)^2)+3*a^2*b/e*(ln(d*x+c)*arctan(d*x
+c)+1/2*I*ln(d*x+c)*ln(1+I*(d*x+c))-1/2*I*ln(d*x+c)*ln(1-I*(d*x+c))+1/2*I*d
ilog(1+I*(d*x+c))-1/2*I*dilog(1-I*(d*x+c))))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{dex+ce}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d*e*x + c*e), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3\operatorname{atan}^3\left(c+dx\right)}{c+dx}\,dx + \int \frac{3ab^2\operatorname{atan}^2\left(c+dx\right)}{c+dx}\,dx + \int \frac{3a^2b\operatorname{atan}\left(c+dx\right)}{c+dx}\,dx}{e}$$

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*atan(c + d*x)**3/(c + d*x), x)
 + Integral(3*a*b**2*atan(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*ata
n(c + d*x)/(c + d*x), x))/e
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{dex+ce}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*log(d*e*x + c*e)/(d*e) + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*b
^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d*x
 + c)^2 + 96*a^2*b*arctan(d*x + c))/(d*e*x + c*e), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c+d\,x)\right)^3}{c\,e+d\,e\,x}\,dx$$

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x), x)
```

3.18 $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx = -\frac{i(a+b\arctan(c+dx))^3}{de^2} - \frac{(a+b\arctan(c+dx))^3}{de^2(c+dx)} + \frac{3b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{3ib^2(a+b\arctan(c+dx))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2} + \frac{3b^3\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^2}$$

```
[Out] -I*(a+b*arctan(d*x+c))^3/d/e^2-(a+b*arctan(d*x+c))^3/d/e^2/(d*x+c)+3*b*(a+b
*arctan(d*x+c))^2*ln(2-2/(1-I*(d*x+c)))/d/e^2-3*I*b^2*(a+b*arctan(d*x+c))*p
olylog(2,-1+2/(1-I*(d*x+c)))/d/e^2+3/2*b^3*polylog(3,-1+2/(1-I*(d*x+c)))/d/
e^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

$= \{5151, 12, 4946, 5044, 4988, 5004, 5112, 6745\}$

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx = -\frac{3ib^2\operatorname{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)(a+b\arctan(c+dx))}{de^2} - \frac{(a+b\arctan(c+dx))^3}{de^2(c+dx)} - \frac{i(a+b\arctan(c+dx))^3}{de^2} + \frac{3b\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))^2}{de^2} + \frac{3b^3\operatorname{PolyLog}\left(3,\frac{2}{1-i(c+dx)}-1\right)}{2de^2}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^3)/(d*e^2) - (a + b*ArcTan[c + d*x])^3/(d*e^2
*(c + d*x)) + (3*b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/
(d*e^2) - ((3*I)*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c +
d*x))])/(d*e^2) + (3*b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{e^2x^2}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{x^2}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{(a+b\arctan(c+dx))^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{i(a+b\arctan(c+dx))^3}{de^2} - \frac{(a+b\arctan(c+dx))^3}{de^2(c+dx)}\\
&\quad + \frac{(3ib)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x(i+x)}\,dx, x, c+dx\right)}{de^2}
\end{aligned}
$$

$$= -\frac{i(a+b\arctan(c+dx))^3}{de^2} - \frac{(a+b\arctan(c+dx))^3}{de^2(c+dx)} + \frac{3b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{(6b^2)\operatorname{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{i(a+b\arctan(c+dx))^3}{de^2} - \frac{(a+b\arctan(c+dx))^3}{de^2(c+dx)} + \frac{3b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{3ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2} + \frac{(3ib^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{i(a+b\arctan(c+dx))^3}{de^2} - \frac{(a+b\arctan(c+dx))^3}{de^2(c+dx)} + \frac{3b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{3ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2} + \frac{3b^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^2}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx$$

$$= -\frac{2a^3}{c+dx} - \frac{6a^2b\arctan(c+dx)}{c+dx} + 6a^2b\log(c+dx) - 3a^2b\log\left(1+c^2+2cdx+d^2x^2\right) + 6ab^2\Big(\arctan(c+dx)\Big((-i$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] ((-2*a^3)/(c + d*x) - (6*a^2*b*ArcTan[c + d*x])/(c + d*x) + 6*a^2*b*Log[c +
 d*x] - 3*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + 6*a*b^2*(ArcTan[c + d*x]
*((-I - (c + d*x)^(-1))*ArcTan[c + d*x] + 2*Log[1 - E^((2*I)*ArcTan[c + d*x
```

```
])]) - I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*((-1/8*I)*Pi^3 + I*
ArcTan[c + d*x]^3 - ArcTan[c + d*x]^3/(c + d*x) + 3*ArcTan[c + d*x]^2*Log[1
 - E^((-2*I)*ArcTan[c + d*x])] + (3*I)*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)
*ArcTan[c + d*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])])/2))/(2*d*e^
2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.33 (sec) , antiderivative size = 2104, normalized size of antiderivative = 12.91

method	result	size
derivativedivides	Expression too large to display	2104
default	Expression too large to display	2104
parts	Expression too large to display	2112

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3/e^2/(d*x+c)+b^3/e^2*(-1/(d*x+c)*arctan(d*x+c)^3+3*ln(d*x+c)*arcta
n(d*x+c)^2-3/2*arctan(d*x+c)^2*ln(1+(d*x+c)^2)+3*arctan(d*x+c)^2*ln((1+I*(d
*x+c))/(1+(d*x+c)^2)^(1/2))-3*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)
^2)-1)-I*arctan(d*x+c)^3+3/4*(2*I*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/
2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2-2*I*Pi*csgn(((1+I*(d*x+c))^2/(1
+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+2*I*Pi+I*Pi*csgn(I*(1+I
*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*
x+c))^2/(1+(d*x+c)^2))^2)^2+2*I*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)
/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3-I*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^
2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))-I*Pi*csgn(I/(1+(1+I*(d*x+
c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)-I*Pi*csgn(I*(1
+I*(d*x+c))^2/(1+(d*x+c)^2))^3-I*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1
+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3+2*I*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x
+c)^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^
2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-2*I*Pi*csgn(I/(1+(1+I
*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+
I*(d*x+c))^2/(1+(d*x+c)^2)))^2-2*I*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)
-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+I*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(
d*x+c)^2)))^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)+2*I*Pi*csgn(((1+I
*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3+I*Pi*csgn
(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2
)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2+2*I*Pi*csgn(I*((1+I*(d*x+c))^2/(1+
(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(
d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-2*I*Pi*csgn(I*(1+(1+I*(d*x+
```

```
c))^2/(1+(d*x+c)^2)))*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2-2*I*Pi*
csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c
)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+I*Pi*csgn(I*(1+(1+I*(d*x+c))^2
/(1+(d*x+c)^2))^2)^3+4*ln(2))*arctan(d*x+c)^2+3*arctan(d*x+c)^2*ln(1+(1+I*(
d*x+c))/(1+(d*x+c)^2)^(1/2))-6*I*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+
(d*x+c)^2)^(1/2))+6*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3*arctan(
d*x+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-6*I*arctan(d*x+c)*polylog(
2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2
)^(1/2)))+3*a*b^2/e^2*(-1/(d*x+c)*arctan(d*x+c)^2+2*ln(d*x+c)*arctan(d*x+c)
-arctan(d*x+c)*ln(1+(d*x+c)^2)-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*
x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(l
n(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+
c+I)*ln(1/2*I*(d*x+c-I)))+I*ln(d*x+c)*ln(1+I*(d*x+c))-I*ln(d*x+c)*ln(1-I*(d
*x+c))+I*dilog(1+I*(d*x+c))-I*dilog(1-I*(d*x+c)))+3*a^2*b/e^2*(-1/(d*x+c)*a
rctan(d*x+c)+ln(d*x+c)-1/2*ln(1+(d*x+c)^2)))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(dex+ce\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx = \frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3\operatorname{atan}^3\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2\operatorname{atan}^2\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b\operatorname{atan}\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*atan(c + d*
x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*atan(c + d*x)**2
/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*atan(c + d*x)/(c**2 +
 2*c*d*x + d**2*x**2), x))/e**2
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx=\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(dex+ce\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e
^2)) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*a^2*b - a^3/(d^2*e^2*x + c*
d*e^2) - 1/32*(4*b^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2 - 32*(d^2*e^2*x + c*d*e^2)*integrate(1/32*(28*(b^3*d
^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*x
^2 + 8*a*b^2*c^2 + b^3*c + 8*a*b^2 + (16*a*b^2*c + b^3)*d*x)*arctan(d*x + c
)^2 - 12*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1) - 3*(b^3*d*x + b^3*c - (b^3*d^2*x^2 + 2*b^3*c*d*x + b^
3*c^2 + b^3)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/(d^4*e^2*
x^4 + 4*c*d^3*e^2*x^3 + (6*c^2 + 1)*d^2*e^2*x^2 + 2*(2*c^3 + c)*d*e^2*x + (
c^4 + c^2)*e^2), x))/(d^2*e^2*x + c*d*e^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)^3}{\left(c\,e+d\,e\,x\right)^2}\,dx$$

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^2, x)
```

3.19 $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 180

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx &= -\frac{3ib(a+b\arctan(c+dx))^2}{2de^3}-\frac{3b(a+b\arctan(c+dx))^2}{2de^3(c+dx)}\\ &\quad-\frac{(a+b\arctan(c+dx))^3}{2de^3}-\frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2}\\ &\quad+\frac{3b^2(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^3}\\ &\quad-\frac{3ib^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{2de^3}\end{aligned}$$

```
[Out] -3/2*I*b*(a+b*arctan(d*x+c))^2/d/e^3-3/2*b*(a+b*arctan(d*x+c))^2/d/e^3/(d*x
+c)-1/2*(a+b*arctan(d*x+c))^3/d/e^3-1/2*(a+b*arctan(d*x+c))^3/d/e^3/(d*x+c)
^2+3*b^2*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^3-3/2*I*b^3*polylog(
2,-1+2/(1-I*(d*x+c)))/d/e^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

= {5151, 12, 4946, 5038, 5044, 4988, 2497, 5004}

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = {} & \frac{3b^2\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{de^3} \\ & - \frac{3b(a+b\arctan(c+dx))^2}{2de^3(c+dx)} - \frac{3ib(a+b\arctan(c+dx))^2}{2de^3} \\ & - \frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2} - \frac{(a+b\arctan(c+dx))^3}{2de^3} \\ & - \frac{3ib^3\,\mathrm{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)}{2de^3}\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c + d*x])^2)/(d*e^3) - (3*b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcTan[c + d*x])^3/(2*d*e^3) - (a + b*
ArcTan[c + d*x])^3/(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcTan[c + d*x])*L
og[2 - 2/(1 - I*(c + d*x))])/(d*e^3) - (((3*I)/2)*b^3*PolyLog[2, -1 + 2/(1
- I*(c + d*x))])/(d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{e^3x^3}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{x^3}\,dx, x, c+dx\right)}{de^3}\\
&= -\frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2}+\frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x^2(1+x^2)}\,dx, x, c+dx\right)}{2de^3}\\
&= -\frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2}+\frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x^2}\,dx, x, c+dx\right)}{2de^3}\\
&\quad -\frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{1+x^2}\,dx, x, c+dx\right)}{2de^3}
\end{aligned}
$$

$$= -\frac{3b(a+b\arctan(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\arctan(c+dx))^3}{2de^3} - \frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{a+b\arctan(x)}{x(1+x^2)}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{3ib(a+b\arctan(c+dx))^2}{2de^3} - \frac{3b(a+b\arctan(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\arctan(c+dx))^3}{2de^3} - \frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2} + \frac{(3ib^2)\,\text{Subst}\left(\int \frac{a+b\arctan(x)}{x(i+x)}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{3ib(a+b\arctan(c+dx))^2}{2de^3} - \frac{3b(a+b\arctan(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\arctan(c+dx))^3}{2de^3} - \frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2} + \frac{3b^2(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^3} - \frac{(3b^3)\,\text{Subst}\left(\int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{3ib(a+b\arctan(c+dx))^2}{2de^3} - \frac{3b(a+b\arctan(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\arctan(c+dx))^3}{2de^3} - \frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2} + \frac{3b^2(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^3} - \frac{3ib^3\,\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{2de^3}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx =$$

$$-\frac{a^3+b^3(1+c^2+2cdx+d^2x^2)\arctan(c+dx)^3+3a^2b(c+dx+(1+(c+dx)^2)\arctan(c+dx))+3ab^2\left(\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a^3 + b^3*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*a^2*b*(
c + d*x + (1 + (c + d*x)^2)*ArcTan[c + d*x]) + 3*a*b^2*(2*(c + d*x)*ArcTan[
c + d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 2*(c + d*x)^2*Log[(c + d*x
)/Sqrt[1 + (c + d*x)^2]]) + 3*b^3*(c + d*x)*(ArcTan[c + d*x]^2 - 2*(c + d*x
)*ArcTan[c + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x])] + I*(c + d*x)*(ArcTan[
c + d*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c + d*x])])))/(d*e^3*(c + d*x)^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(166) = 332.

Time = 1.42 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.35

method	result
derivativedivides	$-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\arctan(dx+c)^3}{2(dx+c)^2}-\frac{3\arctan(dx+c)^2}{2(dx+c)}-\frac{\arctan(dx+c)^3}{2}+3\ln(dx+c)\arctan(dx+c)-\frac{3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2}\right.}{}$
default	$-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\arctan(dx+c)^3}{2(dx+c)^2}-\frac{3\arctan(dx+c)^2}{2(dx+c)}-\frac{\arctan(dx+c)^3}{2}+3\ln(dx+c)\arctan(dx+c)-\frac{3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2}\right.}{}$
parts	$-\frac{a^3}{2e^3(dx+c)^2d}+\frac{b^3\left(-\frac{\arctan(dx+c)^3}{2(dx+c)^2}-\frac{3\arctan(dx+c)^2}{2(dx+c)}-\frac{\arctan(dx+c)^3}{2}+3\ln(dx+c)\arctan(dx+c)-\frac{3\arctan(dx+c)\ln\left(1\right.}{2}\right.}{}$

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^3/e^3/(d*x+c)^2+b^3/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)^3-3/2/(d*
x+c)*arctan(d*x+c)^2-1/2*arctan(d*x+c)^3+3*ln(d*x+c)*arctan(d*x+c)-3/2*arct
an(d*x+c)*ln(1+(d*x+c)^2)-3/4*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I
)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+3/4*I*(ln(d*x
+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*
ln(1/2*I*(d*x+c-I)))+3/2*I*ln(d*x+c)*ln(1+I*(d*x+c))-3/2*I*ln(d*x+c)*ln(1-I
*(d*x+c))+3/2*I*dilog(1+I*(d*x+c))-3/2*I*dilog(1-I*(d*x+c)))+3*a*b^2/e^3*(-
1/2/(d*x+c)^2*arctan(d*x+c)^2-1/(d*x+c)*arctan(d*x+c)-1/2*arctan(d*x+c)^2+l
n(d*x+c)-1/2*ln(1+(d*x+c)^2))+3*a^2*b/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)-1/2
/(d*x+c)-1/2*arctan(d*x+c)))
```

Fricas [F]

$$\int\frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx=\int\frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(dex+ce\right)^3}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)
, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx$$
$$= \frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\operatorname{atan}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*atan(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(3*a*b**2*atan(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3), x) + Integral(3*a**2*b*atan(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3), x))/e**3
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + a
rctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a^2*b - 3/2*(2*d*
(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3))*arctan(d*x
+ c) - (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x +
c))/(d*e^3))*a*b^2 - 3/2*a*b^2*arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3
*x + c^2*d*e^3) - 1/32*(8*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^3 +
 12*(d*x + c)*arctan(d*x + c)^2 - 3*(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2 - 32*(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)*integrate(1/32*(16*(d^
2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^3 + 12*(d^3*x^3 + 3*c*d^2*x^2 +
c^3 + (3*c^2 + 1)*d*x + c)*arctan(d*x + c)^2 + 3*(d^3*x^3 + 3*c*d^2*x^2 + c
^3 + (3*c^2 + 1)*d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 24*(d^2*x^2
+ 2*c*d*x + c^2)*arctan(d*x + c) - 12*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + (10
*c^2 + 1)*d^3*e^3*x^3 + (10*c^3 + 3*c)*d^2*e^3*x^2 + (5*c^4 + 3*c^2)*d*e^3*
x + (c^5 + c^3)*e^3), x))*b^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1
/2*a^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{(c\,e+d\,e\,x)^3}\,dx$$

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^3, x)
```

3.20 $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = & -\frac{b^2(a+b\arctan(c+dx))}{de^4(c+dx)}-\frac{b(a+b\arctan(c+dx))^2}{2de^4}\\ & -\frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2}+\frac{i(a+b\arctan(c+dx))^3}{3de^4}\\ & -\frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3}\\ & +\frac{b^3\log(c+dx)}{de^4}-\frac{b^3\log\left(1+(c+dx)^2\right)}{2de^4}\\ & -\frac{b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^4}\\ & +\frac{ib^2(a+b\arctan(c+dx))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^4}\\ & -\frac{b^3\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^4}\end{aligned}$$

```
[Out] -b^2*(a+b*arctan(d*x+c))/d/e^4/(d*x+c)-1/2*b*(a+b*arctan(d*x+c))^2/d/e^4-1/
2*b*(a+b*arctan(d*x+c))^2/d/e^4/(d*x+c)^2+1/3*I*(a+b*arctan(d*x+c))^3/d/e^4
-1/3*(a+b*arctan(d*x+c))^3/d/e^4/(d*x+c)^3+b^3*ln(d*x+c)/d/e^4-1/2*b^3*ln(1
+(d*x+c)^2)/d/e^4-b*(a+b*arctan(d*x+c))^2*ln(2-2/(1-I*(d*x+c)))/d/e^4+I*b^2
*(a+b*arctan(d*x+c))*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^4-1/2*b^3*polylog(3,
-1+2/(1-I*(d*x+c)))/d/e^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {5151, 12, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$
\begin{aligned}
\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = & \frac{ib^2 \operatorname{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)(a+b\arctan(c+dx))}{de^4} \\
& - \frac{b^2(a+b\arctan(c+dx))}{de^4(c+dx)} \\
& - \frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2} - \frac{b(a+b\arctan(c+dx))^2}{2de^4} \\
& - \frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3} + \frac{i(a+b\arctan(c+dx))^3}{3de^4} \\
& - \frac{b\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))^2}{de^4} \\
& - \frac{b^3 \operatorname{PolyLog}\left(3,\frac{2}{1-i(c+dx)}-1\right)}{2de^4} \\
& + \frac{b^3\log(c+dx)}{de^4} - \frac{b^3\log\left((c+dx)^2+1\right)}{2de^4}
\end{aligned}
$$

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -((b^2*(a + b*ArcTan[c + d*x]))/(d*e^4*(c + d*x))) - (b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) + (
(I/3)*(a + b*ArcTan[c + d*x])^3)/(d*e^4) - (a + b*ArcTan[c + d*x])^3/(3*d*e
^4*(c + d*x)^3) + (b^3*Log[c + d*x])/(d*e^4) - (b^3*Log[1 + (c + d*x)^2])/(
2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^
4) + (I*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(
d*e^4) - (b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{e^4x^4}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{x^4}\,dx, x, c+dx\right)}{de^4}\\
&= -\frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x^3(1+x^2)}\,dx, x, c+dx\right)}{de^4}\\
&= -\frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x^3}\,dx, x, c+dx\right)}{de^4}\\
&\quad - \frac{b\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x(1+x^2)}\,dx, x, c+dx\right)}{de^4}\\
&= -\frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2} + \frac{i(a+b\arctan(c+dx))^3}{3de^4} - \frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3}\\
&\quad - \frac{(ib)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x(i+x)}\,dx, x, c+dx\right)}{de^4} + \frac{b^2\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^2(1+x^2)}\,dx, x, c+dx\right)}{de^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2} + \frac{i(a+b\arctan(c+dx))^3}{3de^4} \\
&\quad - \frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3} - \frac{b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad + \frac{b^2\text{Subst}\left(\int \frac{a+b\arctan(x)}{x^2}\,dx, x, c+dx\right)}{de^4} - \frac{b^2\text{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx, x, c+dx\right)}{de^4} \\
&\quad + \frac{(2b^2)\,\text{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{b^2(a+b\arctan(c+dx))}{de^4(c+dx)} - \frac{b(a+b\arctan(c+dx))^2}{2de^4} \\
&\quad - \frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2} + \frac{i(a+b\arctan(c+dx))^3}{3de^4} \\
&\quad - \frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3} - \frac{b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad + \frac{ib^2(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad - \frac{(ib^3)\,\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{de^4} \\
&\quad + \frac{b^3\text{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx, x, c+dx\right)}{de^4} \\
&= -\frac{b^2(a+b\arctan(c+dx))}{de^4(c+dx)} - \frac{b(a+b\arctan(c+dx))^2}{2de^4} \\
&\quad - \frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2} + \frac{i(a+b\arctan(c+dx))^3}{3de^4} \\
&\quad - \frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3} - \frac{b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad + \frac{ib^2(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^3\,\text{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^4} + \frac{b^3\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, (c+dx)^2\right)}{2de^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2(a+b\arctan(c+dx))}{de^4(c+dx)} - \frac{b(a+b\arctan(c+dx))^2}{2de^4} \\
&\quad - \frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2} + \frac{i(a+b\arctan(c+dx))^3}{3de^4} \\
&\quad - \frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3} - \frac{b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad + \frac{ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^4} + \frac{b^3\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, (c+dx)^2\right)}{2de^4} \\
&\quad - \frac{b^3\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, (c+dx)^2\right)}{2de^4} \\
&= -\frac{b^2(a+b\arctan(c+dx))}{de^4(c+dx)} - \frac{b(a+b\arctan(c+dx))^2}{2de^4} - \frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2} \\
&\quad + \frac{i(a+b\arctan(c+dx))^3}{3de^4} - \frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3} + \frac{b^3\log(c+dx)}{de^4} \\
&\quad - \frac{b^3\log\left(1+(c+dx)^2\right)}{2de^4} - \frac{b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad + \frac{ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^4} \\
&\quad - \frac{b^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.29

$$
\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx
$$

$$
= \frac{-\frac{2a^3}{(c+dx)^3} - \frac{3a^2b}{(c+dx)^2} - \frac{6a^2b\arctan(c+dx)}{(c+dx)^3} - 6a^2b\log(c+dx) + 3a^2b\log\left(1+c^2+2cdx+d^2x^2\right) + 6ab^2\Big(-\frac{(c+dx)^2}{}}{}
$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*a^3)/(c + d*x)^3 - (3*a^2*b)/(c + d*x)^2 - (6*a^2*b*ArcTan[c + d*x])/(
c + d*x)^3 - 6*a^2*b*Log[c + d*x] + 3*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2*x^2
] + 6*a*b^2*(-(((c + d*x)^2 + ArcTan[c + d*x]^2)/(c + d*x)^3) + ArcTan[c +
d*x]*(-1 - (c + d*x)^(-2) + I*ArcTan[c + d*x] - 2*Log[1 - E^((2*I)*ArcTan[c
 + d*x])]) + I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + 6*b^3*((I/24)*Pi^3
```

```
- ArcTan[c + d*x]/(c + d*x) - ArcTan[c + d*x]^2/2 - ArcTan[c + d*x]^2/(2*(c
 + d*x)^2) - (I/3)*ArcTan[c + d*x]^3 - ArcTan[c + d*x]^3/(3*(c + d*x)^3) -
ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] + Log[c + d*x] + Log[
1/Sqrt[1 + (c + d*x)^2]] - I*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)*ArcTan[c
+ d*x])] - PolyLog[3, E^((-2*I)*ArcTan[c + d*x])]/2))/(6*d*e^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.42 (sec) , antiderivative size = 2465, normalized size of antiderivative = 8.59

method	result	size
derivativedivides	Expression too large to display	2465
default	Expression too large to display	2465
parts	Expression too large to display	2473

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3/e^4/(d*x+c)^3+b^3/e^4*(-1/3/(d*x+c)^3*arctan(d*x+c)^3-1/2/(d*
x+c)^2*arctan(d*x+c)^2-ln(d*x+c)*arctan(d*x+c)^2+1/2*arctan(d*x+c)^2*ln(1+(
d*x+c)^2)-arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+arctan(d*x+
c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)-arctan(d*x+c)^2*ln(1+(1+I*(d*x+c))
/(1+(d*x+c)^2)^(1/2))+2*I*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)
^2)^(1/2))-2*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-arctan(d*x+c)^2*
ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+2*I*arctan(d*x+c)*polylog(2,(1+I*(d
*x+c))/(1+(d*x+c)^2)^(1/2))-2*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+
1/12*arctan(d*x+c)*(6*I*Pi*arctan(d*x+c)*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+
c)^2))^2)^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*(d*x+c)+4*I*arctan(d*
x+c)^2*(d*x+c)-3*I*Pi*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*c
sgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2*
(d*x+c)-3*I*Pi*arctan(d*x+c)*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*cs
gn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*(d*x+c)-6*I*Pi*arctan(d*x+c)*csgn
(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csg
n(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*(d*x
+c)-6*I*Pi*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2*csgn(I*(1+
I*(d*x+c))/(1+(d*x+c)^2)^(1/2))*(d*x+c)-6*I*Pi*arctan(d*x+c)*(d*x+c)+3*I*Pi
*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))/(
1+(d*x+c)^2)^(1/2))^2*(d*x+c)+3*I*Pi*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(
1+(d*x+c)^2))^3*(d*x+c)+6*I*Pi*arctan(d*x+c)*csgn(((1+I*(d*x+c))^2/(1+(d*x+
c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*(d*x+c)+6*I*Pi*arctan(d*x+c)*
csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))
^2*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*(d*x+c)+3*I*Pi*arctan(d*x+c)*c
sgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3*
(d*x+c)-6*I*Pi*arctan(d*x+c)*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I
```

```
*(d*x+c))^2/(1+(d*x+c)^2)))^3*(d*x+c)-12*I*(d*x+c)-6*I*Pi*arctan(d*x+c)*csg
n(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3*
(d*x+c)-3*I*Pi*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(
d*x+c))^2/(1+(d*x+c)^2))^2)^2*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*(
d*x+c)+6*I*Pi*arctan(d*x+c)*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+
I*(d*x+c))^2/(1+(d*x+c)^2)))^2*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*(d
*x+c)+3*I*Pi*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+
I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I/(1+(
1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*(d*x+c)+6*I*Pi*arctan(d*x+c)*csgn(I*((1+I*
(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(
d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*(d*x+c)-6*I
*Pi*arctan(d*x+c)*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))
^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/(1+(1+I
*(d*x+c))^2/(1+(d*x+c)^2)))*(d*x+c)-12*ln(2)*arctan(d*x+c)*(d*x+c)-12-3*I*P
i*arctan(d*x+c)*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3*(d*x+c)-6*(d*
x+c)*arctan(d*x+c))/(d*x+c)+ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2)-1)+ln(1+(1
+I*(d*x+c))/(1+(d*x+c)^2)^(1/2)))+3*a*b^2/e^4*(-1/3/(d*x+c)^3*arctan(d*x+c)
^2-1/3/(d*x+c)^2*arctan(d*x+c)-2/3*ln(d*x+c)*arctan(d*x+c)+1/3*arctan(d*x+c
)*ln(1+(d*x+c)^2)+1/6*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilo
g(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))-1/6*I*(ln(d*x+c+I)*ln
(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I
*(d*x+c-I)))-1/3/(d*x+c)-1/3*arctan(d*x+c)-1/3*I*ln(d*x+c)*ln(1+I*(d*x+c))+
1/3*I*ln(d*x+c)*ln(1-I*(d*x+c))-1/3*I*dilog(1+I*(d*x+c))+1/3*I*dilog(1-I*(d
*x+c)))+3*a^2*b/e^4*(-1/3/(d*x+c)^3*arctan(d*x+c)-1/6/(d*x+c)^2-1/3*ln(d*x+
c)+1/6*ln(1+(d*x+c)^2)))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(dex+ce\right)^4}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^
3*d*e^4*x + c^4*e^4), x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx$$
$$= \frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int}{e^4}$$

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*atan(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**
2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atan(c + d*x)**
2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) +
Integral(3*a**2*b*atan(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c
*d**3*x**3 + d**4*x**4), x))/e**4
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(1/(d^4*e^4*x^2 + 2*c*d^3*e^4*x + c^2*d^2*e^4) - log(d^2*x^2 + 2*c*
d*x + c^2 + 1)/(d^2*e^4) + 2*log(d*x + c)/(d^2*e^4)) + 2*arctan(d*x + c)/(d
^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4))*a^2*b - 1/3*a^
3/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/96*(4*b
^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
1)^2 - 96*(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4)*int
egrate(1/32*(28*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c)
^3 + 4*(24*a*b^2*d^2*x^2 + 24*a*b^2*c^2 + b^3*c + 24*a*b^2 + (48*a*b^2*c +
b^3)*d*x)*arctan(d*x + c)^2 - 4*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - (b^3*d*x + b^3*c - 3*(b^3*d^2
*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2)/(d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + (15*c^2 + 1)*d^4*e^4*x^4 + 4
*(5*c^3 + c)*d^3*e^4*x^3 + 3*(5*c^4 + 2*c^2)*d^2*e^4*x^2 + 2*(3*c^5 + 2*c^3
)*d*e^4*x + (c^6 + c^4)*e^4), x))/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^
2*e^4*x + c^3*d*e^4)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{(c\,e+d\,e\,x)^4}\,dx$$

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^4,x)
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^4, x)
```

3.21 $\int \frac{\arctan(1+x)}{2+2x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{\arctan(1+x)}{2+2x} \, dx = \frac{1}{4} i \operatorname{PolyLog}(2,-i(1+x)) - \frac{1}{4} i \operatorname{PolyLog}(2,i(1+x))$$

```
[Out] 1/4*I*polylog(2,-I*(1+x))-1/4*I*polylog(2,I*(1+x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5151, 12, 4940, 2438}

$$\int \frac{\arctan(1+x)}{2+2x} \, dx = \frac{1}{4} i \operatorname{PolyLog}(2,-i(x+1)) - \frac{1}{4} i \operatorname{PolyLog}(2,i(x+1))$$

```
[In] Int[ArcTan[1 + x]/(2 + 2*x),x]
```

```
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\arctan(x)}{2x}\,dx, x, 1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\arctan(x)}{x}\,dx, x, 1+x\right)\\
&= \frac{1}{4}i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, 1+x\right) - \frac{1}{4}i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, 1+x\right)\\
&= \frac{1}{4}i\,\text{PolyLog}(2, -i(1+x)) - \frac{1}{4}i\,\text{PolyLog}(2, i(1+x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(1+x)}{2+2x}\,dx = \frac{1}{4}i\,\text{PolyLog}(2, -i(1+x)) - \frac{1}{4}i\,\text{PolyLog}(2, i(1+x))$$

```
[In] Integrate[ArcTan[1 + x]/(2 + 2*x),x]
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{i\,\mathrm{dilog}(-ix-i+1)}{4}+\frac{i\,\mathrm{dilog}(ix+i+1)}{4}$
derivativedivides	$\frac{\ln(1+x)\arctan(1+x)}{2}+\frac{i\ln(1+x)\ln(1+i(1+x))}{4}-\frac{i\ln(1+x)\ln(1-i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1-i(1+x}{4}$
default	$\frac{\ln(1+x)\arctan(1+x)}{2}+\frac{i\ln(1+x)\ln(1+i(1+x))}{4}-\frac{i\ln(1+x)\ln(1-i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1-i(1+x}{4}$
parts	$\frac{\ln(1+x)\arctan(1+x)}{2}+\frac{i\ln(1+x)\ln(1+i(1+x))}{4}-\frac{i\ln(1+x)\ln(1-i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1-i(1+x}{4}$

```
[In] int(arctan(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
[Out] -1/4*I*dilog(-I*x+1-I)+1/4*I*dilog(I*x+1+I)
```

Fricas [F]

$$\int \frac{\arctan(1+x)}{2+2x}\,dx=\int \frac{\arctan{(x+1)}}{2\,(x+1)}\,dx$$

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="fricas")
[Out] integral(1/2*arctan(x + 1)/(x + 1), x)
```

Sympy [F]

$$\int \frac{\arctan(1+x)}{2+2x}\,dx=\frac{\int \frac{\operatorname{atan}{(x+1)}}{x+1}\,dx}{2}$$

```
[In] integrate(atan(1+x)/(2+2*x),x)
[Out] Integral(atan(x + 1)/(x + 1), x)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17)=34$.

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\arctan(1+x)}{2+2x}\,dx=-\frac{1}{4}\arctan{(x+1,0)}\log{\left(x^2+2\,x+2\right)}+\frac{1}{2}\arctan{(x+1)}\log{(|x+1|)}-\frac{1}{4}i\,\mathrm{Li}_2(i\,x+i+1)+\frac{1}{4}i\,\mathrm{Li}_2(-i\,x-i+1)$$

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="maxima")
[Out] -1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arctan(x + 1)*log(abs(x + 1
)) - 1/4*I*dilog(I*x + I + 1) + 1/4*I*dilog(-I*x - I + 1)
```

Giac [F]

$$\int \frac{\arctan(1+x)}{2+2x}\,dx=\int \frac{\arctan\left(x+1\right)}{2\left(x+1\right)}\,dx$$

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(1+x)}{2+2x}\,dx=-\frac{\mathrm{Li}_2(1-x\,1\mathrm{i}-\mathrm{i})\ 1\mathrm{i}}{4}+\frac{\mathrm{Li}_2(x\,1\mathrm{i}+1+1\mathrm{i})\ 1\mathrm{i}}{4}$$

```
[In] int(atan(x + 1)/(2*x + 2),x)
```

```
[Out] (dilog(x*1i + (1 + 1i))*1i)/4 - (dilog((1 - 1i) - x*1i)*1i)/4
```

3.22 $\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{i\,\text{PolyLog}(2,-i(a+bx))}{2d} - \frac{i\,\text{PolyLog}(2,i(a+bx))}{2d}$$

```
[Out] 1/2*I*polylog(2,-I*(b*x+a))/d-1/2*I*polylog(2,I*(b*x+a))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5151, 12, 4940, 2438}

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{i\,\text{PolyLog}(2,-i(a+bx))}{2d} - \frac{i\,\text{PolyLog}(2,i(a+bx))}{2d}$$

```
[In] Int[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*(a + b*x)])/d - ((I/2)*PolyLog[2, I*(a + b*x)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{b\arctan(x)}{dx}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\arctan(x)}{x}\,dx, x, a+bx\right)}{d}\\ &= \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, a+bx\right)}{2d} - \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, a+bx\right)}{2d}\\ &= \frac{i\,\text{PolyLog}(2,-i(a+bx))}{2d} - \frac{i\,\text{PolyLog}(2,i(a+bx))}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{i(\text{PolyLog}(2,-i(a+bx)) - \text{PolyLog}(2,i(a+bx)))}{2d}$$

```
[In] Integrate[ArcTan[a + b*x]/((a*d)/b + d*x),x]
[Out] ((I/2)*(PolyLog[2, (-I)*(a + b*x)] - PolyLog[2, I*(a + b*x)]))/d
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	s
risch	$-\frac{i \operatorname{dilog}(-ibx-ia+1)}{2d}+\frac{i \operatorname{dilog}(ibx+ia+1)}{2d}$	3
parts	$\frac{\ln(bx+a)\arctan(bx+a)}{d}-\frac{-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\operatorname{dilog}(1+i(bx+a))}{2}+\frac{i\operatorname{dilog}(1-i(bx+a))}{2}}{d}$	9
derivativedivides	$\frac{\frac{b\ln(bx+a)\arctan(bx+a)}{d}-\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\operatorname{dilog}(1+i(bx+a))}{2}+\frac{i\operatorname{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$	9
default	$\frac{\frac{b\ln(bx+a)\arctan(bx+a)}{d}-\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\operatorname{dilog}(1+i(bx+a))}{2}+\frac{i\operatorname{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$	9

```
[In] int(arctan(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
[Out] -1/2*I/d*dilog(1-I*a-I*b*x)+1/2*I/d*dilog(1+I*a+I*b*x)
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\arctan(bx+a)}{dx+\frac{ad}{b}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
[Out] integral(b*arctan(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\operatorname{atan}(a+bx)}{a+bx}\,dx}{d}$$

```
[In] integrate(atan(b*x+a)/(a*d/b+d*x),x)
[Out] b*Integral(atan(a + b*x)/(a + b*x), x)/d
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(29) = 58.

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.00

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\arctan(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d} - \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d} - \frac{\arctan(bx+a,0)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,\arctan(bx+a)\log\left(|bx+a|\right)+i\,\mathrm{Li}_2(i\,bx+i\,a+1)-}{2\,d}$$

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] arctan(b*x + a)*log(d*x + a*d/b)/d - arctan((b^2*x + a*b)/b)*log(d*x + a*d/
b)/d - 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan
(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I
*a + 1))/d
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\arctan(bx+a)}{dx+\frac{ad}{b}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

```
[In] int(atan(a + b*x)/(d*x + (a*d)/b),x)
```

```
[Out] int(atan(a + b*x)/(d*x + (a*d)/b), x)
```

3.23 $\int (a+bx)^2 \sqrt{\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (a+bx)^2 \sqrt{\arctan(a+bx)}\,dx = \text{Int}\left((a+bx)^2 \sqrt{\arctan(a+bx)},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+bx)^2 \sqrt{\arctan(a+bx)}\,dx = \int (a+bx)^2 \sqrt{\arctan(a+bx)}\,dx$$

```
[In] Int[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
```

```
[Out] Defer[Int][(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

Rubi steps

$$\text{integral} = \int (a+bx)^2 \sqrt{\arctan(a+bx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx=\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx$$

```
[In] Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
[Out] Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (bx+a)^2\sqrt{\arctan(bx+a)}dx$$

```
[In] int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
[Out] int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx=\int (a+bx)^2\sqrt{\operatorname{atan}(a+bx)}\,dx$$

```
[In] integrate((b*x+a)**2*atan(b*x+a)**(1/2),x)
[Out] Integral((a + b*x)**2*sqrt(atan(a + b*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 127.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \int (bx+a)^2\sqrt{\arctan(bx+a)}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \int \sqrt{\mathrm{atan}\left(a+b\,x\right)}\,{\left(a+b\,x\right)}^2\,dx$$

```
[In] int(atan(a + b*x)^(1/2)*(a + b*x)^2,x)
[Out] int(atan(a + b*x)^(1/2)*(a + b*x)^2, x)
```

3.24 $\int (e+fx)^3(a+b\arctan(c+dx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 233

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx$$
$$= -\frac{bf(6d^2e^2-12cdef-(1-6c^2)f^2)x}{4d^3}-\frac{bf^2(de-cf)(c+dx)^2}{2d^4}-\frac{bf^3(c+dx)^3}{12d^4}$$
$$-\frac{b(d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+(1-6c^2+c^4)f^4)\arctan(c+dx)}{4d^4f}$$
$$+\frac{(e+fx)^4(a+b\arctan(c+dx))}{4f}$$
$$-\frac{b(de-cf)(de+f-cf)(de-(1+c)f)\log(1+(c+dx)^2)}{2d^4}$$

```
[Out] -1/4*b*f*(6*d^2*e^2-12*c*d*e*f-(-6*c^2+1)*f^2)*x/d^3-1/2*b*f^2*(-c*f+d*e)*(
d*x+c)^2/d^4-1/12*b*f^3*(d*x+c)^3/d^4-1/4*b*(d^4*e^4-4*c*d^3*e^3*f-6*(-c^2+
1)*d^2*e^2*f^2+4*c*(-c^2+3)*d*e*f^3+(c^4-6*c^2+1)*f^4)*arctan(d*x+c)/d^4/f+
1/4*(f*x+e)^4*(a+b*arctan(d*x+c))/f-1/2*b*(-c*f+d*e)*(-c*f+d*e+f)*(d*e-(1+c
)*f)*ln(1+(d*x+c)^2)/d^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{5155, 4972, 716, 649, 209, 266\}$

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \frac{(e+fx)^4(a+b\arctan(c+dx))}{4f}$$
$$-\frac{b\arctan(c+dx)\left(-6(1-c^2)\,d^2e^2f^2+4c(3-c^2)\,def^3+\left(c^4-6c^2+1\right)f^4-4cd^3e^3f+d^4e^4\right)}{4d^4f}$$
$$-\frac{bfx(-(1-6c^2)\,f^2-12cdef+6d^2e^2)}{4d^3}-\frac{bf^2(c+dx)^2(de-cf)}{2d^4}$$
$$-\frac{b(de-cf)(-cf+de+f)(de-(c+1)f)\log\left((c+dx)^2+1\right)}{2d^4}-\frac{bf^3(c+dx)^3}{12d^4}$$

```
[In] Int[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -1/4*(b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/d^3 - (b*f^2*(d*e -
 c*f)*(c + d*x)^2)/(2*d^4) - (b*f^3*(c + d*x)^3)/(12*d^4) - (b*(d^4*e^4 - 4
*c*d^3*e^3*f - 6*(1 - c^2)*d^2*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2
 + c^4)*f^4)*ArcTan[c + d*x])/(4*d^4*f) + ((e + f*x)^4*(a + b*ArcTan[c + d*
x]))/(4*f) - (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c +
d*x)^2])/(2*d^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
```

```
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\Big(\int \big(\frac{de-cf}{d}+\frac{fx}{d}\big)^3 \,(a+b\arctan(x))\,dx, x, c+dx\Big)}{d} \\
&= \frac{(e+fx)^4(a+b\arctan(c+dx))}{4f} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1+x^2}\,dx, x, c+dx\right)}{4f} \\
&= \frac{(e+fx)^4(a+b\arctan(c+dx))}{4f} \\
&\quad - \frac{b\text{Subst}\Big(\int \Big(\frac{f^2(6d^2e^2-12cdef-(1-6c^2)f^2)}{d^4}+\frac{4f^3(de-cf)x}{d^4}+\frac{f^4x^2}{d^4}+\frac{d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+}{d^4(}}{4f} \\
&= -\frac{bf(6d^2e^2-12cdef-(1-6c^2)f^2)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} \\
&\quad - \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4(a+b\arctan(c+dx))}{4f} \\
&\quad - \frac{b\text{Subst}\Big(\int \frac{d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+(1-6c^2+c^4)f^4+4f(de-cf)(de-f-cf)(de+f-cf)x}{1+x^2}\,dx, x, c+}{4d^4f} \\
&= -\frac{bf(6d^2e^2-12cdef-(1-6c^2)f^2)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} \\
&\quad - \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4(a+b\arctan(c+dx))}{4f} \\
&\quad - \frac{(b(de-cf)(de+f-cf)(de-(1+c)f))\text{Subst}\big(\int \frac{x}{1+x^2}\,dx, x, c+dx\big)}{d^4} \\
&\quad - \frac{(b(d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+(1-6c^2+c^4)f^4))\,\text{Subst}\big(\int \frac{1}{1+x^2}\,dx,}{4d^4f}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bf(6d^2e^2-12cdef-(1-6c^2)\,f^2)\,x}{4d^3}-\frac{bf^2(de-cf)(c+dx)^2}{2d^4}-\frac{bf^3(c+dx)^3}{12d^4} \\
&\quad -\frac{b(d^4e^4-4cd^3e^3f-6(1-c^2)\,d^2e^2f^2+4c(3-c^2)\,def^3+(1-6c^2+c^4)\,f^4)\arctan(c+dx)}{4d^4f} \\
&\quad +\frac{(e+fx)^4(a+b\arctan(c+dx))}{4f} \\
&\quad -\frac{b(de-cf)(de+f-cf)(de-(1+c)f)\log\left(1+(c+dx)^2\right)}{2d^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.67

$$
\int (e+fx)^3(a+b\arctan(c+dx))\,dx
$$

$$
= \frac{(e+fx)^4(a+b\arctan(c+dx)) - \frac{b(6df^2(6d^2e^2-12cdef+(-1+6c^2)f^2)x+12f^3(de-cf)(c+dx)^2+2f^4(c+dx)^3-3i(de-(-i+c)f)^4\,\mathrm{lo}}{6d^4}}{4f}
$$

```
[In] Integrate[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^4*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
+ (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^3
- (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^4*L
og[I + c + d*x]))/(6*d^4))/(4*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(221) = 442$.

Time = 0.25 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.99

method	result
parts	$\frac{5bf^2c^2e}{2d^3} - \frac{3bfce^2}{2d^2} + \frac{bf^2\ln\left(1+(dx+c)^2\right)e}{2d^3} + \frac{bf^3\ln\left(1+(dx+c)^2\right)c^3}{2d^4} - \frac{bf^3\ln\left(1+(dx+c)^2\right)c}{2d^4} + bf^2\,\mathrm{arctan}$
derivativedivides	$\frac{a(cf-de-f(dx+c))^4}{4d^3f} - b\Big(-\frac{f^3\arctan(dx+c)c^4}{4} + f^2\arctan(dx+c)c^3de + f^3\arctan(dx+c)c^3(dx+c) - \frac{3f\arctan(dx+c)c^2d^2e^2}{2} - 3f^2\,\mathrm{arc}$
default	$\frac{a(cf-de-f(dx+c))^4}{4d^3f} - b\Big(-\frac{f^3\arctan(dx+c)c^4}{4} + f^2\arctan(dx+c)c^3de + f^3\arctan(dx+c)c^3(dx+c) - \frac{3f\arctan(dx+c)c^2d^2e^2}{2} - 3f^2\,\mathrm{arc}$
parallelrisch	$-42bc^2def^2+36bcd^2e^2f+3xbdf^3-x^3bd^3f^3+6\ln\left(d^2x^2+2cdx+c^2+1\right)bc^3f^3-6\ln\left(d^2x^2+2cdx+c^2+1\right)bd^3e^3-6\ln\big(d^2$
risch	$\frac{x^4f^3a}{4} + xae^3 - \frac{i(fx+e)^4b\ln(1+i(dx+c))}{8f} + \frac{if^3bx^4\ln(1-i(dx+c))}{8} + \frac{ibe^3x\ln(1-i(dx+c))}{2} + \frac{ibe^4\ln\big(d^2x^2+}{16}$

```
[In] int((f*x+e)^3*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*b/d^3*f^2*c^2*e-3/2*b/d^2*f*c*e^2+1/2*b/d^3*f^2*ln(1+(d*x+c)^2)*e+1/2*b
/d^4*f^3*ln(1+(d*x+c)^2)*c^3-1/2*b/d^4*f^3*ln(1+(d*x+c)^2)*c+b*f^2*arctan(d
*x+c)*e*x^3+3/2*b*f*arctan(d*x+c)*e^2*x^2+b*arctan(d*x+c)*x*e^3-1/2*b/d*ln(
1+(d*x+c)^2)*e^3+1/4*b/d^4*f^3*c-13/12*b/d^4*f^3*c^3+1/4*b*f^3*arctan(d*x+c
)*x^4+3/2*b/d^2*f*ln(1+(d*x+c)^2)*c*e^2-3/2*b/d^3*f^2*ln(1+(d*x+c)^2)*c^2*e
-1/12/d*f^3*b*x^3+1/4/d^3*f^3*b*x-1/4/d^4*f^3*b*arctan(d*x+c)+1/d^3*f^2*b*c
^3*e*arctan(d*x+c)-3/d^3*f^2*b*c*e*arctan(d*x+c)-3/2/d^2*f*b*c^2*e^2*arctan
(d*x+c)+2/d^2*f^2*b*c*e*x+1/4*a*(f*x+e)^4/f+1/4/d^2*f^3*b*c*x^2-1/2/d*f^2*b
*e*x^2-3/4/d^3*f^3*b*c^2*x-3/2/d*f*b*e^2*x+1/d*b*c*e^3*arctan(d*x+c)-1/4/d^
4*f^3*b*c^4*arctan(d*x+c)+3/2/d^4*f^3*b*c^2*arctan(d*x+c)+3/2/d^2*f*b*e^2*a
rctan(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.36

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = 3ad^4f^3x^4 + (12ad^4ef^2 - bd^3f^3)x^3 + 3(6ad^4e^2f - 2bd^3ef^2 + bcd^2f^3)x^2 + 3(4ad^4e^3 - 6bd^3e^2f + 8bcd^2$$

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*f^3*x^4 + (12*a*d^4*e*f^2 - b*d^3*f^3)*x^3 + 3*(6*a*d^4*e^2*f
 - 2*b*d^3*e*f^2 + b*c*d^2*f^3)*x^2 + 3*(4*a*d^4*e^3 - 6*b*d^3*e^2*f + 8*b*
c*d^2*e*f^2 - (3*b*c^2 - b)*d*f^3)*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3
 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x + 4*b*c*d^3*e^3 - 6*(b*c^2 - b)*d^2*e^
2*f + 4*(b*c^3 - 3*b*c)*d*e*f^2 - (b*c^4 - 6*b*c^2 + b)*f^3)*arctan(d*x + c
) - 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + (3*b*c^2 - b)*d*e*f^2 - (b*c^3 - b*c)*
f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.48

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \frac{1}{4}\,af^3x^4 + aef^2x^3 + \frac{3}{2}\,ae^2fx^2$$
$$+\frac{3}{2}\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)be^2f$$
$$+\frac{1}{2}\Bigg(2\,x^3\arctan(dx+c) - d\Bigg(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log(d^2x^2+2\,cdx+}{d^4}$$
$$+\frac{1}{12}\Bigg(3\,x^4\arctan(dx+c) - d\Bigg(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4} + \frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - 6\,(c^3$$
$$+ae^3x + \frac{\left(2\,(dx+c)\arctan(dx+c) - \log\left((dx+c)^2+1\right)\right)be^3}{2\,d}$$

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/2*(x^2*arctan(d*x + c) -
d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/
d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/d^4))*b*e*f^2 + 1/12*(3*x^4*arctan(d*x + c) - d*((d^2*x
```

```
^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d^2*x +
 c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*f^3 + a
*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e^3/d
```

Giac [F]

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx=\int (fx+e)^3(b\arctan(dx+c)+a)\,dx$$

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.38

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \operatorname{atan}(c+d\,x)\,\left(b\,e^3\,x+\frac{3\,b\,e^2\,f\,x^2}{2}+b\,e\,f^2\,x^3+\frac{b\,f^3\,x^4}{4}\right)$$

$$+\,x\left(\frac{e\,(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2-3\,b\,d\,e\,f+6\,a\,f^2)}{2\,d^2} - \frac{(4\,c^2+4)\,\left(\frac{f^2\,(8\,a\,c\,f-b\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{4\,d^2} + \frac{2\,c\,\left(\frac{2\,c\,\left(\frac{f^2\,(8\,a\,c\,f-b\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f-4\,b\,d\,e\,f^2+4\,a\,f^3}{4\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{4\,d^2}\right)}{d}\right)$$

$$-\,x^2\left(\frac{c\,\left(\frac{f^2\,(8\,a\,c\,f-b\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d} - \frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f-4\,b\,d\,e\,f^2+4\,a\,f^3}{8\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{8\,d^2}\right)$$

$$+\,x^3\left(\frac{f^2\,(8\,a\,c\,f-b\,f+12\,a\,d\,e)}{12\,d}-\frac{2\,a\,c\,f^3}{3\,d}\right)+\frac{a\,f^3\,x^4}{4}$$

$$-\,\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\,\left(-64\,b\,c^3\,d^4\,f^3+192\,b\,c^2\,d^5\,e\,f^2-192\,b\,c\,d^6\,e^2\,f+64\,b\,c\,d^4\,f^3+64\,b\,d^7\,e^3-6\right.}{128\,d^8}$$

$$-\,\frac{b\operatorname{atan}\left(\frac{4\,d^3\,\left(\frac{c\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3\right)}{4\,d^3}+\frac{x\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\right.}{4\,d^2}\right.}{c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3}\right.}{4\,d^4}$$

```
[In] int((e + f*x)^3*(a + b*atan(c + d*x)),x)
```

```
[Out] atan(c + d*x)*((b*f^3*x^4)/4 + b*e^3*x + (3*b*e^2*f*x^2)/2 + b*e*f^2*x^3) +
 x*((e*(6*a*f^2 + 6*a*c^2*f^2 + 2*a*d^2*e^2 - 3*b*d*e*f + 12*a*c*d*e*f))/(2
*d^2) - ((4*c^2 + 4)*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/
d))/(4*d^2) + (2*c*((2*c*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f
^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 - 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*
d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 + 4))/(4*d^2)))/d) - x^2*((c*((f^2*(8*a*c*
f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 - 4
```

```
*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f^3*(4*c^2 + 4))
/(8*d^2)) + x^3*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(12*d) - (2*a*c*f^3)/(3*d
)) + (a*f^3*x^4)/4 - (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(64*b*d^7*e^3 - 64*b
*c^3*d^4*f^3 + 64*b*c*d^4*f^3 - 64*b*d^5*e*f^2 - 192*b*c*d^6*e^2*f + 192*b*
c^2*d^5*e*f^2))/(128*d^8) - (b*atan((4*d^3*((c*(f^3 - 6*c^2*f^3 + c^4*f^3 -
 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2
))/(4*d^3) + (x*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*
c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^2)))/(f^3 - 6*c^2*f^3 +
 c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c
^3*d*e*f^2))*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2
*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^4)
```

3.25 $\int (e+fx)^2(a+b\arctan(c+dx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 155

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx$$

$$=-\frac{bf(de-cf)x}{d^2}-\frac{bf^2(c+dx)^2}{6d^3}-\frac{b(de-cf)\left(d^2e^2-2cdef-(3-c^2)\,f^2\right)\arctan(c+dx)}{3d^3f}$$

$$+\frac{(e+fx)^3(a+b\arctan(c+dx))}{3f}-\frac{b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\log\left(1+(c+dx)^2\right)}{6d^3}$$

```
[Out] -b*f*(-c*f+d*e)*x/d^2-1/6*b*f^2*(d*x+c)^2/d^3-1/3*b*(-c*f+d*e)*(d^2*e^2-2*c
*d*e*f-(-c^2+3)*f^2)*arctan(d*x+c)/d^3/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c))/
f-1/6*b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*ln(1+(d*x+c)^2)/d^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx$$

$$=\frac{(e+fx)^3(a+b\arctan(c+dx))}{3f}$$

$$-\frac{b\arctan(c+dx)(de-cf)\left(-(3-c^2)\,f^2-2cdef+d^2e^2\right)}{3d^3f}$$

$$-\frac{b(-(1-3c^2)\,f^2-6cdef+3d^2e^2)\log\left((c+dx)^2+1\right)}{6d^3}-\frac{bf^2(c+dx)^2}{6d^3}-\frac{bfx(de-cf)}{d^2}$$

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -((b*f*(d*e - c*f)*x)/d^2) - (b*f^2*(c + d*x)^2)/(6*d^3) - (b*(d*e - c*f)*(
d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*ArcTan[c + d*x])/(3*d^3*f) + ((e + f*x
)^3*(a + b*ArcTan[c + d*x]))/(3*f) - (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2
)*f^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 (a+b\arctan(x))\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3(a+b\arctan(c+dx))}{3f} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1+x^2}\,dx, x, c+dx\right)}{3f}$$

$$= \frac{(e+fx)^3(a+b\arctan(c+dx))}{3f} - \frac{b\text{Subst}\left(\int \left(\frac{3f^2(de-cf)}{d^3}+\frac{f^3x}{d^3}+\frac{(de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-(1-3c^2)f^2)x}{d^3(1+x^2)}\right)dx, x, c+dx\right)}{3f}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3(a+b\arctan(c+dx))}{3f} - \frac{b\text{Subst}\left(\int \frac{(de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-(1-3c^2)f^2)x}{1+x^2}\,dx, x, c+dx\right)}{3d^3f}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3(a+b\arctan(c+dx))}{3f} - \frac{(b(3d^2e^2-6cdef-(1-3c^2)f^2))\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{3d^3} - \frac{(b(de-cf)(d^2e^2-2cdef-(3-c^2)f^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{3d^3f}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} - \frac{b(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)\arctan(c+dx)}{3d^3f} + \frac{(e+fx)^3(a+b\arctan(c+dx))}{3f} - \frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\log(1+(c+dx)^2)}{6d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.76

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx$$
$$= \frac{(e+fx)^3(a+b\arctan(c+dx)) - \frac{b(6df^2(de-cf)x+f^3(c+dx)^2-i(de-(-i+c)f)^3\log(i-c-dx)+i(de-(i+c)f)^3\log(i+c+dx))}{2d^3}}{3f}$$

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^3*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c +
 d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3*L
og[I + c + d*x]))/(2*d^3))/(3*f)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.71

method	result
parts	$\frac{a(fx+e)^3}{3f}-\frac{f^2bc\arctan(dx+c)}{d^3}+\frac{fbe\arctan(dx+c)}{d^2}+\frac{b\,f^2\arctan(dx+c)x^3}{3}+b\arctan\left(dx+c\right)x\,e^2-$
derivativedivides	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\,f^2\arctan(dx+c)c^2(dx+c)}{d^2}-\frac{2bf\arctan(dx+c)ce(dx+c)}{d}-\frac{b\,f^2\arctan(dx+c)c(dx+c)^2}{d^2}+b\arctan(dx+c)e}{}$
default	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\,f^2\arctan(dx+c)c^2(dx+c)}{d^2}-\frac{2bf\arctan(dx+c)ce(dx+c)}{d}-\frac{b\,f^2\arctan(dx+c)c(dx+c)^2}{d^2}+b\arctan(dx+c)e}{}$
parallelrisch	$-\frac{-\ln\left(d^2x^2+2cdx+c^2+1\right)b\,f^2+6\arctan(dx+c)b\,c^2def-6\ln\left(d^2x^2+2cdx+c^2+1\right)bcdef-6x^2\arctan(dx+c)b\,d^3ef-6x\,a}{}$
risch	$-\frac{i(fx+e)^3b\ln(1+i(dx+c))}{6f}+\frac{ib\,e^3\ln\left(d^2x^2+2cdx+c^2+1\right)}{12f}+\frac{fbce\ln\left(d^2x^2+2cdx+c^2+1\right)}{d^2}+\frac{if^2b\,x^3\ln(1-i(dx+c)}{6}$

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(f*x+e)^3/f-f^2/d^3*b*c*arctan(d*x+c)+f/d^2*b*e*arctan(d*x+c)+1/3*b*f
^2*arctan(d*x+c)*x^3+b*arctan(d*x+c)*x*e^2-b/d^2*f*c*e+1/3*f^2/d^3*b*c^3*ar
ctan(d*x+c)+2/3*f^2/d^2*b*c*x-f/d*b*e*x-1/2*b/d^3*f^2*ln(1+(d*x+c)^2)*c^2-1
/2*b*e^2*ln(1+(d*x+c)^2)/d+b*f*arctan(d*x+c)*e*x^2+b/d^2*f*ln(1+(d*x+c)^2)*
c*e+5/6*b/d^3*f^2*c^2-f/d^2*b*c^2*e*arctan(d*x+c)+1/6*b/d^3*f^2*ln(1+(d*x+c
)^2)+1/d*b*c*e^2*arctan(d*x+c)-1/6*f^2/d*b*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx$$
$$=\frac{2\,ad^3f^2x^3+(6\,ad^3ef-bd^2f^2)x^2+2\,(3\,ad^3e^2-3\,bd^2ef+2\,bcdf^2)x+2\,(bd^3f^2x^3+3\,bd^3efx^2+3\,bd^3e^2x}{}$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f - b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 - 3*b
*d^2*e*f + 2*b*c*d*f^2)*x + 2*(b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^
2*x + 3*b*c*d^2*e^2 - 3*(b*c^2 - b)*d*e*f + (b*c^3 - 3*b*c)*f^2)*arctan(d*x
 + c) - (3*b*d^2*e^2 - 6*b*c*d*e*f + (3*b*c^2 - b)*f^2)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1))/d^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 131.97 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.43

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx$$
$$=\begin{cases} ae^2x+aefx^2+\frac{af^2x^3}{3}+\frac{bc^3f^2\operatorname{atan}(c+dx)}{3d^3}-\frac{bc^2ef\operatorname{atan}(c+dx)}{d^2}-\frac{bc^2f^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3}+\frac{ibc^2f^2\operatorname{atan}(c+dx)}{d^3}+\frac{bce^2\operatorname{atan}(c+dx}{d} \\ (a+b\operatorname{atan}(c))\left(e^2x+efx^2+\frac{f^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*atan(c + d*x
)/(3*d**3) - b*c**2*e*f*atan(c + d*x)/d**2 - b*c**2*f**2*log(c/d + x - I/d)
/d**3 + I*b*c**2*f**2*atan(c + d*x)/d**3 + b*c*e**2*atan(c + d*x)/d + 2*b*c
*e*f*log(c/d + x - I/d)/d**2 - 2*I*b*c*e*f*atan(c + d*x)/d**2 + 2*b*c*f**2*
x/(3*d**2) - b*c*f**2*atan(c + d*x)/d**3 + b*e**2*x*atan(c + d*x) + b*e*f*x
**2*atan(c + d*x) + b*f**2*x**3*atan(c + d*x)/3 - b*e**2*log(c/d + x - I/d)
/d + I*b*e**2*atan(c + d*x)/d - b*e*f*x/d - b*f**2*x**2/(6*d) + b*e*f*atan(
c + d*x)/d**2 + b*f**2*log(c/d + x - I/d)/(3*d**3) - I*b*f**2*atan(c + d*x)
/(3*d**3), Ne(d, 0)), ((a + b*atan(c))*(e**2*x + e*f*x**2 + f**2*x**3/3), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.42

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx=\frac{1}{3}\,af^2x^3+aefx^2$$
$$+\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)bef$$
$$+\frac{1}{6}\left(2\,x^3\arctan(dx+c)-d\left(\frac{dx^2-4\,cx}{d^3}-\frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4}+\frac{(3\,c^2-1)\log(d^2x^2+2\,cdx+}{d^4}\right.\right.$$
$$+ae^2x+\frac{\left(2\,(dx+c)\arctan(dx+c)-\log\left((dx+c)^2+1\right)\right)be^2}{2\,d}$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arc
tan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*e*f +
```

```
 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan
((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*
b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*
b*e^2/d
```

Giac [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx=\int (fx+e)^2(b\arctan(dx+c)+a)\,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.65

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx=x^2\left(\frac{f\,(6\,a\,c\,f-b\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right)$$
$$-x\left(\frac{2\,c\left(\frac{f\,(6\,a\,c\,f-b\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}\right.$$
$$\left.-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2-3\,b\,d\,e\,f+3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2+3)}{3\,d^2}\right)$$
$$+\operatorname{atan}(c+d\,x)\,\left(b\,e^2\,x+b\,e\,f\,x^2+\frac{b\,f^2\,x^3}{3}\right)+\frac{a\,f^2\,x^3}{3}$$
$$-\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\,\left(36\,b\,c^2\,d^3\,f^2-72\,b\,c\,d^4\,e\,f+36\,b\,d^5\,e^2-12\,b\,d^3\,f^2\right)}{72\,d^6}$$
$$+\frac{b\,\operatorname{atan}\left(\frac{3\,d^2\left(\frac{c\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d^2}+\frac{x\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d}\right)}{c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f}\right)\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d\right.}{3\,d^3}$$

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x)),x)
```

```
[Out] x^2*((f*(6*a*c*f - b*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - x*((2*c*((f*(6*a*
c*f - b*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*f^2 + 3*a*c^2*f^2 +
3*a*d^2*e^2 - 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 + 3))/(3*d^
2)) + atan(c + d*x)*((b*f^2*x^3)/3 + b*e^2*x + b*e*f*x^2) + (a*f^2*x^3)/3 -
 (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(36*b*d^5*e^2 - 12*b*d^3*f^2 + 36*b*c^2*
```

```
d^3*f^2 - 72*b*c*d^4*e*f))/(72*d^6) + (b*atan((3*d^2*((c*(c^3*f^2 - 3*c*f^2
 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d^2) + (x*(c^3*f^2 - 3*c*f^2 +
3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d)))/(c^3*f^2 - 3*c*f^2 + 3*c*d^2*
e^2 + 3*d*e*f - 3*c^2*d*e*f))*(c^3*f^2 - 3*c*f^2 + 3*c*d^2*e^2 + 3*d*e*f -
3*c^2*d*e*f))/(3*d^3)
```

3.26 $\int (e+fx)(a+b\arctan(c+dx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int (e+fx)(a+b\arctan(c+dx))\,dx = -\frac{bfx}{2d} - \frac{b(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2d^2f} + \frac{(e+fx)^2(a+b\arctan(c+dx))}{2f} - \frac{b(de-cf)\log\left(1+(c+dx)^2\right)}{2d^2}$$

```
[Out] -1/2*b*f*x/d-1/2*b*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/d^2/f+1/2*(f*x+
e)^2*(a+b*arctan(d*x+c))/f-1/2*b*(-c*f+d*e)*ln(1+(d*x+c)^2)/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\int (e+fx)(a+b\arctan(c+dx))\,dx = \frac{(e+fx)^2(a+b\arctan(c+dx))}{2f} - \frac{b\arctan(c+dx)(-cf+de+f)(de-(c+1)f)}{2d^2f} - \frac{b(de-cf)\log\left((c+dx)^2+1\right)}{2d^2} - \frac{bfx}{2d}$$

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -1/2*(b*f*x)/d - (b*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d
^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x]))/(2*f) - (b*(d*e - c*f)*Log[1
+ (c + d*x)^2])/(2*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)(a+b\arctan(x))\,dx, x, c+dx\right)}{d}$$

$$
\begin{aligned}
&= \frac{(e+fx)^2(a+b\arctan(c+dx))}{2f} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1+x^2}\,dx,x,c+dx\right)}{2f} \\
&= \frac{(e+fx)^2(a+b\arctan(c+dx))}{2f} \\
&\quad - \frac{b\text{Subst}\left(\int \left(\frac{f^2}{d^2}+\frac{(de-f-cf)(de+f-cf)+2f(de-cf)x}{d^2(1+x^2)}\right)dx,x,c+dx\right)}{2f} \\
&= -\frac{bfx}{2d} + \frac{(e+fx)^2(a+b\arctan(c+dx))}{2f} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{(de-f-cf)(de+f-cf)+2f(de-cf)x}{1+x^2}\,dx,x,c+dx\right)}{2d^2f} \\
&= -\frac{bfx}{2d} + \frac{(e+fx)^2(a+b\arctan(c+dx))}{2f} - \frac{(b(de-cf))\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d^2} \\
&\quad - \frac{(b(de+f-cf)(de-(1+c)f))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{2d^2f} \\
&= -\frac{bfx}{2d} - \frac{b(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2d^2f} \\
&\quad + \frac{(e+fx)^2(a+b\arctan(c+dx))}{2f} - \frac{b(de-cf)\log\left(1+(c+dx)^2\right)}{2d^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$
\begin{aligned}
&\int (e+fx)(a+b\arctan(c+dx))\,dx \\
&= aex + \frac{1}{2}afx^2 + bex\arctan(c+dx) \\
&\quad + \frac{bf\left(\frac{1}{2}d\left(-\frac{c}{d}+\frac{c+dx}{d}\right)^2\arctan(c+dx) - \frac{1}{2}d\left(\frac{x}{d}-\frac{i(i-c)^2\log(i-c-dx)}{2d^2}+\frac{i(i+c)^2\log(i+c+dx)}{2d^2}\right)\right)}{d} \\
&\quad - \frac{be\left(-2c\arctan(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right)\right)}{2d}
\end{aligned}
$$

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
[Out] a*e*x + (a*f*x^2)/2 + b*e*x*ArcTan[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)/
d)^2*ArcTan[c + d*x])/2 - (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^2
+ ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d - (b*e*(-2*c*ArcTan[c + d*
x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

method	result
parts	$a\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b\left(\frac{\arctan(dx+c)(dx+c)^2 f}{2d}-\frac{\arctan(dx+c)cf(dx+c)}{d}+\arctan(dx+c)e(dx+c)-\frac{f(dx+c)+\frac{(-2cf+2de)\ln\left(\right.}{2}}{}\right.}{d}$
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\left(\arctan(dx+c)fc(dx+c)-\arctan(dx+c)ed(dx+c)-\frac{\arctan(dx+c)f(dx+c)^2}{2}+\frac{f(dx+c)}{2}-\frac{(2c}{}\right.}{d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\left(\arctan(dx+c)fc(dx+c)-\arctan(dx+c)ed(dx+c)-\frac{\arctan(dx+c)f(dx+c)^2}{2}+\frac{f(dx+c)}{2}-\frac{(2c}{}\right.}{d}}{d}$
parallelrisch	$\frac{\arctan(dx+c)b\,d^2f\,x^2+a\,d^2f\,x^2+2x\arctan(dx+c)b\,d^2e+2a\,d^2ex-\arctan(dx+c)b\,c^2f+2bcde\arctan(dx+c)+bcf\ln\left(d^2x^2-\right.}{2d^2}$
risch	$-\frac{ib\left(f\,x^2+2ex\right)\ln(1+i(dx+c))}{4}+\frac{ibf\,x^2\ln(1-i(dx+c))}{4}+\frac{ibex\ln(1-i(dx+c))}{2}+\frac{af\,x^2}{2}-\frac{\arctan(dx+c)b\,c^2f}{2d^2}+\frac{b}{}$

```
[In] int((f*x+e)*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*f*x^2+e*x)+b/d*(1/2/d*arctan(d*x+c)*(d*x+c)^2*f-1/d*arctan(d*x+c)*c*
f*(d*x+c)+arctan(d*x+c)*e*(d*x+c)-1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(1+
(d*x+c)^2)-f*arctan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int (e+fx)(a+b\arctan(c+dx))\,dx$$
$$=\frac{ad^2fx^2+(2\,ad^2e-bdf)x+(bd^2fx^2+2\,bd^2ex+2\,bcde-(bc^2-b)f)\arctan\left(dx+c\right)-(bde-bcf)\log\left(d^2\right.}{2\,d^2}$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + (2*a*d^2*e - b*d*f)*x + (b*d^2*f*x^2 + 2*b*d^2*e*x + 2*b
*c*d*e - (b*c^2 - b)*f)*arctan(d*x + c) - (b*d*e - b*c*f)*log(d^2*x^2 + 2*c
*d*x + c^2 + 1))/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int (e+fx)(a+b\arctan(c+dx))\,dx$$
$$=\begin{cases} aex+\frac{afx^2}{2}-\frac{bc^2f\operatorname{atan}(c+dx)}{2d^2}+\frac{bce\operatorname{atan}(c+dx)}{d}+\frac{bcf\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2}-\frac{ibcf\operatorname{atan}(c+dx)}{d^2}+bex\operatorname{atan}(c+dx)+\frac{bfx^2\operatorname{ata}}{2} \\ (a+b\operatorname{atan}(c))\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*atan(c + d*x)/(2*d**2) + b*c*e*ata
n(c + d*x)/d + b*c*f*log(c/d + x - I/d)/d**2 - I*b*c*f*atan(c + d*x)/d**2 +
 b*e*x*atan(c + d*x) + b*f*x**2*atan(c + d*x)/2 - b*e*log(c/d + x - I/d)/d
+ I*b*e*atan(c + d*x)/d - b*f*x/(2*d) + b*f*atan(c + d*x)/(2*d**2), Ne(d, 0
)), ((a + b*atan(c))*(e*x + f*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int (e+fx)(a+b\arctan(c+dx))\,dx=\frac{1}{2}afx^2$$
$$+\frac{1}{2}\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)bf$$
$$+aex+\frac{\left(2\,(dx+c)\arctan(dx+c)-\log\left((dx+c)^2+1\right)\right)be}{2\,d}$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*e*x + 1/2
*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e/d
```

Giac [F]

$$\int (e+fx)(a+b\arctan(c+dx))\,dx=\int (fx+e)(b\arctan\left(dx+c\right)+a)\,dx$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\begin{aligned}\int (e+fx)(a+b\arctan(c+dx))\,dx &= a\,e\,x+\frac{a\,f\,x^2}{2}-\frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}\\&\quad+\frac{b\,f\,\mathrm{atan}(c+d\,x)}{2\,d^2}+\frac{b\,f\,x^2\,\mathrm{atan}(c+d\,x)}{2}-\frac{b\,f\,x}{2\,d}\\&\quad+b\,e\,x\,\mathrm{atan}(c+d\,x)-\frac{b\,c^2\,f\,\mathrm{atan}(c+d\,x)}{2\,d^2}\\&\quad+\frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d^2}\\&\quad+\frac{b\,c\,e\,\mathrm{atan}(c+d\,x)}{d}\end{aligned}$$

```
[In] int((e + f*x)*(a + b*atan(c + d*x)),x)
```

```
[Out] a*e*x + (a*f*x^2)/2 - (b*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*f*a
tan(c + d*x))/(2*d^2) + (b*f*x^2*atan(c + d*x))/2 - (b*f*x)/(2*d) + b*e*x*a
tan(c + d*x) - (b*c^2*f*atan(c + d*x))/(2*d^2) + (b*c*f*log(c^2 + d^2*x^2 +
 2*c*d*x + 1))/(2*d^2) + (b*c*e*atan(c + d*x))/d
```

3.27 $\int (a+b\arctan(c+dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int (a+b\arctan(c+dx))\,dx=ax+\frac{b(c+dx)\arctan(c+dx)}{d}-\frac{b\log\left(1+(c+dx)^2\right)}{2d}$$

```
[Out] a*x+b*(d*x+c)*arctan(d*x+c)/d-1/2*b*ln(1+(d*x+c)^2)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5147, 4930, 266}

$$\int (a+b\arctan(c+dx))\,dx=ax+\frac{b(c+dx)\arctan(c+dx)}{d}-\frac{b\log\left((c+dx)^2+1\right)}{2d}$$

```
[In] Int[a + b*ArcTan[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcTan[c + d*x])/d - (b*Log[1 + (c + d*x)^2])/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \arctan(c+dx)\,dx \\
&= ax + \frac{b\text{Subst}\left(\int \arctan(x)\,dx, x, c+dx\right)}{d} \\
&= ax + \frac{b(c+dx)\arctan(c+dx)}{d} - \frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{d} \\
&= ax + \frac{b(c+dx)\arctan(c+dx)}{d} - \frac{b\log\left(1+(c+dx)^2\right)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+b\arctan(c+dx))\,dx = ax + bx\arctan(c+dx) - \frac{b\left(-2c\arctan(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right)\right)}{2d}$$

```
[In] Integrate[a + b*ArcTan[c + d*x],x]
[Out] a*x + b*x*ArcTan[c + d*x] - (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d*
x + d^2*x^2]))/(2*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$ax+\frac{b\left((dx+c)\arctan(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	35
parts	$ax+\frac{b\left((dx+c)\arctan(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	35
derivativedivides	$\frac{(dx+c)a+b\left((dx+c)\arctan(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	40
parallelrisch	$-\frac{b\left(-2\arctan(dx+c)x\,d^2-2c\arctan(dx+c)d+\ln(d^2x^2+2cdx+c^2+1)d\right)}{2d^2}+ax$	54
risch	$ax-\frac{ibx\ln(1+i(dx+c))}{2}+\frac{ibx\ln(1-i(dx+c))}{2}+\frac{bc\arctan(dx+c)}{d}-\frac{b\ln(d^2x^2+2cdx+c^2+1)}{2d}$	73

```
[In] int(a+b*arctan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b/d*((d*x+c)*arctan(d*x+c)-1/2*ln(1+(d*x+c)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int(a+b\arctan(c+dx))\,dx=\frac{2\,adx+2\,(bdx+bc)\arctan\left(dx+c\right)-b\log\left(d^2x^2+2\,cdx+c^2+1\right)}{2\,d}$$

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + 2*(b*d*x + b*c)*arctan(d*x + c) - b*log(d^2*x^2 + 2*c*d*x +
c^2 + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int(a+b\arctan(c+dx))\,dx=ax+b\left(\begin{cases}\frac{c\,\mathrm{atan}\,(c+dx)}{d}+x\,\mathrm{atan}\,(c+dx)-\frac{\log\left(c^2+2cdx+d^2x^2+1\right)}{2d} & \text{for } d\neq 0\\ x\,\mathrm{atan}\,(c) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a+b*atan(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*atan(c + d*x)/d + x*atan(c + d*x) - log(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*atan(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a+b\arctan(c+dx))\,dx=ax+\frac{\left(2\,(dx+c)\arctan\left(dx+c\right)-\log\left((dx+c)^2+1\right)\right)b}{2\,d}$$

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a+b\arctan(c+dx))\,dx=ax+\frac{\left(2\,(dx+c)\arctan\left(dx+c\right)-\log\left((dx+c)^2+1\right)\right)b}{2\,d}$$

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+b\arctan(c+dx))\,dx=a\,x+b\,x\,\mathrm{atan}(c+d\,x)-\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}+\frac{b\,c\,\mathrm{atan}(c+d\,x)}{d}$$

```
[In] int(a + b*atan(c + d*x),x)
```

```
[Out] a*x + b*x*atan(c + d*x) - (b*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*c
*atan(c + d*x))/d
```

3.28 $\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\begin{aligned}\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx = & -\frac{(a+b\arctan(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f}\\ & +\frac{(a+b\arctan(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}\\ & +\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}\\ & -\frac{ib\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}\end{aligned}$$

```
[Out] -(a+b*arctan(d*x+c))*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))*ln(2*d*(f*x+
e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+1/2*I*b*polylog(2,1-2/(1-I*(d*x+c)))/f-1/
2*I*b*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used

$= \{5155, 4966, 2449, 2352, 2497\}$

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx = \frac{(a+b\arctan(c+dx))\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} - \frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{f} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[c
+ d*x])*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f + ((I
/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - ((I/2)*b*PolyLog[2, 1 - (2*d
*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\arctan(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}\\
&= -\frac{(a+b\arctan(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{(a+b\arctan(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{f}-\frac{b\text{Subst}\left(\int \frac{\log\left(\frac{2\left(\frac{de-cf}{d}+\frac{fx}{d}\right)}{\left(\frac{if}{d}+\frac{de-cf}{d}\right)(1-ix)}\right)}{1+x^2}\,dx, x, c+dx\right)}{f}\\
&= -\frac{(a+b\arctan(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{(a+b\arctan(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}\\
&\quad-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}+\frac{(ib)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-i(c+dx)}\right)}{f}\\
&= -\frac{(a+b\arctan(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{(a+b\arctan(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}\\
&\quad+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$
\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx
$$

$$
= \frac{2a\log(d(e+fx))+ib\log\left(\frac{d(e+fx)}{de-(i+c)f}\right)\log(1-i(c+dx))-ib\log\left(\frac{d(e+fx)}{de+if-cf}\right)\log(1+i(c+dx))-ib\,\text{PolyL}}{2f}
$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

```
[Out] (2*a*Log[d*(e + f*x)] + I*b*Log[(d*(e + f*x))/(d*e - (I + c)*f)]*Log[1 - I*
(c + d*x)] - I*b*Log[(d*(e + f*x))/(d*e + I*f - c*f)]*Log[1 + I*(c + d*x)]
- I*b*PolyLog[2, (f*(-I + c + d*x))/(-(d*e) + (-I + c)*f)] + I*b*PolyLog[2,
 (f*(I + c + d*x))/(-(d*e) + (I + c)*f)])/(2*f)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.22

method	result
parts	$\frac{a\ln(fx+e)}{f}+\frac{b\left(\frac{d\ln(f(dx+c)-cf+de)\arctan(dx+c)}{f}-d\left(-\frac{i\ln(f(dx+c)-cf+de)\left(\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)-\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)\right)}{2f}-\frac{i\left(\mathrm{di}\right.}{}\right.\right.}{d}$
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\arctan(dx+c)}{f}+\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\right)}{2f}+\right.}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\arctan(dx+c)}{f}+\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\right)}{2f}+\right.}{d}$
risch	$\frac{a\ln(icf-ide+(-idx-ic+1)f-f)}{f}+\frac{ib\,\mathrm{dilog}\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}+\frac{ib\ln(-idx-ic+1)\ln\left(\frac{icf-ide+(-idx-ic+1)}{icf-ide-f}\right.}{2f}$

```
[In] int((a+b*arctan(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(f*x+e)/f+b/d*(d*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)-d*(-1/2*I*ln(f*(
d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*f-
d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-dilog((I*f+f*(d*x+
c))/(c*f-d*e+I*f)))/f))
```

Fricas [F]

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx=\int \frac{b\arctan\left(dx+c\right)+a}{fx+e}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx = \int \frac{b\arctan\left(dx+c\right)+a}{fx+e}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx = \int \frac{b\arctan\left(dx+c\right)+a}{fx+e}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx = \int \frac{a+b\,\mathrm{atan}(c+d\,x)}{e+f\,x}\,dx$$

```
[In] int((a + b*atan(c + d*x))/(e + f*x),x)
```

```
[Out] int((a + b*atan(c + d*x))/(e + f*x), x)
```

3.29 $\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 151

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx=\frac{bd(de-cf)\arctan(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}-\frac{a+b\arctan(c+dx)}{f(e+fx)}+\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)\,f^2}-\frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}$$

```
[Out] b*d*(-c*f+d*e)*arctan(d*x+c)/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+(-a-b*arctan
(d*x+c))/f/(f*x+e)+b*d*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-1/2*b*d*ln
(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5153, 2007, 719, 31, 648, 632, 210, 642}

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx=-\frac{a+b\arctan(c+dx)}{f(e+fx)}+\frac{bd\arctan(c+dx)(de-cf)}{f\left((c^2+1)\,f^2-2cdef+d^2e^2\right)}-\frac{bd\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left((c^2+1)\,f^2-2cdef+d^2e^2\right)}+\frac{bd\log(e+fx)}{(c^2+1)\,f^2-2cdef+d^2e^2}$$

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])/(f*(e + f*x)) + (b*d*Log[e + f*x])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) - (b*d*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
```

```
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(c+dx)}{f(e+fx)}+\frac{(bd)\int\frac{1}{(e+fx)(1+(c+dx)^2)}\,dx}{f}\\
&= -\frac{a+b\arctan(c+dx)}{f(e+fx)}+\frac{(bd)\int\frac{1}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}\\
&= -\frac{a+b\arctan(c+dx)}{f(e+fx)}+\frac{(bd)\int\frac{d^2e-2cdf-d^2fx}{1+c^2+2cdx+d^2x^2}\,dx}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}+\frac{(bdf)\int\frac{1}{e+fx}\,dx}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&= -\frac{a+b\arctan(c+dx)}{f(e+fx)}+\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&\quad-\frac{(bd)\int\frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2}\,dx}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}+\frac{(bd^2(de-cf))\int\frac{1}{1+c^2+2cdx+d^2x^2}\,dx}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}\\
&= -\frac{a+b\arctan(c+dx)}{f(e+fx)}+\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&\quad-\frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}\\
&\quad-\frac{(2bd^2(de-cf))\,\text{Subst}\left(\int\frac{1}{-4d^2-x^2}\,dx,x,2cd+2d^2x\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}\\
&= \frac{bd(de-cf)\arctan(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}-\frac{a+b\arctan(c+dx)}{f(e+fx)}\\
&\quad+\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)\,f^2}-\frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int\frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx=\frac{-\frac{a+b\arctan(c+dx)}{e+fx}+\frac{bd(i(-de+(i+c)f)\log(i-c-dx)+i(de+if-cf)\log(i+c+dx)+2f\log(d(e+fx)))}{2(d^2e^2-2cdef+(1+c^2)f^2)}}{f}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)) + (b*d*(I*(-(d*e) + (I + c)*f)*Log[I
- c - d*x] + I*(d*e + I*f - c*f)*Log[I + c + d*x] + 2*f*Log[d*(e + f*x)]))/
(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.06

method	result
parts	$-\frac{a}{(fx+e)f}+\frac{b\left(-\frac{d^{2}\arctan(dx+c)}{(f(dx+c)-cf+de)f}+\frac{d^{2}\left(\frac{f\ln(f(dx+c)-cf+de)}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}+\frac{-\frac{f\ln\left(1+(dx+c)^{2}\right)}{2}+(-cf+de)\arctan(dx+c)}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}\right)}{f}\right)}{d}$
derivativedivides	$\frac{\frac{a\,d^{2}}{(cf-de-f(dx+c))f}+b\,d^{2}\left(\frac{\arctan(dx+c)}{(cf-de-f(dx+c))f}-\frac{\frac{\frac{f\ln\left(1+(dx+c)^{2}\right)}{2}+(cf-de)\arctan(dx+c)}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}-\frac{f\ln(cf-de-f(dx+c))}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}}{f}\right)}{d}$
default	$\frac{\frac{a\,d^{2}}{(cf-de-f(dx+c))f}+b\,d^{2}\left(\frac{\arctan(dx+c)}{(cf-de-f(dx+c))f}-\frac{\frac{\frac{f\ln\left(1+(dx+c)^{2}\right)}{2}+(cf-de)\arctan(dx+c)}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}-\frac{f\ln(cf-de-f(dx+c))}{c^{2}f^{2}-2cdef+d^{2}e^{2}+f^{2}}}{f}\right)}{d}$
parallelrisch	$\frac{-2x\arctan(dx+c)bc\,d^{3}f^{2}+2x\arctan(dx+c)b\,d^{4}ef+2\ln(fx+e)xb\,d^{3}f^{2}-\ln\left(d^{2}x^{2}+2cdx+c^{2}+1\right)xb\,d^{3}f^{2}-2\arctan(dx+c}{2(fx}$
risch	$\frac{ib\ln(1+i(dx+c))}{2f(fx+e)}+\frac{-ib\,f^{2}\ln(1-i(dx+c))-i\ln\left((cdf-d^{2}e-3idf)x-2icf-ide+c^{2}f-cde+3f\right)bcd\,f^{2}x-i\ln\left((cdf-d^{2}e-\right.}{}$

```
[In] int((a+b*arctan(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(f*x+e)/f+b/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)+d^2/f*(1/(c^2*f^
2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)*(-1/2*f*ln(1+(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.26

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx = -\frac{2\,ad^{2}e^{2}-4\,acdef+2\,(ac^{2}+a)f^{2}-2\,(bcdef-(bc^{2}+b)f^{2}+(bd^{2}ef-bcdf^{2})x)\arctan\left(dx+c\right)+(bdf}{2\,(d^{2}e^{3}f-2\,cde^{2}f^{2}+(c^{2}+1)ef^{3}+(d^{2}e^{2}f^{2}-2\,cde}$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 + a)*f^2 - 2*(b*c*d*e*f - (b*c^2
 + b)*f^2 + (b*d^2*e*f - b*c*d*f^2)*x)*arctan(d*x + c) + (b*d*f^2*x + b*d*e
*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b*d*f^2*x + b*d*e*f)*log(f*x + e)
)/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^2 + 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3
 + (c^2 + 1)*f^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.17

$$\begin{aligned}&\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx\\&=\frac{1}{2}\left(d\left(\frac{2\,(d^2e-cdf)\arctan\left(\frac{d^2x+cd}{d}\right)}{(d^2e^2f-2\,cdef^2+(c^2+1)\,f^3)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+(c^2+1)\,f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+(c^2+1)\,f^2}\right)-2\right.\\&\quad-\frac{a}{f^2x+ef}\end{aligned}$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2
 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*
f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
- 2*arctan(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx=\int \frac{b\arctan\left(dx+c\right)+a}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx = \frac{b\,d\,\ln\left(e+f\,x\right)}{d^2\,e^2-2\,c\,d\,e\,f+\left(c^2+1\right)\,f^2}-\frac{b\,\mathrm{atan}(c+d\,x)}{f\,\left(e+f\,x\right)}-\frac{a}{x\,f^2+e\,f} -\frac{b\,d\,\ln\left(c+d\,x-\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(d\,e-c\,f+f\,1\mathrm{i}\right)}-\frac{b\,d\,\ln\left(c+d\,x+1\mathrm{i}\right)}{2\,f\,\left(f-c\,f\,1\mathrm{i}+d\,e\,1\mathrm{i}\right)}$$

```
[In] int((a + b*atan(c + d*x))/(e + f*x)^2,x)
```

```
[Out] (b*d*log(e + f*x))/(f^2*(c^2 + 1) + d^2*e^2 - 2*c*d*e*f) - (b*atan(c + d*x)
)/(f*(e + f*x)) - a/(e*f + f^2*x) - (b*d*log(c + d*x - 1i)*1i)/(2*f*(f*1i -
 c*f + d*e)) - (b*d*log(c + d*x + 1i))/(2*f*(f - c*f*1i + d*e*1i))
```

3.30 $\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 227

$$\begin{aligned}\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx = &-\frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)(e+fx)}\\&+\frac{bd^2(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2}\\&-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}+\frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2}\\&-\frac{bd^2(de-cf)\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2}\end{aligned}$$

```
[Out] -1/2*b*d/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)/(f*x+e)+1/2*b*d^2*(-c*f+d*e+f)*(d*
e-(1+c)*f)*arctan(d*x+c)/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^2+1/2*(-a-b*arct
an(d*x+c))/f/(f*x+e)^2+b*d^2*(-c*f+d*e)*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1
)*f^2)^2-1/2*b*d^2*(-c*f+d*e)*ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+
(c^2+1)*f^2)^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

= {5153, 2007, 723, 814, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx = &-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}\\ &+\frac{bd^2\arctan(c+dx)(-cf+de+f)(de-(c+1)f)}{2f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}\\ &-\frac{bd^2(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}\\ &-\frac{bd}{2(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}\\ &+\frac{bd^2(de-cf)\log(e+fx)}{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

```
[Out] -1/2*(b*d)/((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) + (b*d^2*(d*e
+ f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (
1 + c^2)*f^2)^2) - (a + b*ArcTan[c + d*x])/(2*f*(e + f*x)^2) + (b*d^2*(d*e
- c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 - (b*d^2*(d*e
- c*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int\frac{1}{(e+fx)^2(1+(c+dx)^2)}\,dx}{2f}\\
&= -\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int\frac{1}{(e+fx)^2(1+c^2+2cdx+d^2x^2)}\,dx}{2f}\\
&= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}\\
&\quad -\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int\frac{d(de-2cf)-d^2fx}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{2f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}\\
&= -\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}\\
&\quad +\frac{(bd)\int\left(\frac{2df^2(de-cf)}{(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)}+\frac{d^2\left(d^2e^2-4cdef-(1-3c^2)f^2-2df(de-cf)x\right)}{(d^2e^2-2cdef+(1+c^2)f^2)(1+c^2+2cdx+d^2x^2)}\right)dx}{2f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}
\end{aligned}$$

$$= -\frac{bd}{2\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)(e+fx)} - \frac{a + b\arctan(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2} + \frac{(bd^3)\int \frac{d^2e^2-4cdef-(1-3c^2)f^2-2df(de-cf)x}{1+c^2+2cdx+d^2x^2}\,dx}{2f\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2}$$

$$= -\frac{bd}{2\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)(e+fx)} - \frac{a + b\arctan(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2} - \frac{(bd^2(de-cf))\int \frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2}\,dx}{2\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2} + \frac{(bd(4cd^2f(de-cf)+2d^2(d^2e^2-4cdef-(1-3c^2)\,f^2)))\int \frac{1}{1+c^2+2cdx+d^2x^2}\,dx}{4f\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2}$$

$$= -\frac{bd}{2\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)(e+fx)} - \frac{a + b\arctan(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2} - \frac{bd^2(de-cf)\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2} - \frac{(bd(4cd^2f(de-cf)+2d^2(d^2e^2-4cdef-(1-3c^2)\,f^2)))\,\mathrm{Subst}\left(\int \frac{1}{-4d^2-x^2}\,dx, x, 2cd+2d^2x\right)}{2f\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2}$$

$$= -\frac{bd}{2\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)(e+fx)} + \frac{bd^2(de-f-cf)(de+f-cf)\arctan(c+dx)}{2f\left(d^2e^2 - 2cdef + f^2 + c^2f^2\right)^2} - \frac{a + b\arctan(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2} - \frac{bd^2(de-cf)\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2 - 2cdef + (1+c^2)\,f^2\right)^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.77

$$\int \frac{a + b\arctan(c+dx)}{(e+fx)^3}\,dx = \frac{-\frac{a+b\arctan(c+dx)}{(e+fx)^2} + \frac{1}{2}bd^2\left(-\frac{2f}{d(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)} - \frac{i\log(i-c-dx)}{(de-(-i+c)f)^2} + \frac{i\log(i+c+dx)}{(de-(i+c)f)^2} - \frac{4f(-de+cf)\log(d(e+fx))}{(d^2e^2-2cdef+(1+c^2)f^2)^2}\right)}{2f}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)^2) + (b*d^2*((-2*f)/(d*(d^2*e^2 - 2*c*
d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (I*Log[I - c - d*x])/(d*e - (-I + c)*f)
^2 + (I*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (4*f*(-(d*e) + c*f)*Log[d*(
e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2))/2)/(2*f)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.08

method	result
parts	$-\frac{a}{2(fx+e)^2 f}+\frac{b\left(-\frac{d^3\arctan(dx+c)}{2(f(dx+c)-cf+de)^2 f}+\frac{d^3\left(-\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(f(dx+c)-cf+de)}-\frac{2(cf-de)f\ln(f(dx+c)-cf+de}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}\right.}{2f}\right.}{d}$
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\arctan(dx+c)}{2(cf-de-f(dx+c))^2 f}-\frac{\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(dx+}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}}{}\right.}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\arctan(dx+c)}{2(cf-de-f(dx+c))^2 f}-\frac{\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(dx+}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}}{}\right.}{d}$
parallelrisch	$-\frac{2x^2\arctan(dx+c)bc\,d^5e\,f^4-2x\arctan(dx+c)b\,c^2d^4e\,f^4+4x\arctan(dx+c)bc\,d^5e^2f^3+4\ln(fx+e)xbc\,d^4e\,f^4-2\ln\left(d^2x^2+\right.}{}$
risch	Expression too large to display

```
[In] int((a+b*arctan(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(f*x+e)^2/f+b/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arctan(d*x+c)+1/2*
d^3/f*(-f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(f*(d*x+c)-c*f+d*e)-2*(c*f-d*e)*f
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e
*f+d^2*e^2+f^2)^2*(1/2*(2*c*f^2-2*d*e*f)*ln(1+(d*x+c)^2)+(c^2*f^2-2*c*d*e*f
+d^2*e^2-f^2)*arctan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(219) = 438$.

Time = 1.22 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.00

$$\int\frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx =$$
$$-\frac{ad^4e^4-(4\,ac-b)d^3e^3f+2\,(3\,ac^2-bc+a)d^2e^2f^2-(4\,ac^3-bc^2+4\,ac-b)def^3+(ac^4+2\,ac^2+a)f^4}{}$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^4*e^4 - (4*a*c - b)*d^3*e^3*f + 2*(3*a*c^2 - b*c + a)*d^2*e^2*f^2
 - (4*a*c^3 - b*c^2 + 4*a*c - b)*d*e*f^3 + (a*c^4 + 2*a*c^2 + a)*f^4 + (b*d
^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 + b)*d*f^4)*x - (2*b*c*d^3*e^3*f - (5
*b*c^2 + 3*b)*d^2*e^2*f^2 + 4*(b*c^3 + b*c)*d*e*f^3 - (b*c^4 + 2*b*c^2 + b)
*f^4 + (b*d^4*e^2*f^2 - 2*b*c*d^3*e*f^3 + (b*c^2 - b)*d^2*f^4)*x^2 + 2*(b*d
^4*e^3*f - 2*b*c*d^3*e^2*f^2 + (b*c^2 - b)*d^2*e*f^3)*x)*arctan(d*x + c) +
(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3
*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b*d^3*e^
3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2
- b*c*d^2*e*f^3)*x)*log(f*x + e))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 +
 1)*d^2*e^4*f^3 - 4*(c^3 + c)*d*e^3*f^4 + (c^4 + 2*c^2 + 1)*e^2*f^5 + (d^4*
e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 + 1)*d^2*e^2*f^5 - 4*(c^3 + c)*d*e*f^6
 + (c^4 + 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2
 + 1)*d^2*e^3*f^4 - 4*(c^3 + c)*d*e^2*f^5 + (c^4 + 2*c^2 + 1)*e*f^6)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(c + dx)}{(e + fx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.80

$$\int \frac{a + b\arctan(c + dx)}{(e + fx)^3}\,dx =$$

$$-\frac{1}{2}\left(d\left(\frac{(d^2e - cdf)\log\left(d^2x^2 + 2\,cdx + c^2 + 1\right)}{d^4e^4 - 4\,cd^3e^3f + 2\,(3\,c^2 + 1)d^2e^2f^2 - 4\,(c^3 + c)def^3 + (c^4 + 2\,c^2 + 1)f^4} - \frac{}{d^4e^4 - 4\,cd^3e^3f + 2}\right.\right.$$
$$- \frac{a}{2\,(f^3x^2 + 2\,ef^2x + e^2f)}$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*((d^2*e - c*d*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^4*e^4 - 4*c*d^
3*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 +
1)*f^4) - 2*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^
2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - (d^4*e^
```

```
2 - 2*c*d^3*e*f + (c^2 - 1)*d^2*f^2)*arctan((d^2*x + c*d)/d)/((d^4*e^4*f -
4*c*d^3*e^3*f^2 + 2*(3*c^2 + 1)*d^2*e^2*f^3 - 4*(c^3 + c)*d*e*f^4 + (c^4 +
2*c^2 + 1)*f^5)*d) + 1/(d^2*e^3 - 2*c*d*e^2*f + (c^2 + 1)*e*f^2 + (d^2*e^2*
f - 2*c*d*e*f^2 + (c^2 + 1)*f^3)*x)) + arctan(d*x + c)/(f^3*x^2 + 2*e*f^2*x
 + e^2*f))*b - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx = \int \frac{b\arctan\left(dx+c\right)+a}{\left(fx+e\right)^3}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 8.02 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.76

$$\begin{aligned}
\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx &= \frac{b\,d^3\,e\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2}\\
&\quad -\frac{a\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}\\
&\quad -\frac{b\,d\,e}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}\\
&\quad -\frac{a\,c^2\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}\\
&\quad -\frac{b\,\mathrm{atan}(c+d\,x)}{2\,f\,{\left(e+f\,x\right)}^2}-\frac{b\,c\,d^2\,f\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2}\\
&\quad +\frac{a\,c\,d\,e}{{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}\\
&\quad -\frac{b\,d\,f\,x}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}\\
&\quad -\frac{a\,d^2\,e^2}{2\,f\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}\\
&\quad -\frac{b\,d^2\,\ln\left(c+d\,x-\mathrm{i}\right)\,1{}\mathrm{i}}{4\,f\,{\left(d\,e-c\,f+f\,1{}\mathrm{i}\right)}^2}+\frac{b\,d^2\,\ln\left(c+d\,x+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4\,f\,{\left(c\,f-d\,e+f\,1{}\mathrm{i}\right)}^2}
\end{aligned}$$

```
[In] int((a + b*atan(c + d*x))/(e + f*x)^3,x)
```

```
[Out] (b*d^2*log(c + d*x + 1i)*1i)/(4*f*(f*1i + c*f - d*e)^2) - (a*f)/(2*(e + f*x
)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d*e)/(2*(e + f*x)^2*(f^2 +
```

```
c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d^2*log(c + d*x - 1i)*1i)/(4*f*(f*1i -
 c*f + d*e)^2) - (b*atan(c + d*x))/(2*f*(e + f*x)^2) - (a*c^2*f)/(2*(e + f*
x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) + (b*d^3*e*log(e + f*x))/(f^2 +
 c^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 - (b*c*d^2*f*log(e + f*x))/(f^2 + c^2*f^2
 + d^2*e^2 - 2*c*d*e*f)^2 + (a*c*d*e)/((e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2
 - 2*c*d*e*f)) - (b*d*f*x)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*
e*f)) - (a*d^2*e^2)/(2*f*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f))
```

3.31 $\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 382

$$\begin{aligned}
&\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx \\
&= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad - \frac{2b^2f(de-cf)(c+dx)\arctan(c+dx)}{d^3} - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^2}{3d^3} \\
&\quad - \frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\,(a+b\arctan(c+dx))^2}{3d^3f} \\
&\quad + \frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f} \\
&\quad + \frac{2b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&\quad + \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&\quad + \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}$$

```
[Out] 1/3*b^2*f^2*x/d^2-2*a*b*f*(-c*f+d*e)*x/d^2-1/3*b^2*f^2*arctan(d*x+c)/d^3-2*
b^2*f*(-c*f+d*e)*(d*x+c)*arctan(d*x+c)/d^3-1/3*b*f^2*(d*x+c)^2*(a+b*arctan(
d*x+c))/d^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))^
2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arctan(d*x+c))^2
/d^3/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c))^2/f+2/3*b*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d^3+b^2*f*(-c*f+d*e)*l
n(1+(d*x+c)^2)/d^3+1/3*I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(2
,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5155, 4974, 4930, 266, 4946, 327, 209, 5104, 5004, 5040, 4964, 2449, 2352}

$$\begin{aligned}
&\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx \\
&= \frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)(a+b\arctan(c+dx))^2}{3d^3} \\
&\quad - \frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)(a+b\arctan(c+dx))^2}{3d^3f} \\
&\quad + \frac{2b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))}{3d^3} \\
&\quad - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3} + \frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f} \\
&\quad - \frac{2abfx(de-cf)}{d^2} - \frac{2b^2f(c+dx)\arctan(c+dx)(de-cf)}{d^3} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad + \frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{3d^3} \\
&\quad + \frac{b^2f(de-cf)\log\left((c+dx)^2+1\right)}{d^3} + \frac{b^2f^2x}{3d^2}
\end{aligned}$$

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) - (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTan[c + d*x
])/(3*d^3) - (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(
c + d*x)^2*(a + b*ArcTan[c + d*x]))/(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f
 - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2
- 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/(3*d^3*f) + ((e + f
*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*f) + (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/(3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 (a+b\arctan(x))^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f}$$

$$-\frac{(2b)\text{Subst}\Bigg(\int \Bigg(\frac{3f^2(de-cf)(a+b\arctan(x))}{d^3}+\frac{f^3x(a+b\arctan(x))}{d^3}+\frac{((de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cde}{d^3(1+x^2)}}{3f}$$

$$=\frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f}-\frac{(2b)\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-(1-3c^2)f^2)x)(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{3d^3f}-\frac{(2bf^2)\,\text{Subst}(\int x(a+b\arctan(x))\,dx,x,c+dx)}{3d^3}-\frac{(2bf(de-cf))\text{Subst}(\int (a+b\arctan(x))\,dx,x,c+dx)}{d^3}$$

$$=-\frac{2abf(de-cf)x}{d^2}-\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3}+\frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f}-\frac{(2b)\text{Subst}\Bigg(\int \left(\frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)(a+b\arctan(x))}{1+x^2}+\frac{f(3d^2e^2-6cdef-(1-3c^2)f^2)x(a+b\arctan(x))}{1+x^2}\right)\,dx,x}{3d^3f}+\frac{(b^2f^2)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,c+dx\right)}{3d^3}-\frac{(2b^2f(de-cf))\,\text{Subst}(\int \arctan(x)\,dx,x,c+dx)}{d^3}$$

$$=\frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{2b^2f(de-cf)(c+dx)\arctan(c+dx)}{d^3}-\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3}+\frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f}-\frac{(b^2f^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{3d^3}+\frac{(2b^2f(de-cf))\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d^3}-\frac{(2b(3d^2e^2-6cdef-(1-3c^2)f^2))\,\text{Subst}\left(\int \frac{x(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{3d^3}-\frac{(2b(de-cf)(d^2e^2-2cdef-(3-c^2)f^2))\,\text{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx,x,c+dx\right)}{3d^3f}$$

$$=\frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{b^2f^2\arctan(c+dx)}{3d^3}-\frac{2b^2f(de-cf)(c+dx)\arctan(c+dx)}{d^3}-\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3}+\frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)(a+b\arctan(c+dx))^2}{3d^3}-\frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)(a+b\arctan(c+dx))^2}{3d^3f}+\frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f}+\frac{b^2f(de-cf)\log(1+(c+dx)^2)}{d^3}+\frac{(2b(3d^2e^2-6cdef-(1-3c^2)f^2))\,\text{Subst}\left(\int \frac{a+b\arctan(x)}{i-x}\,dx,x,c+dx\right)}{3d^3}$$

$$
\begin{aligned}
&= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad - \frac{2b^2f(de-cf)(c+dx)\arctan(c+dx)}{d^3} - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^2}{3d^3} \\
&\quad - \frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\,(a+b\arctan(c+dx))^2}{3d^3f} \\
&\quad + \frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f} \\
&\quad + \frac{2b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&\quad + \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&\quad - \frac{(2b^2(3d^2e^2-6cdef-(1-3c^2)\,f^2))\operatorname{Subst}\left(\int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{3d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad - \frac{2b^2f(de-cf)(c+dx)\arctan(c+dx)}{d^3} - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^2}{3d^3} \\
&\quad - \frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\,(a+b\arctan(c+dx))^2}{3d^3f} \\
&\quad + \frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f} \\
&\quad + \frac{2b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&\quad + \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&\quad + \frac{(2ib^2(3d^2e^2-6cdef-(1-3c^2)\,f^2))\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad - \frac{2b^2f(de-cf)(c+dx)\arctan(c+dx)}{d^3} - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^2}{3d^3} \\
&\quad - \frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\,(a+b\arctan(c+dx))^2}{3d^3f} \\
&\quad + \frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f} \\
&\quad + \frac{2b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&\quad + \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&\quad + \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)\,f^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 801 vs. 2(382) = 764.

Time = 7.96 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.10

$$
\begin{aligned}
&\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx = a^2e^2x + a^2efx^2 + \frac{1}{3}a^2f^2x^3 \\
&\quad + \frac{ab(-dfx(6de-4cf+dfx)+2(3def-3c^2def+c^3f^2+3c(d^2e^2-f^2)+d^3x(3e^2+3efx+f^2x^2))\,\mathrm{arcta}}{3d^3} \\
&\quad + \frac{b^2e^2\left(\arctan(c+dx)\left((-i+c+dx)\arctan(c+dx)+2\log\left(1+e^{2i\arctan(c+dx)}\right)\right)-i\operatorname{PolyLog}\left(2,-e^{2i\,\mathrm{arctar}}\right.\right.}{d} \\
&\quad + \frac{b^2ef\left((1+2ic-c^2+d^2x^2)\arctan(c+dx)^2-2\arctan(c+dx)\left(c+dx+2c\log\left(1+e^{2i\arctan(c+dx)}\right)\right)+\mathrm{lo}\right.}{d^2} \\
&\quad + \frac{b^2f^2(1+(c+dx)^2)^{3/2}\left(\frac{c+dx}{\sqrt{1+(c+dx)^2}}+\frac{6c(c+dx)\arctan(c+dx)}{\sqrt{1+(c+dx)^2}}+\frac{3(c+dx)\arctan(c+dx)^2}{\sqrt{1+(c+dx)^2}}+\frac{3c^2(c+dx)\arctan(c+dx)^2}{\sqrt{1+(c+dx)^2}}+i\,\mathrm{ar}\right.}{}
\end{aligned}
$$

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(-(d*f*x*(6*d*e - 4*c*f +
d*f*x)) + 2*(3*d*e*f - 3*c^2*d*e*f + c^3*f^2 + 3*c*(d^2*e^2 - f^2) + d^3*x*
(3*e^2 + 3*e*f*x + f^2*x^2))*ArcTan[c + d*x] + (-3*d^2*e^2 + 6*c*d*e*f + (1
 - 3*c^2)*f^2)*Log[1 + (c + d*x)^2]))/(3*d^3) + (b^2*e^2*(ArcTan[c + d*x]*(
(-I + c + d*x)*ArcTan[c + d*x] + 2*Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*
```

```
PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/d + (b^2*e*f*((1 + (2*I)*c - c^2 +
 d^2*x^2)*ArcTan[c + d*x]^2 - 2*ArcTan[c + d*x]*(c + d*x + 2*c*Log[1 + E^((
2*I)*ArcTan[c + d*x])]) + Log[1 + (c + d*x)^2] + (2*I)*c*PolyLog[2, -E^((2*
I)*ArcTan[c + d*x])]))/d^2 + (b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c + d*x)/Sq
rt[1 + (c + d*x)^2] + (6*c*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2]
 + (3*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)
*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + I*ArcTan[c + d*x]^2*Cos[3*ArcTa
n[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - 2*ArcTan
[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c^2
*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])]
+ 6*c*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 + (c + d*x)^2]] + ((3*I - 12*c -
(9*I)*c^2)*ArcTan[c + d*x]^2 + 2*ArcTan[c + d*x]*(-2 + (-3 + 9*c^2)*Log[1 +
 E^((2*I)*ArcTan[c + d*x])]) + 18*c*Log[1/Sqrt[1 + (c + d*x)^2]])/Sqrt[1 +
(c + d*x)^2] - ((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/
(1 + (c + d*x)^2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6*c*ArcTan[c + d*x]*Sin[
3*ArcTan[c + d*x]] - ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTa
n[c + d*x]^2*Sin[3*ArcTan[c + d*x]]))/(12*d^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1017 vs. 2(362) = 724.

Time = 0.67 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.66

method	result	size
parts	Expression too large to display	1018
derivativedivides	Expression too large to display	1042
default	Expression too large to display	1042
risch	Expression too large to display	2416

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*(f*x+e)^3/f+b^2/d*(1/3/d^2*f^2*arctan(d*x+c)^2*(d*x+c)^3-1/d^2*f^2*
arctan(d*x+c)^2*(d*x+c)^2*c+1/d*f*arctan(d*x+c)^2*(d*x+c)^2*e+1/d^2*f^2*arc
tan(d*x+c)^2*(d*x+c)*c^2-2/d*f*arctan(d*x+c)^2*(d*x+c)*c*e+arctan(d*x+c)^2*
(d*x+c)*e^2-1/3/d^2*f^2*arctan(d*x+c)^2*c^3+1/d*f*arctan(d*x+c)^2*c^2*e-arc
tan(d*x+c)^2*c*e^2+1/3*d/f*arctan(d*x+c)^2*e^3-2/3/d^2/f*(1/2*arctan(d*x+c)
*f^3*(d*x+c)^2-3*arctan(d*x+c)*c*f^3*(d*x+c)+3*arctan(d*x+c)*d*e*f^2*(d*x+c
)+3/2*arctan(d*x+c)*ln(1+(d*x+c)^2)*c^2*f^3-3*arctan(d*x+c)*ln(1+(d*x+c)^2)
*c*d*e*f^2+3/2*arctan(d*x+c)*ln(1+(d*x+c)^2)*d^2*e^2*f-1/2*arctan(d*x+c)*ln
(1+(d*x+c)^2)*f^3-arctan(d*x+c)^2*c^3*f^3+3*arctan(d*x+c)^2*c^2*d*e*f^2-3*a
rctan(d*x+c)^2*c*d^2*e^2*f+arctan(d*x+c)^2*d^3*e^3+3*arctan(d*x+c)^2*c*f^3-
3*arctan(d*x+c)^2*d*e*f^2-1/2*f^2*(f*(d*x+c)+1/2*(-6*c*f+6*d*e)*ln(1+(d*x+c
)^2)-f*arctan(d*x+c))-1/2*f*(3*c^2*f^2-6*c*d*e*f+3*d^2*e^2-f^2)*(-1/2*I*(ln
```

```
(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+
c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+
I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))-1/4*(-2*c^3*f
^3+6*c^2*d*e*f^2-6*c*d^2*e^2*f+2*d^3*e^3+6*c*f^3-6*d*e*f^2)*arctan(d*x+c)^2
))+1/3*a*b/d^3*f^2*ln(1+(d*x+c)^2)+2/d*a*b*c*e^2*arctan(d*x+c)+2/3/d^3*f^2*
b*a*c^3*arctan(d*x+c)+4/3/d^2*c*f^2*x*b*a-2/d*e*x*f*b*a-a*b/d^3*f^2*ln(1+(d
*x+c)^2)*c^2-a*b/d*ln(1+(d*x+c)^2)*e^2-2/d^3*c*f^2*b*a*arctan(d*x+c)-1/3/d*
f^2*b*a*x^2-2*b/d^2*arctan(d*x+c)*a*c^2*e*f+2*a*b/d^2*f*ln(1+(d*x+c)^2)*c*e
+2*a*b*f*arctan(d*x+c)*e*x^2-2/d^2*c*f*e*b*a+5/3/d^3*c^2*f^2*b*a+2/d^2*e*f*
b*a*arctan(d*x+c)+2/3*a*b*f^2*arctan(d*x+c)*x^3+2*a*b*arctan(d*x+c)*x*e^2
```

Fricas [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\int (fx+e)^2(b\arctan\left(dx+c\right)+a)^2\,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x +
 b^2*e^2)*arctan(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arcta
n(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\int (fx+e)^2(b\arctan\left(dx+c\right)+a)^2\,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan
(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^
2*e^2 + 1/3*a^2*f^2*x^3 + 36*b^2*d^2*f^2*integrate(1/48*x^4*arctan(d*x + c)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*f^2*integrate(1/48*x^4*log
```

```
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*
d^2*e*f*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) + 72*b^2*c*d*f^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 4*b^2*d^2*f^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*d^2*e*f*integrate(1/48*
x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) +
6*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*d^2*e^2*integrate(1/48*x^2*arctan(d*x +
 c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144*b^2*c*d*e*f*integrate(1/48*x^
2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*c^2*f^2*inte
grate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2
*d^2*e*f*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 4*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*e^2*integrate(1/
48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 12*b^2*c*d*e*f*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*f^2*integrate(1/48*x^2*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*c*d*e^
2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72
*b^2*c^2*e*f*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 12*b^2*d^2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e*f*integrate(1/48*x^2*log(
d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*
e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 6*b^2*c^2*e*f*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*e^2*integrate(1/48*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2
*c^2*e^2*integrate(1/48*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + a^2*e*f*x^2 + 3/4*b^2*e^2*arctan(d*x + c)^2*arctan((d^2
*x + c*d)/d)/d - 8*b^2*d*f^2*integrate(1/48*x^3*arctan(d*x + c)/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) - 24*b^2*d*e*f*integrate(1/48*x^2*arctan(d*x + c)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^2*d*e^2*integrate(1/48*x*arctan(d*x
+ c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 1/4*(3*arctan(d*x + c)*arctan((d^2
*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*e^2 + 2*(x^2*arctan(d*x
 + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 +
2*c*d*x + c^2 + 1)/d^3))*a*b*e*f + 1/3*(2*x^3*arctan(d*x + c) - d*((d*x^2 -
 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d
^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*f^2 + a^2*e^2*x + 36*b^2*f^2*integrat
e(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*f^2*
integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 72*b^2*e*f*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 6*b^2*e*f*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*e^2*integrate(1/48*log(d
^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + (2*(d*x +
 c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*e^2/d + 1/12*(b^2*f^2*x^3 +
```

```
 3*b^2*e*f*x^2 + 3*b^2*e^2*x)*arctan(d*x + c)^2 - 1/48*(b^2*f^2*x^3 + 3*b^2
*e*f*x^2 + 3*b^2*e^2*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Giac [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\int (fx+e)^2(b\arctan(dx+c)+a)^2\,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\int (e+f\,x)^2\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^2\,dx$$

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x))^2,x)
[Out] int((e + f*x)^2*(a + b*atan(c + d*x))^2, x)
```

3.32 $\int (e+fx)(a+b\arctan(c+dx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 222

$$\begin{aligned}
&\int (e+fx)(a+b\arctan(c+dx))^2\,dx \\
&= -\frac{abfx}{d}-\frac{b^2f(c+dx)\arctan(c+dx)}{d^2}+\frac{i(de-cf)(a+b\arctan(c+dx))^2}{d^2} \\
&\quad -\frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^2}{2d^2f} \\
&\quad +\frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}+\frac{2b(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad +\frac{b^2f\log\left(1+(c+dx)^2\right)}{2d^2}+\frac{ib^2(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}
\end{aligned}$$

```
[Out] -a*b*f*x/d-b^2*f*(d*x+c)*arctan(d*x+c)/d^2+I*(-c*f+d*e)*(a+b*arctan(d*x+c))
^2/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a+b*arctan(d*x+c))^2/d^2/f+1/2*(f*x+
e)^2*(a+b*arctan(d*x+c))^2/f+2*b*(-c*f+d*e)*(a+b*arctan(d*x+c))*ln(2/(1+I*(
d*x+c)))/d^2+1/2*b^2*f*ln(1+(d*x+c)^2)/d^2+I*b^2*(-c*f+d*e)*polylog(2,1-2/(
1+I*(d*x+c)))/d^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5155, 4974, 4930, 266, 5104, 5004, 5040, 4964, 2449, 2352}

$$\begin{aligned}
&\int (e+fx)(a+b\arctan(c+dx))^2\,dx \\
&= \frac{i(de-cf)(a+b\arctan(c+dx))^2}{d^2} \\
&\quad - \frac{(-cf+de+f)(de-(c+1)f)(a+b\arctan(c+dx))^2}{2d^2f} \\
&\quad + \frac{2b(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))}{d^2} \\
&\quad + \frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f} - \frac{abfx}{d} - \frac{b^2f(c+dx)\arctan(c+dx)}{d^2} \\
&\quad + \frac{ib^2(de-cf)\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{d^2} + \frac{b^2f\log\left((c+dx)^2+1\right)}{2d^2}
\end{aligned}$$

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] -((a*b*f*x)/d) - (b^2*f*(c + d*x)*ArcTan[c + d*x])/d^2 + (I*(d*e - c*f)*(a
+ b*ArcTan[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*Arc
Tan[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*f)
+ (2*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 +
(b^2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/
(1 + I*(c + d*x))])/d^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)(a+b\arctan(x))^2\,dx,x,c+dx\right)}{d}\\
&= \frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}\\
&\quad -\frac{b\text{Subst}\left(\int \left(\frac{f^2(a+b\arctan(x))}{d^2}+\frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b\arctan(x))}{d^2(1+x^2)}\right)\,dx,x,c+dx\right)}{f}\\
&= \frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}\\
&\quad -\frac{b\text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{d^2f}\\
&\quad -\frac{(bf)\text{Subst}(\int (a+b\arctan(x))\,dx,x,c+dx)}{d^2}\\
&= -\frac{abfx}{d}+\frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}\\
&\quad -\frac{b\text{Subst}\left(\int \left(\frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(x))}{1+x^2}-\frac{2f(-de+cf)x(a+b\arctan(x))}{1+x^2}\right)\,dx,x,c+dx\right)}{d^2f}\\
&\quad -\frac{(b^2f)\,\text{Subst}(\int \arctan(x)\,dx,x,c+dx)}{d^2}\\
&= -\frac{abfx}{d}-\frac{b^2f(c+dx)\arctan(c+dx)}{d^2}\\
&\quad +\frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}+\frac{(b^2f)\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d^2}\\
&\quad -\frac{(2b(de-cf))\text{Subst}\left(\int \frac{x(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{d^2}\\
&\quad -\frac{(b(de+f-cf)(de-(1+c)f))\text{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx,x,c+dx\right)}{d^2f}\\
&= -\frac{abfx}{d}-\frac{b^2f(c+dx)\arctan(c+dx)}{d^2}+\frac{i(de-cf)(a+b\arctan(c+dx))^2}{d^2}\\
&\quad -\frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^2}{2d^2f}\\
&\quad +\frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}+\frac{b^2f\log\left(1+(c+dx)^2\right)}{2d^2}\\
&\quad +\frac{(2b(de-cf))\text{Subst}\left(\int \frac{a+b\arctan(x)}{i-x}\,dx,x,c+dx\right)}{d^2}
\end{aligned}
$$

$$= -\frac{abfx}{d} - \frac{b^2 f(c+dx)\arctan(c+dx)}{d^2} + \frac{i(de-cf)(a+b\arctan(c+dx))^2}{d^2}$$
$$- \frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^2}{2d^2 f}$$
$$+ \frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}$$
$$+ \frac{2b(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}$$
$$+ \frac{b^2 f\log\left(1+(c+dx)^2\right)}{2d^2} - \frac{(2b^2(de-cf))\,\mathrm{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d^2}$$

$$= -\frac{abfx}{d} - \frac{b^2 f(c+dx)\arctan(c+dx)}{d^2} + \frac{i(de-cf)(a+b\arctan(c+dx))^2}{d^2}$$
$$- \frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^2}{2d^2 f}$$
$$+ \frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}$$
$$+ \frac{2b(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}$$
$$+ \frac{b^2 f\log\left(1+(c+dx)^2\right)}{2d^2} + \frac{(2ib^2(de-cf))\,\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+i(c+dx)}\right)}{d^2}$$

$$= -\frac{abfx}{d} - \frac{b^2 f(c+dx)\arctan(c+dx)}{d^2} + \frac{i(de-cf)(a+b\arctan(c+dx))^2}{d^2}$$
$$- \frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^2}{2d^2 f}$$
$$+ \frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}$$
$$+ \frac{2b(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}$$
$$+ \frac{b^2 f\log\left(1+(c+dx)^2\right)}{2d^2} + \frac{ib^2(de-cf)\,\mathrm{PolyLog}\left(2, 1-\frac{2}{1+i(c+dx)}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.19

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx$$

$$= \frac{2a^2cde-2abcf-a^2c^2f+2a^2d^2ex-2abdfx+a^2d^2fx^2+b^2(-i+c+dx)(2de+if-cf+dfx)\arctan(c}{}$$

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (2*a^2*c*d*e - 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x - 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(-I + c + d*x)*(2*d*e + I*f - c*f + d*f*x)*ArcTan[c + d*x]^2
- 2*b*ArcTan[c + d*x]*(b*f*(c + d*x) + a*(-2*c*d*e + c^2*f - 2*d^2*e*x - f*
(1 + d^2*x^2)) - 2*b*(d*e - c*f)*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + 4*a*
b*d*e*Log[1/Sqrt[1 + (c + d*x)^2]] - 2*b^2*f*Log[1/Sqrt[1 + (c + d*x)^2]] -
 4*a*b*c*f*Log[1/Sqrt[1 + (c + d*x)^2]] - (2*I)*b^2*(d*e - c*f)*PolyLog[2,
-E^((2*I)*ArcTan[c + d*x])])/(2*d^2)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.86

method	result
parts	$a^2\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b^2\left(\frac{\arctan(dx+c)^2(dx+c)^2f}{2d}-\frac{\arctan(dx+c)^2cf(dx+c)}{d}+\arctan(dx+c)^2e(dx+c)-\frac{-\ln\left(1+(dx+c)^2\right)}{}\right.}{}$
derivativedivides	$\frac{-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\arctan(dx+c)^2fc(dx+c)-\arctan(dx+c)^2ed(dx+c)-\frac{\arctan(dx+c)^2f(dx+c)^2}{2}-\ln\left(1+\right.\right.}{}}{}$
default	$\frac{-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\arctan(dx+c)^2fc(dx+c)-\arctan(dx+c)^2ed(dx+c)-\frac{\arctan(dx+c)^2f(dx+c)^2}{2}-\ln\left(1+\right.\right.}{}}{}$
risch	Expression too large to display

```
[In] int((f*x+e)*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*f*x^2+e*x)+b^2/d*(1/2/d*arctan(d*x+c)^2*(d*x+c)^2*f-1/d*arctan(d*x
+c)^2*c*f*(d*x+c)+arctan(d*x+c)^2*e*(d*x+c)-1/d*(-ln(1+(d*x+c)^2)*arctan(d*
x+c)*c*f+ln(1+(d*x+c)^2)*arctan(d*x+c)*d*e-1/2*arctan(d*x+c)^2*f+arctan(d*x
+c)*(d*x+c)*f-1/2*f*ln(1+(d*x+c)^2)-1/2*(-2*c*f+2*d*e)*(-1/2*I*(ln(d*x+c-I)
*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-
1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilo
g(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))))+2*a*b/d*(1/2/d*arcta
n(d*x+c)*(d*x+c)^2*f-1/d*arctan(d*x+c)*c*f*(d*x+c)+arctan(d*x+c)*e*(d*x+c)-
1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(1+(d*x+c)^2)-f*arctan(d*x+c)))
```

Fricas [F]

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx=\int (fx+e)(b\arctan(dx+c)+a)^2\,dx$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arctan(d*x + c)^2 + 2*(a*b*f*x
 + a*b*e)*arctan(d*x + c), x)
```

Sympy [F]

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx=\int \left(a+b\operatorname{atan}\left(c+dx\right)\right)^2\left(e+fx\right)\,dx$$

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2*(e + f*x), x)
```

Maxima [F]

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx=\int (fx+e)(b\arctan\left(dx+c\right)+a)^2\,dx$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan(d
*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*
e + 12*b^2*d^2*f*integrate(1/16*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + b^2*d^2*f*integrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*e*integrate(1/16*x^2*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*f*integrate(1
/16*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*d^2*f*i
ntegrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*
e*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12
*b^2*c^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 4*b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*e*integr
ate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + b^2*c^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*e*integrate(1/16*x*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^2*c^2*e*integrate(1/
16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1
/2*a^2*f*x^2 + 3/4*b^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*b^
2*d*f*integrate(1/16*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
- 8*b^2*d*e*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x +
c*d)/d)^3/d)*b^2*e + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^
2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f + a^2*e*
```

```
x + 12*b^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + b^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
 + 2*c*d*x + c^2 + 1), x) + b^2*e*integrate(1/16*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + (2*(d*x + c)*arctan(d*x + c) -
 log((d*x + c)^2 + 1))*a*b*e/d + 1/8*(b^2*f*x^2 + 2*b^2*e*x)*arctan(d*x + c
)^2 - 1/32*(b^2*f*x^2 + 2*b^2*e*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Giac [F]

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx = \int (fx+e)(b\arctan(dx+c)+a)^2\,dx$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx = \int (e+f\,x)\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^2\,dx$$

```
[In] int((e + f*x)*(a + b*atan(c + d*x))^2,x)
```

```
[Out] int((e + f*x)*(a + b*atan(c + d*x))^2, x)
```

3.33 $\int (a+b\arctan(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int (a+b\arctan(c+dx))^2\,dx=\frac{i(a+b\arctan(c+dx))^2}{d}+\frac{(c+dx)(a+b\arctan(c+dx))^2}{d}+\frac{2b(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}$$

```
[Out] I*(a+b*arctan(d*x+c))^2/d+(d*x+c)*(a+b*arctan(d*x+c))^2/d+2*b*(a+b*arctan(d
*x+c))*ln(2/(1+I*(d*x+c)))/d+I*b^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5147, 4930, 5040, 4964, 2449, 2352}

$$\int (a+b\arctan(c+dx))^2\,dx=\frac{(c+dx)(a+b\arctan(c+dx))^2}{d}+\frac{i(a+b\arctan(c+dx))^2}{d}+\frac{2b\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))}{d}+\frac{ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{d}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (I*(a + b*ArcTan[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^2)/d +
 (2*b*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d + (I*b^2*PolyLog[
2, 1 - 2/(1 + I*(c + d*x))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+b\arctan(x))^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)(a+b\arctan(c+dx))^2}{d} - \frac{(2b)\text{Subst}\left(\int \frac{x(a+b\arctan(x))}{1+x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{i(a+b\arctan(c+dx))^2}{d} + \frac{(c+dx)(a+b\arctan(c+dx))^2}{d} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\arctan(x)}{i-x}\,dx, x, c+dx\right)}{d}$$

$$= \frac{i(a+b\arctan(c+dx))^2}{d} + \frac{(c+dx)(a+b\arctan(c+dx))^2}{d} + \frac{2b(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{i(a+b\arctan(c+dx))^2}{d} + \frac{(c+dx)(a+b\arctan(c+dx))^2}{d} + \frac{2b(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d} + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+i(c+dx)}\right)}{d}$$

$$= \frac{i(a+b\arctan(c+dx))^2}{d} + \frac{(c+dx)(a+b\arctan(c+dx))^2}{d} + \frac{2b(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d} + \frac{ib^2\,\text{PolyLog}\left(2, 1-\frac{2}{1+i(c+dx)}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int (a+b\arctan(c+dx))^2\,dx$$

$$= \frac{b^2(-i+c+dx)\arctan(c+dx)^2 + 2b\arctan(c+dx)\left(ac+adx+b\log\left(1+e^{2i\arctan(c+dx)}\right)\right) + a\Big(ac+adx}{d}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(a*c + a*d*x +
b*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + a*(a*c + a*d*x + 2*b*Log[1/Sqrt[1 +
 (c + d*x)^2]]) - I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36

method	result
parts	$a^{2}x+\frac{b^{2}\left(\arctan(dx+c)^{2}(dx+c+i)+2\arctan(dx+c)\ln\left(1+\frac{(1+i(dx+c))^{2}}{1+(dx+c)^{2}}\right)-2i\arctan(dx+c)^{2}-i\operatorname{polylog}\left(2,-\frac{(1+i(dx}{1+(dx}\right.\right.}{d}$
derivativedivides	$\frac{(dx+c)a^{2}-i\arctan(dx+c)^{2}b^{2}+\arctan(dx+c)^{2}b^{2}(dx+c)-i\operatorname{polylog}\left(2,-\frac{(1+i(dx+c))^{2}}{1+(dx+c)^{2}}\right)b^{2}+2\arctan(dx+c)\ln\left(1+\frac{(1+i(d}{1+(d}\right.}{d}$
default	$\frac{(dx+c)a^{2}-i\arctan(dx+c)^{2}b^{2}+\arctan(dx+c)^{2}b^{2}(dx+c)-i\operatorname{polylog}\left(2,-\frac{(1+i(dx+c))^{2}}{1+(dx+c)^{2}}\right)b^{2}+2\arctan(dx+c)\ln\left(1+\frac{(1+i(d}{1+(d}\right.}{d}$
risch	$-\frac{b^{2}(dx+c-i)\ln(1+i(dx+c))^{2}}{4d}-\frac{\ln(-idx-ic+1)^{2}b^{2}c}{4d}-\frac{\ln(-idx-ic+1)ab}{d}+\left(\frac{b^{2}x\ln(1-i(dx+c))}{2}-\frac{ib(2axd-b}{}\right.$

```
[In] int((a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+b^2/d*(arctan(d*x+c)^2*(d*x+c+I)+2*arctan(d*x+c)*ln(1+(1+I*(d*x+c))^2
/(1+(d*x+c)^2))-2*I*arctan(d*x+c)^2-I*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)
^2)))+2*a*b/d*((d*x+c)*arctan(d*x+c)-1/2*ln(1+(d*x+c)^2))
```

Fricas [F]

$$\int (a+b\arctan(c+dx))^2\,dx=\int (b\arctan(dx+c)+a)^2\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2, x)
```

Sympy [F]

$$\int (a+b\arctan(c+dx))^2\,dx=\int (a+b\operatorname{atan}(c+dx))^2\,dx$$

```
[In] integrate((a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2, x)
```

Maxima [F]

$$\int (a+b\arctan(c+dx))^2\,dx=\int (b\arctan(dx+c)+a)^2\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(12*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*(3*arctan(d*x
+ c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*c^2 + 4*x*a
rctan(d*x + c)^2 + 192*d^2*integrate(1/16*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 16*d^2*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*c*d*integrate(1/16*x*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 64*d^2*integrate(1/16*x^
2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 32*c
*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
 c^2 + 1), x) + 64*c*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*c^2*integrate(1/16*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - x*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2 + 12*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*arctan(
d*x + c)*arctan((d^2*x + c*d)/d)^2/d + 4*arctan((d^2*x + c*d)/d)^3/d - 128*
d*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*i
ntegrate(1/16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x))*b^2 + a^2*x + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))
*a*b/d
```

Giac [F]

$$\int (a+b\arctan(c+dx))^2\,dx=\int (b\arctan(dx+c)+a)^2\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\arctan(c+dx))^2\,dx=\int (a+b\,\mathrm{atan}(c+d\,x))^2\,dx$$

```
[In] int((a + b*atan(c + d*x))^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^2, x)
```

3.34 $\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 261

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx &= -\frac{(a+b\arctan(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\ &\quad+\frac{(a+b\arctan(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &\quad+\frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{f} \\ &\quad-\frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &\quad-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}\end{aligned}$$

```
[Out] -(a+b*arctan(d*x+c))^2*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))^2*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+I*b*(a+b*arctan(d*x+c))*polylog(2,1-2
/(1-I*(d*x+c)))/f-I*b*(a+b*arctan(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-
c*f)/(1-I*(d*x+c)))/f-1/2*b^2*polylog(3,1-2/(1-I*(d*x+c)))/f+1/2*b^2*polylo
g(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5155, 4968}

$$\begin{aligned}
&\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx \\
&= -\frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{f} \\
&\quad + \frac{(a+b\arctan(c+dx))^2\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} \\
&\quad + \frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{f} \\
&\quad - \frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))^2}{f} \\
&\quad + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}
\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (I*b*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*
(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) +
(b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])
/(2*f)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
```

```
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}\\ &= -\frac{(a+b\arctan(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f}\\ &\quad+\frac{(a+b\arctan(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}\\ &\quad+\frac{ib(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{f}\\ &\quad-\frac{ib(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}\\ &\quad-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}\end{aligned}$$

Mathematica [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
[Out] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.59 (sec) , antiderivative size = 1877, normalized size of antiderivative = 7.19

method	result	size
derivativedivides	Expression too large to display	1877
default	Expression too large to display	1877
parts	Expression too large to display	1998

```
[In] int((a+b*arctan(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f-b^2*d*(-ln(c*f-d*e-f*(d*x+c))/f*arctan(d
*x+c)^2+2/f*(1/2*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)
-1/4*I*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^
2/(1+(d*x+c)^2)))*(csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+
c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I/
(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2
)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c
*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*
x+c)^2)))-csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))+csgn(I*(
I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+
I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^
2)*arctan(d*x+c)^2-1/2*I*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c
)^2))+1/4*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-1/2*f/(c*f-d*e+I*f)*arc
tan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c
*f))-1/2*I*f/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))
^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/4*I*f/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*
f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/2*I*c*f/(c*f-d*e+I*f)*arc
tan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c
*f))-1/2*c*f/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))
^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/4*c*f/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*
f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-I*d*e*arctan(d*x+c)*polylog
(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f-
2*d*e)+1/2*d*e/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/4*d*e/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+
I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))))-2*a*b*d*(-ln(c*f-d*e-f*
(d*x+c))/f*arctan(d*x+c)+1/2*I*ln(c*f-d*e-f*(d*x+c))*(ln((I*f+f*(d*x+c))/(c
*f-d*e+I*f))-ln((I*f-f*(d*x+c))/(d*e+I*f-c*f)))/f+1/2*I*(dilog((I*f+f*(d*x+
c))/(c*f-d*e+I*f))-dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f)))/f))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx=\int \frac{(b\arctan{(dx+c)}+a)^2}{fx+e}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f*x + e), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(d*x+c))**2/(f*x+e),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{fx+e}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="maxima")
[Out] a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(f*x + e), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^2}{e+f\,x}\,dx$$

```
[In] int((a + b*atan(c + d*x))^2/(e + f*x),x)
[Out] int((a + b*atan(c + d*x))^2/(e + f*x), x)
```

3.35 $\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 568

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx &= \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)}+\frac{ib^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&+\frac{b^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}-\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}\\
&+\frac{2abd\log(e+fx)}{f^2+(de-cf)^2}-\frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&+\frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&+\frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&-\frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2}+\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&+\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\end{aligned}$$

```
[Out] 2*a*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)+I*b^2*d*arctan(d*x+c)
^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^2*d*(-c*f+d*e)*arctan(d*x+c)^2/f/(d^2*
e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arctan(d*x+c))^2/f/(f*x+e)+2*a*b*d*ln(f*x+e
)/(f^2+(-c*f+d*e)^2)-2*b^2*d*arctan(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+2*b^2*d*arctan(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I
```

```
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+2*b^2*d*arctan(d*x+c)*ln(2/(1+I*
(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-a*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+
d*e)^2)+I*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2
)-I*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+I*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f
+(c^2+1)*f^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5153, 2007, 719, 31, 648, 632, 210, 642, 6873, 5165, 720, 649, 209, 266, 6820, 12, 6857, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964}

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx = & \frac{2abd\arctan(c+dx)(de-cf)}{f\left((de-cf)^2+f^2\right)} \\ & -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}+\frac{2abd\log(e+fx)}{(de-cf)^2+f^2} \\ & -\frac{abd\log\left((c+dx)^2+1\right)}{(de-cf)^2+f^2}+\frac{ib^2d\arctan(c+dx)^2}{\left(c^2+1\right)f^2-2cdef+d^2e^2} \\ & +\frac{b^2d\arctan(c+dx)^2(de-cf)}{f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)} \\ & -\frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2} \\ & +\frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(de+(-c+i)f)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2} \\ & +\frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2} \\ & +\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2} \\ & -\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2} \\ & +\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (2*a*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + (I*b^2*d*
ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(d*e - c*
```

```
f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (a + b*Ar
cTan[c + d*x])^2/(f*(e + f*x)) + (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) - (2*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) + (2*b^2*d*ArcTan[c + d*x]*Log[(2*d*(e + f*x))/((d*e + (
I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*b
^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) - (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
 (I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5165

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}+\frac{(2bd)\int \frac{a+b\arctan(c+dx)}{(e+fx)(1+(c+dx)^2)}\,dx}{f}\\
&= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}+\frac{(2bd)\int \frac{a+b\arctan(c+dx)}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}\\
&= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}+\frac{(2b)\text{Subst}\left(\int \frac{a+b\arctan(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx,x,c+dx\right)}{f}\\
&= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}+\frac{(2b)\text{Subst}\left(\int \frac{d(a+b\arctan(x))}{(de-cf+fx)(1+x^2)}\,dx,x,c+dx\right)}{f}
\end{aligned}$$

$$= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)} + \frac{(2bd)\text{Subst}\left(\int \frac{a+b\arctan(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)} + \frac{(2bd)\text{Subst}\left(\int \left(\frac{a}{(de-cf+fx)(1+x^2)} + \frac{b\arctan(x)}{(de-cf+fx)(1+x^2)}\right)\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)} + \frac{(2abd)\text{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} + \frac{(2b^2d)\,\text{Subst}\left(\int \frac{\arctan(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)} + \frac{(2b^2d)\,\text{Subst}\left(\int \left(\frac{f^2\arctan(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} + \frac{(de-cf-fx)\arctan(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(1+x^2)}\right)\,dx, x, c+dx\right)}{f} + \frac{(2abd)\text{Subst}\left(\int \frac{de-cf-fx}{1+x^2}\,dx, x, c+dx\right)}{f\left(f^2+(de-cf)^2\right)} + \frac{(2abdf)\text{Subst}\left(\int \frac{1}{de-cf+fx}\,dx, x, c+dx\right)}{f^2+(de-cf)^2}$$

$$= -\frac{(a+b\arctan(c+dx))^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{(2b^2d)\,\text{Subst}\left(\int \frac{(de-cf-fx)\arctan(x)}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} + \frac{(2b^2df)\,\text{Subst}\left(\int \frac{\arctan(x)}{de-cf+fx}\,dx, x, c+dx\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} - \frac{(2abd)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{f^2+(de-cf)^2} + \frac{(2abd(de-cf))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{f\left(f^2+(de-cf)^2\right)}$$

$$
\begin{aligned}
&= \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{(a+b\arctan(c+dx))^2}{f(e+fx)} \\
&\quad + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad + \frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad - \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{(2b^2d)\,\mathrm{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad - \frac{(2b^2d)\,\mathrm{Subst}\left(\int \frac{\log\left(\frac{2(de-cf+fx)}{(de+if-cf)(1-ix)}\right)}{1+x^2}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad + \frac{(2b^2d)\,\mathrm{Subst}\left(\int \left(\frac{de\left(1-\frac{cf}{de}\right)\arctan(x)}{1+x^2} - \frac{fx\arctan(x)}{1+x^2}\right)dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{(a+b\arctan(c+dx))^2}{f(e+fx)} \\
&\quad + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad + \frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad - \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} - \frac{ib^2d\,\mathrm{PolyLog}\left(2, 1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad + \frac{(2ib^2d)\,\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{(2b^2d)\,\mathrm{Subst}\left(\int \frac{x\arctan(x)}{1+x^2}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad + \frac{(2b^2d(de-cf))\,\mathrm{Subst}\left(\int \frac{\arctan(x)}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}
\end{aligned}
$$

$$= \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{(a+b\arctan(c+dx))^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{(2b^2d)\,\text{Subst}\left(\int\frac{\arctan(x)}{i-x}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$= \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{(a+b\arctan(c+dx))^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{(2b^2d)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$= \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$+ \frac{b^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} - \frac{(a+b\arctan(c+dx))^2}{f(e+fx)}$$

$$+ \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$+ \frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$+ \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2}$$

$$+ \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$+ \frac{(2ib^2d)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$= \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{ib^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$+ \frac{b^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} - \frac{(a+b\arctan(c+dx))^2}{f(e+fx)}$$

$$+ \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$+ \frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$+ \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$- \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$- \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

Mathematica [A] (verified)

Time = 5.05 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx$$

$$= \frac{-\frac{a^2}{f}+\frac{2ab\left(-\left(\left(-cde+f+c^2f-d^2ex+cdfx\right)\arctan(c+dx)\right)+d(e+fx)\log\left(\frac{d(e+fx)}{\sqrt{1+(c+dx)^2}}\right)\right)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(e+fx)\left(-\frac{e^{i\arctan\left(\frac{de-cf}{f}\right)}\arctan(c+dx)^2}{f\sqrt{1+\frac{(de-cf)^2}{f^2}}}+\right.}{}}{}$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (-(a^2/f) + (2*a*b*(-((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcTan[c +
 d*x]) + d*(e + f*x)*Log[(d*(e + f*x))/Sqrt[1 + (c + d*x)^2]]))/(d^2*e^2 -
2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(-((E^(I*ArcTan[(d*e - c*f)/f
])*ArcTan[c + d*x]^2)/(f*Sqrt[1 + (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTan[
c + d*x]^2)/(d*(e + f*x)) - ((d*e - c*f)*((-I)*(Pi - 2*ArcTan[(d*e - c*f)/f
])*ArcTan[c + d*x] - Pi*Log[1 + E^((-2*I)*ArcTan[c + d*x])] - 2*(ArcTan[(d*
e - c*f)/f] + ArcTan[c + d*x])*Log[1 - E^((2*I)*(ArcTan[(d*e - c*f)/f] + Ar
cTan[c + d*x]))] + Pi*Log[1/Sqrt[1 + (c + d*x)^2]] + 2*ArcTan[(d*e - c*f)/f
]*Log[Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2, E^((2*I)
*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))]))/(f^2*(1 + (d*e - c*f)^2/f^2)
)))/(d*e - c*f))/(e + f*x)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.37

method	result
parts	$-\frac{a^2}{(fx+e)f}+\frac{b^2\Big(-\frac{d^2\arctan(dx+c)^2}{(f(dx+c)-cf+de)f}+\frac{2d^2\Big(\frac{\arctan(dx+c)f\ln(f(dx+c)-cf+de)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{\arctan(dx+c)f\ln\left(1+(dx+c)^2\right)}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)}-\frac{\mathrm{arcta}}{c^2f^2-2c}}{}}{}$
derivativedivides	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\Big(\frac{\arctan(dx+c)^2}{(cf-de-f(dx+c))f}-\frac{2\Big(\frac{\arctan(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{\arctan(dx+c)^2cf}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{\arctan(dx+c)^2}{c^2f^2-2cdef+d^2e}}{}}{}$
default	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\Big(\frac{\arctan(dx+c)^2}{(cf-de-f(dx+c))f}-\frac{2\Big(\frac{\arctan(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{\arctan(dx+c)^2cf}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{\arctan(dx+c)^2}{c^2f^2-2cdef+d^2e}}{}}{}$

```
[In] int((a+b*arctan(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/(f*x+e)/f+b^2/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)^2+2*d^2/f*(a
rctan(d*x+c)*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)-1/2*ar
ctan(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)-1/(c^2*f^2-2*
c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*c*f+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*
arctan(d*x+c)^2*d*e-f^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*I*ln(f*(d*x+c
)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*f-d*e+I
*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-dilog((I*f+f*(d*x+c))/(
c*f-d*e+I*f)))/f)+1/2*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*I*(ln(d*x+c-I
```

```
)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(
-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dil
og(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))+1/2/(c^2*f^2-2*c*d*e*
f+d^2*e^2+f^2)*(c*f-d*e)*arctan(d*x+c)^2))+2*a*b/d*(-d^2/(f*(d*x+c)-c*f+d*e
)/f*arctan(d*x+c)+d^2/f*(1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c
*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*f*ln(1+(d*x+c)^2)+(-c*f+d*e
)*arctan(d*x+c))))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx=\int \frac{(b\arctan\left(dx+c\right)+a)^2}{(fx+e)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f^2*x^2 + 2
*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*atan(d*x+c))**2/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx=\int \frac{(b\arctan\left(dx+c\right)+a)^2}{(fx+e)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] (d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2 + (
c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*f +
(c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2)) - 2*
arctan(d*x + c)/(f^2*x + e*f))*a*b - 1/16*(4*arctan(d*x + c)^2 - 16*(f^2*x
+ e*f)*integrate(1/16*(12*(d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*arctan(d*x
+ c)^2 + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2 + 8*(d*f*x + d*e)*arctan(d*x + c) - 4*(d^2*f*x^2 + c*d*e + (d^2*e +
```

```
c*d*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f
+ 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x
^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)*x), x) - log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2)*b^2/(f^2*x + e*f) - a^2/(f^2*x + e*f)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^2}{{\left(e+f\,x\right)}^2}\,dx$$

```
[In] int((a + b*atan(c + d*x))^2/(e + f*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(e + f*x)^2, x)
```

3.36 $\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 564

$$
\begin{aligned}
&\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx \\
&= \frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\arctan(c+dx)}{d^3}-\frac{bf^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad -\frac{3ibf(de-cf)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad -\frac{3bf(de-cf)(c+dx)(a+b\arctan(c+dx))^2}{d^3}-\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad +\frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^3}{3d^3} \\
&\quad -\frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\,(a+b\arctan(c+dx))^3}{3d^3f} \\
&\quad +\frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f} \\
&\quad -\frac{6b^2f(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad +\frac{b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad -\frac{b^3f^2\log\left(1+(c+dx)^2\right)}{2d^3}-\frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad +\frac{ib^2(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad +\frac{b^3(3d^2e^2-6cdef-(1-3c^2)\,f^2)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^3}
\end{aligned}
$$

```
[Out] a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arctan(d*x+c)/d^3-1/2*b*f^2*(a+b*arctan(d*x
+c))^2/d^3-3*I*b*f*(-c*f+d*e)*(a+b*arctan(d*x+c))^2/d^3-3*b*f*(-c*f+d*e)*(d
*x+c)*(a+b*arctan(d*x+c))^2/d^3-1/2*b*f^2*(d*x+c)^2*(a+b*arctan(d*x+c))^2/d
^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))^3/d^3-1/3
*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arctan(d*x+c))^3/d^3/f+1/
3*(f*x+e)^3*(a+b*arctan(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arctan(d*x+c))*
ln(2/(1+I*(d*x+c)))/d^3+b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(
d*x+c))^2*ln(2/(1+I*(d*x+c)))/d^3-1/2*b^3*f^2*ln(1+(d*x+c)^2)/d^3-3*I*b^3*f
*(-c*f+d*e)*polylog(2,1-2/(1+I*(d*x+c)))/d^3+I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^3+1/2*b^3*(
3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(3,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5155, 4974, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 5104, 5114, 6745}

$$\begin{aligned}
&\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx \\
&= \frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))}{d^3} \\
&\quad -\frac{6b^2f(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))}{d^3} \\
&\quad +\frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)(a+b\arctan(c+dx))^3}{3d^3} \\
&\quad -\frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)(a+b\arctan(c+dx))^3}{3d^3f} \\
&\quad +\frac{b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad -\frac{3ibf(de-cf)(a+b\arctan(c+dx))^2}{d^3}-\frac{3bf(c+dx)(de-cf)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad -\frac{bf^2(a+b\arctan(c+dx))^2}{2d^3}-\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad +\frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f}+\frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\arctan(c+dx)}{d^3} \\
&\quad +\frac{b^3\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\operatorname{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)}{2d^3} \\
&\quad -\frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{d^3}-\frac{b^3f^2\log\left((c+dx)^2+1\right)}{2d^3}
\end{aligned}$$

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(a + b
*ArcTan[c + d*x])^2)/(2*d^3) - ((3*I)*b*f*(d*e - c*f)*(a + b*ArcTan[c + d*x
])^2)/d^3 - (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/d^3 - (
b*f^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/(3*d^3*f)
+ ((e + f*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
 b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 + (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 - (b^3*f^2*Log[1 + (c + d*x)^2])/(2*d^3) - ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
^3 + (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/(2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 (a+b\arctan(x))^3\,dx, x, c+dx\right)}{d}\\
&= \frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f}\\
&\quad -\frac{b\text{Subst}\left(\int \left(\frac{3f^2(de-cf)(a+b\arctan(x))^2}{d^3}+\frac{f^3x(a+b\arctan(x))^2}{d^3}+\frac{((de-cf)\left(d^2e^2-2cdef-3f^2+c^2f^2\right)+f\left(3d^2e^2-6cdef-(\right.}{d^3(1+x^2)}\right.\right.}{f}\\
&= \frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f}\\
&\quad -\frac{b\text{Subst}\left(\int \frac{((de-cf)\left(d^2e^2-2cdef-3f^2+c^2f^2\right)+f\left(3d^2e^2-6cdef-\left(1-3c^2\right)f^2\right)x)(a+b\arctan(x))^2}{1+x^2}\,dx, x, c+dx\right)}{d^3f}\\
&\quad -\frac{(bf^2)\,\text{Subst}\left(\int x(a+b\arctan(x))^2\,dx, x, c+dx\right)}{d^3}\\
&\quad -\frac{(3bf(de-cf))\text{Subst}\left(\int (a+b\arctan(x))^2\,dx, x, c+dx\right)}{d^3}\\
&= -\frac{3bf(de-cf)(c+dx)(a+b\arctan(c+dx))^2}{d^3}\\
&\quad -\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3}+\frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f}\\
&\quad -\frac{b\text{Subst}\left(\int \left(\frac{(de-cf)\left(d^2e^2-2cdef-\left(3-c^2\right)f^2\right)(a+b\arctan(x))^2}{1+x^2}+\frac{f\left(3d^2e^2-6cdef-\left(1-3c^2\right)f^2\right)x(a+b\arctan(x))^2}{1+x^2}\right)\,dx, x,\right.}{d^3f}\\
&\quad +\frac{(b^2f^2)\,\text{Subst}\left(\int \frac{x^2(a+b\arctan(x))}{1+x^2}\,dx, x, c+dx\right)}{d^3}\\
&\quad +\frac{(6b^2f(de-cf))\,\text{Subst}\left(\int \frac{x(a+b\arctan(x))}{1+x^2}\,dx, x, c+dx\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3ibf(de-cf)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad - \frac{3bf(de-cf)(c+dx)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3} + \frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f} \\
&\quad + \frac{(b^2f^2)\operatorname{Subst}(\int (a+b\arctan(x))\,dx,x,c+dx)}{d^3} \\
&\quad - \frac{(b^2f^2)\operatorname{Subst}\left(\int \frac{a+b\arctan(x)}{1+x^2}\,dx,x,c+dx\right)}{d^3} \\
&\quad - \frac{(6b^2f(de-cf))\operatorname{Subst}\left(\int \frac{a+b\arctan(x)}{i-x}\,dx,x,c+dx\right)}{d^3} \\
&\quad - \frac{(b(3d^2e^2-6cdef-(1-3c^2)f^2))\operatorname{Subst}\left(\int \frac{x(a+b\arctan(x))^2}{1+x^2}\,dx,x,c+dx\right)}{d^3} \\
&\quad - \frac{(b(de-cf)(d^2e^2-2cdef-(3-c^2)f^2))\operatorname{Subst}\left(\int \frac{(a+b\arctan(x))^2}{1+x^2}\,dx,x,c+dx\right)}{d^3f}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ab^2f^2x}{d^2} - \frac{bf^2(a+b\arctan(c+dx))^2}{2d^3} - \frac{3ibf(de-cf)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad - \frac{3bf(de-cf)(c+dx)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)(a+b\arctan(c+dx))^3}{3d^3} \\
&\quad - \frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)(a+b\arctan(c+dx))^3}{3d^3f} \\
&\quad + \frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f} \\
&\quad - \frac{6b^2f(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad + \frac{(b^3f^2)\operatorname{Subst}(\int \arctan(x)\,dx,x,c+dx)}{d^3} \\
&\quad + \frac{(6b^3f(de-cf))\operatorname{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^3} \\
&\quad + \frac{(b(3d^2e^2-6cdef-(1-3c^2)f^2))\operatorname{Subst}\left(\int \frac{(a+b\arctan(x))^2}{i-x}\,dx,x,c+dx\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\arctan(c+dx)}{d^3}-\frac{bf^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad -\frac{3ibf(de-cf)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad -\frac{3bf(de-cf)(c+dx)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad -\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad +\frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^3}{3d^3} \\
&\quad -\frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\,(a+b\arctan(c+dx))^3}{3d^3f} \\
&\quad +\frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f} \\
&\quad -\frac{6b^2f(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad +\frac{b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad -\frac{(b^3f^2)\,\mathrm{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,c+dx\right)}{d^3} \\
&\quad -\frac{(6ib^3f(de-cf))\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{d^3} \\
&\quad -\frac{(2b^2(3d^2e^2-6cdef-(1-3c^2)\,f^2))\,\mathrm{Subst}\left(\int\frac{(a+b\arctan(x))\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\arctan(c+dx)}{d^3} - \frac{bf^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad - \frac{3ibf(de-cf)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad - \frac{3bf(de-cf)(c+dx)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^3}{3d^3} \\
&\quad - \frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\,(a+b\arctan(c+dx))^3}{3d^3f} \\
&\quad + \frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f} \\
&\quad - \frac{6b^2f(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad + \frac{b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad - \frac{b^3f^2\log\left(1+(c+dx)^2\right)}{2d^3} - \frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad + \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad - \frac{(ib^3(3d^2e^2-6cdef-(1-3c^2)\,f^2))\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\arctan(c+dx)}{d^3} - \frac{bf^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad - \frac{3ibf(de-cf)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad - \frac{3bf(de-cf)(c+dx)(a+b\arctan(c+dx))^2}{d^3} \\
&\quad - \frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^3}{3d^3} \\
&\quad - \frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\,(a+b\arctan(c+dx))^3}{3d^3f} \\
&\quad + \frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f} \\
&\quad - \frac{6b^2f(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad + \frac{b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad - \frac{b^3f^2\log\left(1+(c+dx)^2\right)}{2d^3} - \frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad + \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)\,f^2)\,(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&\quad + \frac{b^3(3d^2e^2-6cdef-(1-3c^2)\,f^2)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^3}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1844 vs. $2(564) = 1128$.

Time = 13.86 (sec) , antiderivative size = 1844, normalized size of antiderivative = 3.27

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx = \frac{a^2(ad^2e^2-3bdef+2bcf^2)\,x}{d^2} - \frac{a^2f(-2ade+bf)x^2}{2d}$$

$$+\frac{1}{3}a^3f^2x^3 + \frac{(3a^2bcd^2e^2+3a^2bdef-3a^2bc^2def-3a^2bcf^2+a^2bc^3f^2)\arctan(c+dx)}{d^3}$$

$$+a^2bx\left(3e^2+3efx+f^2x^2\right)\arctan(c+dx)$$

$$+\frac{(-3a^2bd^2e^2+6a^2bcdef+a^2bf^2-3a^2bc^2f^2)\log\left(1+c^2+2cdx+d^2x^2\right)}{2d^3}$$

$$+\frac{3ab^2e^2\big(-i\arctan(c+dx)^2+(c+dx)\arctan(c+dx)^2+2\arctan(c+dx)\log\left(1+e^{2i\arctan(c+dx)}\right)-i\,\mathrm{Po}}{d}$$

$$+6ab^2ef\Bigg(-\frac{(c+dx)\arctan(c+dx)}{d^2}+\frac{ic\arctan(c+dx)^2}{d^2}-\frac{c(c+dx)\arctan(c+dx)^2}{d^2}$$

$$+\frac{(1+(c+dx)^2)\arctan(c+dx)^2}{2d^2}-\frac{2c\arctan(c+dx)\log\left(1+e^{2i\arctan(c+dx)}\right)}{d^2}$$

$$-\frac{\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)}{d^2}+\frac{ic\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(c+dx)}\right)}{d^2}\Bigg)$$

$$+\frac{b^3e^2\big(-i\arctan(c+dx)^3+(c+dx)\arctan(c+dx)^3+3\arctan(c+dx)^2\log\left(1+e^{2i\arctan(c+dx)}\right)-3i\,\mathrm{arc}}{d}$$

$$+\,b^3ef\left(\arctan(c+dx)\left(3i\arctan(c+dx)+2ic\arctan(c+dx)^2+(1+(c+dx)^2)\arctan(c+dx)^2-(c+\right.\right.$$

$$+\,ab^2f^2(1+(c+dx)^2)^{3/2}\Bigg(\frac{c+dx}{\sqrt{1+(c+dx)^2}}+\frac{6c(c+dx)\arctan(c+dx)}{\sqrt{1+(c+dx)^2}}+\frac{3(c+dx)\arctan(c+dx)^2}{\sqrt{1+(c+dx)^2}}+\frac{3c^2(c+dx)\arctan(c+dx)^2}{\sqrt{1+(c+dx)^2}}+$$

$$+\,b^3f^2\Bigg(-i(3c-\arctan(c+dx)+3c^2\arctan(c+dx))\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(c+dx)}\right)+\frac{1}{12}(1+(c+dx)^2)^{3/}$$

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a^2*(a*d^2*e^2 - 3*b*d*e*f + 2*b*c*f^2)*x)/d^2 - (a^2*f*(-2*a*d*e + b*f)*x
^2)/(2*d) + (a^3*f^2*x^3)/3 + ((3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 3*a^2*b
*c^2*d*e*f - 3*a^2*b*c*f^2 + a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + a^2*b*x*
(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[c + d*x] + ((-3*a^2*b*d^2*e^2 + 6*a^2*b*
c*d*e*f + a^2*b*f^2 - 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2
```

```
*d^3) + (3*a*b^2*e^2*((-I)*ArcTan[c + d*x]^2 + (c + d*x)*ArcTan[c + d*x]^2
+ 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] - I*PolyLog[2, -E^((
2*I)*ArcTan[c + d*x])]))/d + 6*a*b^2*e*f*(-(((c + d*x)*ArcTan[c + d*x])/d^2
) + (I*c*ArcTan[c + d*x]^2)/d^2 - (c*(c + d*x)*ArcTan[c + d*x]^2)/d^2 + ((1
 + (c + d*x)^2)*ArcTan[c + d*x]^2)/(2*d^2) - (2*c*ArcTan[c + d*x]*Log[1 + E
^((2*I)*ArcTan[c + d*x])])/d^2 - Log[1/Sqrt[1 + (c + d*x)^2]]/d^2 + (I*c*Po
lyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d^2) + (b^3*e^2*((-I)*ArcTan[c + d*x]
^3 + (c + d*x)*ArcTan[c + d*x]^3 + 3*ArcTan[c + d*x]^2*Log[1 + E^((2*I)*Arc
Tan[c + d*x])] - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x]
)] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/d + (b^3*e*f*(ArcTan[c
+ d*x]*((3*I)*ArcTan[c + d*x] + (2*I)*c*ArcTan[c + d*x]^2 + (1 + (c + d*x)^
2)*ArcTan[c + d*x]^2 - (c + d*x)*ArcTan[c + d*x]*(3 + 2*c*ArcTan[c + d*x])
- 6*Log[1 + E^((2*I)*ArcTan[c + d*x])] - 6*c*ArcTan[c + d*x]*Log[1 + E^((2*
I)*ArcTan[c + d*x])]) + (3*I)*(1 + 2*c*ArcTan[c + d*x])*PolyLog[2, -E^((2*I
)*ArcTan[c + d*x])] - 3*c*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])]))/d^2 + (a
*b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c + d*x)/Sqrt[1 + (c + d*x)^2] + (6*c*(c
 + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2] + (3*(c + d*x)*ArcTan[c + d*
x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 +
(c + d*x)^2] + I*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTa
n[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - 2*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*
x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c^2*ArcTan[c + d*x]*Cos[3*ArcTan
[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c*Cos[3*ArcTan[c + d*x]]*
Log[1/Sqrt[1 + (c + d*x)^2]] + (ArcTan[c + d*x]*(-4 + (3*I - 12*c - (9*I)*c
^2)*ArcTan[c + d*x]) + 6*(-1 + 3*c^2)*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcT
an[c + d*x])] + 18*c*Log[1/Sqrt[1 + (c + d*x)^2]])/Sqrt[1 + (c + d*x)^2] -
((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/(1 + (c + d*x)^
2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6*c*ArcTan[c + d*x]*Sin[3*ArcTan[c + d*
x]] - ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTan[c + d*x]^2*Si
n[3*ArcTan[c + d*x]]))/(4*d^3) + (b^3*f^2*((-I)*(3*c - ArcTan[c + d*x] + 3*
c^2*ArcTan[c + d*x])*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + ((1 + (c + d*
x)^2)^(3/2)*((3*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2] + (9*c*(c
+ d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*(c + d*x)*ArcTan[c + d
*x]^3)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)*ArcTan[c + d*x]^3)/Sqrt[1 +
 (c + d*x)^2] - (9*I)*c*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] + I*ArcTan
[c + d*x]^3*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^3*Cos[3*ArcT
an[c + d*x]] + 18*c*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)
*ArcTan[c + d*x])] - 3*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]]*Log[1 + E^(
(2*I)*ArcTan[c + d*x])] + 9*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]]*Lo
g[1 + E^((2*I)*ArcTan[c + d*x])] + 3*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 +
(c + d*x)^2]] + (3*(ArcTan[c + d*x]^2*(-2 - (9*I)*c + I*ArcTan[c + d*x] - 4
*c*ArcTan[c + d*x] - (3*I)*c^2*ArcTan[c + d*x]) + 3*ArcTan[c + d*x]*(6*c -
ArcTan[c + d*x] + 3*c^2*ArcTan[c + d*x])*Log[1 + E^((2*I)*ArcTan[c + d*x])]
 + 3*Log[1/Sqrt[1 + (c + d*x)^2]]))/Sqrt[1 + (c + d*x)^2] + (6*(-1 + 3*c^2)
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/(1 + (c + d*x)^2)^(3/2) + 3*ArcTan
[c + d*x]*Sin[3*ArcTan[c + d*x]] + 9*c*ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d
```

```
*x]] - ArcTan[c + d*x]^3*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTan[c + d*x]^3*S
in[3*ArcTan[c + d*x]]))/12))/d^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 28.49 (sec) , antiderivative size = 5843, normalized size of antiderivative = 10.36

method	result	size
derivativedivides	Expression too large to display	5843
default	Expression too large to display	5843
parts	Expression too large to display	6026

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx=\int (fx+e)^2(b\arctan(dx+c)+a)^3\,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x +
 b^3*e^2)*arctan(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^2)
*arctan(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arctan(d
*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx=\int (fx+e)^2(b\arctan\left(dx+c\right)+a)^3\,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e
^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan(
(d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e^2 - 7/32*(6
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((
d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e^2 + 1/3*a^3*f^
2*x^3 + 7/8*b^3*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^
2*f^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 3*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*f^2*integrate
(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*
e*f*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
+ 56*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 4*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2
 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*e*f*int
egrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c
)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 19
2*a*b^2*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 192*a*b^2*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x
+ c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*e*f*integrate(1/32*x
^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*c^2*f^2*int
egrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^
3*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^3*c*d*f^2*integrate(1/32*x^3*arc
tan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*f*integrat
e(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*
c*d*x + c^2 + 1), x) + 3*b^3*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^
2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1
), x) + 384*a*b^2*c*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 96*a*b^2*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arct
an(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c^2*e*f*integrate(
1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*e^2
```

```
*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*f*integrate(1/32*x^2*arctan(d*x
+ c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6
*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*e*f*integrate(1/32*x*ar
ctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*e*f*integrate(1/32*x*arctan(d*x + c)^
2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e^2*integrate(1/32*x*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^3*e*f*x^2 + 3*a*b^2*e^2*arc
tan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*b^3*d*f^2*integrate(1/32*x^3*a
rctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d*f^2*integrate(1/
32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 - 12*b^3*d*e*f*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 3*b^3*d*e*f*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 +
1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*d*e^2*integrate(1/32*x*arct
an(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*e^2*integrate(1/3
2*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) -
(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/
d)*a*b^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*ar
ctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^
3*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/
d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*e*f + 1/2*(2*x^3*ar
ctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)
/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b*f^2 + a^
3*e^2*x + 28*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 3*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2
 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*f^2*in
tegrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b
^3*e*f*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 6*b^3*e*f*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*e*f*integrate(1/32*x*ar
ctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*e^2*integrate(1/3
2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*
b*e^2/d + 1/24*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(d*x + c)^
3 - 1/32*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(d*x + c)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Giac [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx=\int (fx+e)^2(b\arctan\left(dx+c\right)+a)^3\,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx=\int {\left(e+f\,x\right)}^2\,{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3 \,d x$$

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((e + f*x)^2*(a + b*atan(c + d*x))^3, x)
```

3.37 $\int (e+fx)(a+b\arctan(c+dx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 337

$$
\begin{aligned}
\int (e+fx)(a+b\arctan(c+dx))^3\,dx
&= -\frac{3ibf(a+b\arctan(c+dx))^2}{2d^2}-\frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2}\\
&\quad+\frac{i(de-cf)(a+b\arctan(c+dx))^3}{d^2}\\
&\quad-\frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^3}{2d^2f}\\
&\quad+\frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f}-\frac{3b^2f(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}\\
&\quad+\frac{3b(de-cf)(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}-\frac{3ib^3f\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}\\
&\quad+\frac{3ib^2(de-cf)(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}\\
&\quad+\frac{3b^3(de-cf)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}
\end{aligned}
$$

```
[Out] -3/2*I*b*f*(a+b*arctan(d*x+c))^2/d^2-3/2*b*f*(d*x+c)*(a+b*arctan(d*x+c))^2/
d^2+I*(-c*f+d*e)*(a+b*arctan(d*x+c))^3/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(
a+b*arctan(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arctan(d*x+c))^3/f-3*b^2*f*(a
+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d^2+3*b*(-c*f+d*e)*(a+b*arctan(d*x+c)
)^2*ln(2/(1+I*(d*x+c)))/d^2-3/2*I*b^3*f*polylog(2,1-2/(1+I*(d*x+c)))/d^2+3*
I*b^2*(-c*f+d*e)*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^2+3/2*b
^3*(-c*f+d*e)*polylog(3,1-2/(1+I*(d*x+c)))/d^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5155, 4974, 4930, 5040, 4964, 2449, 2352, 5104, 5004, 5114, 6745}

$$\begin{aligned}
&\int (e+fx)(a+b\arctan(c+dx))^3\,dx \\
&= \frac{3ib^2(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))}{d^2} \\
&\quad -\frac{3b^2f\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))}{d^2}+\frac{i(de-cf)(a+b\arctan(c+dx))^3}{d^2} \\
&\quad -\frac{(-cf+de+f)(de-(c+1)f)(a+b\arctan(c+dx))^3}{2d^2f} \\
&\quad +\frac{3b(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2}{d^2}-\frac{3ibf(a+b\arctan(c+dx))^2}{2d^2} \\
&\quad -\frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2}+\frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f} \\
&\quad +\frac{3b^3(de-cf)\operatorname{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)}{2d^2}-\frac{3ib^3f\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{2d^2}
\end{aligned}$$

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*f*(a + b*ArcTan[c + d*x])^2)/d^2 - (3*b*f*(c + d*x)*(a + b*Ar
cTan[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^3)/d^2 -
 ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^3)/(2*d^2*f) +
((e + f*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcTan[c +
d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (3*b*(d*e - c*f)*(a + b*ArcTan[c + d*
x])^2*Log[2/(1 + I*(c + d*x))])/d^2 - (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcTan[c + d*x])*PolyL
og[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(
1 + I*(c + d*x))])/(2*d^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
```

```
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)(a+b\arctan(x))^3\,dx,x,c+dx\right)}{d}\\
&= \frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f}\\
&\quad -\frac{(3b)\text{Subst}\left(\int \left(\frac{f^2(a+b\arctan(x))^2}{d^2}+\frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b\arctan(x))^2}{d^2(1+x^2)}\right)\,dx,x,c+dx\right)}{2f}\\
&= \frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f}\\
&\quad -\frac{(3b)\text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b\arctan(x))^2}{1+x^2}\,dx,x,c+dx\right)}{2d^2f}\\
&\quad -\frac{(3bf)\text{Subst}\left(\int (a+b\arctan(x))^2\,dx,x,c+dx\right)}{2d^2}\\
&= -\frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2}+\frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f}\\
&\quad -\frac{(3b)\text{Subst}\left(\int \left(\frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(x))^2}{1+x^2}-\frac{2f(-de+cf)x(a+b\arctan(x))^2}{1+x^2}\right)\,dx,x,c+dx\right)}{2d^2f}\\
&\quad +\frac{(3b^2f)\,\text{Subst}\left(\int \frac{x(a+b\arctan(x))}{1+x^2}\,dx,x,c+dx\right)}{d^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3ibf(a+b\arctan(c+dx))^2}{2d^2} - \frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2} \\
&\quad + \frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f} - \frac{(3b^2f)\,\text{Subst}\left(\int \frac{a+b\arctan(x)}{i-x}\,dx, x, c+dx\right)}{d^2} \\
&\quad - \frac{(3b(de-cf))\text{Subst}\left(\int \frac{x(a+b\arctan(x))^2}{1+x^2}\,dx, x, c+dx\right)}{d^2} \\
&\quad - \frac{(3b(de+f-cf)(de-(1+c)f))\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{1+x^2}\,dx, x, c+dx\right)}{2d^2f} \\
&= -\frac{3ibf(a+b\arctan(c+dx))^2}{2d^2} - \frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2} \\
&\quad + \frac{i(de-cf)(a+b\arctan(c+dx))^3}{d^2} \\
&\quad - \frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^3}{2d^2f} \\
&\quad + \frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f} - \frac{3b^2f(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad + \frac{(3b^3f)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d^2} \\
&\quad + \frac{(3b(de-cf))\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{i-x}\,dx, x, c+dx\right)}{d^2} \\
&= -\frac{3ibf(a+b\arctan(c+dx))^2}{2d^2} - \frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2} \\
&\quad + \frac{i(de-cf)(a+b\arctan(c+dx))^3}{d^2} \\
&\quad - \frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^3}{2d^2f} \\
&\quad + \frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f} - \frac{3b^2f(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad + \frac{3b(de-cf)(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{(3ib^3f)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{(6b^2(de-cf))\,\text{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3ibf(a+b\arctan(c+dx))^2}{2d^2} - \frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2} \\
&\quad + \frac{i(de-cf)(a+b\arctan(c+dx))^3}{d^2} \\
&\quad - \frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^3}{2d^2f} \\
&\quad + \frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f} - \frac{3b^2f(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad + \frac{3b(de-cf)(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{3ib^3f\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2} \\
&\quad + \frac{3ib^2(de-cf)(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{(3ib^3(de-cf))\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}dx,x,c+dx\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3ibf(a+b\arctan(c+dx))^2}{2d^2} - \frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2} \\
&\quad + \frac{i(de-cf)(a+b\arctan(c+dx))^3}{d^2} \\
&\quad - \frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^3}{2d^2f} \\
&\quad + \frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f} - \frac{3b^2f(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad + \frac{3b(de-cf)(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{3ib^3f\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2} \\
&\quad + \frac{3ib^2(de-cf)(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad + \frac{3b^3(de-cf)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.07 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.76

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx$$

$$= \frac{a^2(2ade-3bf-2acf)(c+dx)+a^3f(c+dx)^2+3a^2bf\arctan(c+dx)-3a^2b(c+dx)(cf-d(2e+fx))}{}$$

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a^2*(2*a*d*e - 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 + 3*a^2*b*f*
ArcTan[c + d*x] - 3*a^2*b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcTan[c + d*x] +
 6*a*b^2*f*(-((c + d*x)*ArcTan[c + d*x]) + ((1 + (c + d*x)^2)*ArcTan[c + d*
x]^2)/2 - Log[1/Sqrt[1 + (c + d*x)^2]]) - 3*a^2*b*(d*e - c*f)*Log[1 + (c +
d*x)^2] + 6*a*b^2*d*e*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*
Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x
])]) - 6*a*b^2*c*f*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*Log
[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]
) + b^3*f*(ArcTan[c + d*x]*((3*I)*ArcTan[c + d*x] - 3*(c + d*x)*ArcTan[c +
d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 6*Log[1 + E^((2*I)*ArcTan[c +
d*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*d*e*(ArcTan
[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan[c +
 d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + (
3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2) - 2*b^3*c*f*(ArcTan[c + d*x]^2
*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan[c + d*x])]) -
(3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + (3*PolyLog[3
, -E^((2*I)*ArcTan[c + d*x])])/2))/(2*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.33 (sec) , antiderivative size = 8267, normalized size of antiderivative = 24.53

method	result	size
parts	Expression too large to display	8267
derivativedivides	Expression too large to display	8269
default	Expression too large to display	8269

```
[In] int((f*x+e)*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx = \int (fx+e)(b\arctan\left(dx+c\right)+a)^3\,dx$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arctan(d*x + c)^3 + 3*(a*b^2*f
*x + a*b^2*e)*arctan(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arctan(d*x + c),
x)
```

Sympy [F]

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx = \int \left(a+b\operatorname{atan}\left(c+dx\right)\right)^3\left(e+fx\right)\,dx$$

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3*(e + f*x), x)
```

Maxima [F]

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx = \int (fx+e)(b\arctan\left(dx+c\right)+a)^3\,dx$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e*a
rctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan((d^2
*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e - 7/32*(6*arcta
n(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x
+ c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e + 7/8*b^3*e*arctan(d
*x + c)^3*arctan((d^2*x + c*d)/d)/d + 56*b^3*d^2*f*integrate(1/64*x^3*arcta
n(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*f*integrate(1/64
*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 192*a*b^2*d^2*f*integrate(1/64*x^3*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x +
 c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*f*integrate(1/64*x^2*
arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*f*integrat
e(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*
d*x + c^2 + 1), x) + 6*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*
```

```
f*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*e*integrate(1/64*x^2*arctan(
d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*f*integrate(1/
64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*e*
integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b
^3*c^2*f*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 24*b^3*d^2*e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*f*integrate(1/64*
x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 12*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2
*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c^2*f*integra
te(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 384*a*b^2*c*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*f*integrate(1/64*x*arctan(d
*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c*d*e*integrate(1/64*x
*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 6*b^3*c^2*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/2*a^3*f*x^2 + 3*a*b^2*e
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*f*integrate(1/64*x^
2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*f*integrate
(1/64*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) - 24*b^3*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 6*b^3*d*e*integrate(1/64*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x + c)*arctan((d^2*x + c*d)
/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*e - 7/32*(6*arctan(d*x + c)^2*
arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d
 + arctan((d^2*x + c*d)/d)^4/d)*b^3*e + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2
 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 +
1)/d^3))*a^2*b*f + a^3*e*x + 56*b^3*f*integrate(1/64*x*arctan(d*x + c)^3/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*f*integrate(1/64*x*arctan(d*x + c)*
log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*
a*b^2*f*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 6*b^3*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - l
og((d*x + c)^2 + 1))*a^2*b*e/d + 1/16*(b^3*f*x^2 + 2*b^3*e*x)*arctan(d*x +
c)^3 - 3/64*(b^3*f*x^2 + 2*b^3*e*x)*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^2
```

Giac [F]

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx=\int (fx+e)(b\arctan\left(dx+c\right)+a)^3\,dx$$

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx=\int (e+f\,x)\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^3\,dx$$

```
[In] int((e + f*x)*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((e + f*x)*(a + b*atan(c + d*x))^3, x)
```

3.38 $\int (a+b\arctan(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 143

$$\int (a+b\arctan(c+dx))^3\,dx=\frac{i(a+b\arctan(c+dx))^3}{d}+\frac{(c+dx)(a+b\arctan(c+dx))^3}{d}+\frac{3b(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{3ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}+\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}$$

```
[Out] I*(a+b*arctan(d*x+c))^3/d+(d*x+c)*(a+b*arctan(d*x+c))^3/d+3*b*(a+b*arctan(d
*x+c))^2*ln(2/(1+I*(d*x+c)))/d+3*I*b^2*(a+b*arctan(d*x+c))*polylog(2,1-2/(1
+I*(d*x+c)))/d+3/2*b^3*polylog(3,1-2/(1+I*(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used

$= \{5147, 4930, 5040, 4964, 5004, 5114, 6745\}$

$$\begin{aligned}\int (a+b\arctan(c+dx))^3\,dx = {} & \frac{3ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))}{d} \\ & + \frac{(c+dx)(a+b\arctan(c+dx))^3}{d} + \frac{i(a+b\arctan(c+dx))^3}{d} \\ & + \frac{3b\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2}{d} \\ & + \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)}{2d}\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (I*(a + b*ArcTan[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^3)/d +
 (3*b*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d + ((3*I)*b^2*(a
 + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d + (3*b^3*PolyL
og[3, 1 - 2/(1 + I*(c + d*x))])/(2*d)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+b\arctan(x))^3\,dx,x,c+dx\right)}{d}\\
&= \frac{(c+dx)(a+b\arctan(c+dx))^3}{d}-\frac{(3b)\text{Subst}\left(\int \frac{x(a+b\arctan(x))^2}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{i(a+b\arctan(c+dx))^3}{d}+\frac{(c+dx)(a+b\arctan(c+dx))^3}{d}\\
&\quad+\frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{i-x}\,dx,x,c+dx\right)}{d}\\
&= \frac{i(a+b\arctan(c+dx))^3}{d}+\frac{(c+dx)(a+b\arctan(c+dx))^3}{d}\\
&\quad+\frac{3b(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d}\\
&\quad-\frac{\left(6b^2\right)\text{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{i(a+b\arctan(c+dx))^3}{d} + \frac{(c+dx)(a+b\arctan(c+dx))^3}{d} \\
&\quad + \frac{3b(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad + \frac{3ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad - \frac{(3ib^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{i(a+b\arctan(c+dx))^3}{d} + \frac{(c+dx)(a+b\arctan(c+dx))^3}{d} \\
&\quad + \frac{3b(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad + \frac{3ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} \\
&\quad + \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.48

$$
\int (a+b\arctan(c+dx))^3\,dx
$$

$$
= \frac{2a^3(c+dx)+6a^2b(c+dx)\arctan(c+dx)-3a^2b\log\left(1+(c+dx)^2\right)+6ab^2(\arctan(c+dx)((-i+c+dx}{}
$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcTan[c + d*x] - 3*a^2*b*Log[1 + (c +
 d*x)^2] + 6*a*b^2*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*Log
[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]
) + 2*b^3*(ArcTan[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^
((2*I)*ArcTan[c + d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcT
an[c + d*x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/(2*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(136) = 272$.

Time = 0.78 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.95

method	result
derivativedivides	$(dx+c)a^3+b^3\left(\arctan(dx+c)^3(dx+c+i)-2i\arctan(dx+c)^3+3\arctan(dx+c)^2\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-3i\arctan(dx+c)\right.$
default	$(dx+c)a^3+b^3\left(\arctan(dx+c)^3(dx+c+i)-2i\arctan(dx+c)^3+3\arctan(dx+c)^2\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-3i\arctan(dx+c)\right.$
parts	$a^3x+\frac{b^3\left(\arctan(dx+c)^3(dx+c+i)-2i\arctan(dx+c)^3+3\arctan(dx+c)^2\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-3i\arctan(dx+c)\right.}{d}$

```
[In] int((a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3+b^3*(arctan(d*x+c)^3*(d*x+c+I)-2*I*arctan(d*x+c)^3+3*arcta
n(d*x+c)^2*ln(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I*arctan(d*x+c)*polylog(2,
-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/2*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2
)))+3*a*b^2*(arctan(d*x+c)^2*(d*x+c+I)+2*arctan(d*x+c)*ln(1+(1+I*(d*x+c))^2
/(1+(d*x+c)^2))-2*I*arctan(d*x+c)^2-I*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)
^2)))+3*a^2*b*((d*x+c)*arctan(d*x+c)-1/2*ln(1+(d*x+c)^2)))
```

Fricas [F]

$$\int (a+b\arctan(c+dx))^3\,dx = \int (b\arctan(dx+c)+a)^3\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arctan
(d*x + c) + a^3, x)
```

Sympy [F]

$$\int (a+b\arctan(c+dx))^3\,dx = \int \left(a+b\operatorname{atan}\left(c+dx\right)\right)^3\,dx$$

```
[In] integrate((a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3, x)
```

Maxima [F]

$$\int (a+b\arctan(c+dx))^3\,dx = \int \left(b\arctan\left(dx+c\right)+a\right)^3\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 1/8*b^3*x*arctan(
d*x + c)^3 + 3*a*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 3/32
*b^3*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - (3*arctan(d*x +
 c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2 -
7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*a
rctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2 + 7/8*b^3
*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^2*integrate(1/32*x^
2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*integrate
(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 96*a*b^2*d^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*integrate(1/32*x*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*integrate(1/32*x^2*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 6*b^3*c*d*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*integrate(1/32*x*
arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*integrate(
1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 3*b^3*c^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c
*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*a*b^2*arctan(d*x +
c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*integrate(1/32*x*arctan(d*x + c)^
2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*integrate(1/32*x*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x +
c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2 - 7/32*
(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan
((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3 + a^3*x + 3*b^3*in
tegrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2
 + 1))*a^2*b/d
```

Giac [F]

$$\int (a+b\arctan(c+dx))^3\,dx=\int (b\arctan(dx+c)+a)^3\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\arctan(c+dx))^3\,dx=\int (a+b\,\mathrm{atan}(c+d\,x))^3\,dx$$

```
[In] int((a + b*atan(c + d*x))^3,x)
```

```
[Out] int((a + b*atan(c + d*x))^3, x)
```

3.39 $\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 372

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx &= -\frac{(a+b\arctan(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\ &+\frac{(a+b\arctan(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &+\frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\ &-\frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\ &-\frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\ &+\frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\ &-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{4f}\end{aligned}$$

```
[Out] -(a+b*arctan(d*x+c))^3*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))^3*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+3/2*I*b*(a+b*arctan(d*x+c))^2*polylog
(2,1-2/(1-I*(d*x+c)))/f-3/2*I*b*(a+b*arctan(d*x+c))^2*polylog(2,1-2*d*(f*x+
e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2
/(1-I*(d*x+c)))/f+3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2*d*(f*x+e)/(d*e+
```

```
I*f-c*f)/(1-I*(d*x+c)))/f-3/4*I*b^3*polylog(4,1-2/(1-I*(d*x+c)))/f+3/4*I*b^
3*polylog(4,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5155, 4970}

$$
\begin{aligned}
&\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx \\
&=\frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f} \\
&-\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{2f} \\
&-\frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f} \\
&+\frac{(a+b\arctan(c+dx))^3\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} \\
&+\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))^2}{2f} \\
&-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))^3}{f} \\
&+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{4f}-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}
\end{aligned}
$$

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))]
)/f - (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))
/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (3*b^2*(a + b*ArcTan[c + d*x])
*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2
*f) - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f + (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^3)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^3*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[3
*I*b*(a + b*ArcTan[c*x])^2*(PolyLog[2, 1 - 2/(1 - I*c*x)]/(2*e)), x] - Simp
[3*I*b*(a + b*ArcTan[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1
 - I*c*x)))]/(2*e)), x] - Simp[3*b^2*(a + b*ArcTan[c*x])*(PolyLog[3, 1 - 2/
(1 - I*c*x)]/(2*e)), x] + Simp[3*b^2*(a + b*ArcTan[c*x])*(PolyLog[3, 1 - 2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x] - Simp[3*I*b^3*(PolyLog
[4, 1 - 2/(1 - I*c*x)]/(4*e)), x] + Simp[3*I*b^3*(PolyLog[4, 1 - 2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{(a+b\arctan(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\
&\quad + \frac{(a+b\arctan(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\
&\quad + \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\
&\quad - \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\
&\quad - \frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\
&\quad + \frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\
&\quad - \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f} + \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{4f}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx = \int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.95 (sec) , antiderivative size = 3817, normalized size of antiderivative = 10.26

method	result	size
derivativedivides	Expression too large to display	3817
default	Expression too large to display	3817
parts	Expression too large to display	4072

```
[In] int((a+b*arctan(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*d*ln(c*f-d*e-f*(d*x+c))/f-b^3*d*(-ln(c*f-d*e-f*(d*x+c))/f*arctan(d
*x+c)^3+3/f*(1/3*arctan(d*x+c)^3*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)
-I*d*e*arctan(d*x+c)^2*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2
)/(d*e+I*f-c*f))/(2*I*f+2*c*f-2*d*e)-1/2*I*arctan(d*x+c)^2*polylog(2,-(1+I*
(d*x+c))^2/(1+(d*x+c)^2))+1/2*arctan(d*x+c)*polylog(3,-(1+I*(d*x+c))^2/(1+(
d*x+c)^2))+1/2*I*c*f/(c*f-d*e+I*f)*arctan(d*x+c)^2*polylog(2,(c*f-d*e+I*f)*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/4*I*polylog(4,-(1+I*(d*x+c))
^2/(1+(d*x+c)^2))-1/2*I*f/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(3,(c*f-d*e+I*
f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*f/(c*f-d*e+I*f)*arctan(
d*x+c)^2*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f
))+1/4*f/(c*f-d*e+I*f)*polylog(4,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2
)/(d*e+I*f-c*f))-1/3*c*f/(c*f-d*e+I*f)*arctan(d*x+c)^3*ln(1-(c*f-d*e+I*f)*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*c*f/(c*f-d*e+I*f)*arctan(d*
x+c)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1
/6*I*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/
(1+(d*x+c)^2)))*(csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I/(1
+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+
c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f
-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+
```

```
c)^2)))-csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))
^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))+csgn(I*(I*
f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2)
*arctan(d*x+c)^3-1/3*I*f/(c*f-d*e+I*f)*arctan(d*x+c)^3*ln(1-(c*f-d*e+I*f)*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/3*d*e/(c*f-d*e+I*f)*arctan(d*
x+c)^3*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/2*
d*e/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)/(d*e+I*f-c*f))+I*d*e*polylog(4,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)/(d*e+I*f-c*f))/(4*I*f+4*c*f-4*d*e)-1/4*I*c*f/(c*f-d*e+I*f)*polyl
og(4,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))))-3*a*b^2*d
*(-ln(c*f-d*e-f*(d*x+c))/f*arctan(d*x+c)^2+2/f*(1/2*arctan(d*x+c)^2*ln(I*f*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d
*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)-1/4*I*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*(csgn(I*(I*f*(1+I*(d*x
+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(
I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)
))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(I*(I*f*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+
c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*
(d*x+c))^2/(1+(d*x+c)^2)))+csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1
+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/
(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2)*arctan(d*x+c)^2-1/2*I*arctan(d*x+c)*p
olylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/4*polylog(3,-(1+I*(d*x+c))^2/(1+
(d*x+c)^2))-1/2*f/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*I*f/(c*f-d*e+I*f)*arctan(d*x+c)
^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/4*I*f/
(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*
f-c*f))+1/2*I*c*f/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*c*f/(c*f-d*e+I*f)*arctan(d*x+c)
^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/4*c*f/
(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*
f-c*f))-I*d*e*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f-2*d*e)+1/2*d*e/(c*f-d*e+I*f)*arctan(d*x+
c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/4*d*
e/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+
I*f-c*f))))-3*a^2*b*d*(-ln(c*f-d*e-f*(d*x+c))/f*arctan(d*x+c)+1/2*I*ln(c*f-
d*e-f*(d*x+c))*(ln((I*f+f*(d*x+c))/(c*f-d*e+I*f))-ln((I*f-f*(d*x+c))/(d*e+I
*f-c*f)))/f+1/2*I*(dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f))-dilog((I*f-f*(d*x+c
```

```
))/(d*e+I*f-c*f)))/f))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx=\int \frac{(b\arctan{(dx+c)}+a)^3}{fx+e}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*atan(d*x+c))**3/(f*x+e),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx=\int \frac{(b\arctan{(dx+c)}+a)^3}{fx+e}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*b^3*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d*x + c)^2
+ 96*a^2*b*arctan(d*x + c))/(f*x + e), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3}{e+f\,x}\,dx$$

```
[In] int((a + b*atan(c + d*x))^3/(e + f*x),x)
[Out] int((a + b*atan(c + d*x))^3/(e + f*x), x)
```

3.40 $\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 1233

$$
\begin{aligned}
\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx =\; & \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)}+\frac{3iab^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3ab^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} \\
& + \frac{ib^3d\arctan(c+dx)^3}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{b^3d(de-cf)\arctan(c+dx)^3}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}-\frac{(a+b\arctan(c+dx))^3}{f(e+fx)} \\
& + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2}-\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)} \\
& + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}
\end{aligned}
$$

```
[Out] 3*a^2*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)+I*b^3*d*arctan(d*x+
c)^3/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*a*b^2*d*(-c*f+d*e)*arctan(d*x+c)^2/f
/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(
d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^3*d*(-c*f+d*e)*arctan(d*x+c)^3/f/(d^2*e^2-
2*c*d*e*f+(c^2+1)*f^2)-(a+b*arctan(d*x+c))^3/f/(f*x+e)+3*a^2*b*d*ln(f*x+e)/
(f^2+(-c*f+d*e)^2)-6*a*b^2*d*arctan(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)-3*b^3*d*arctan(d*x+c)^2*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*
c*d*e*f+(c^2+1)*f^2)+6*a*b^2*d*arctan(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(
1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*b^3*d*arctan(d*x+c)^2*ln(2*
d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+6*a*
b^2*d*arctan(d*x+c)*ln(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*b
^3*d*arctan(d*x+c)^2*ln(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/
2*a^2*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+d*e)^2)+3*I*a*b^2*d*polylog(2,1-2/(1-I
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*I*b^3*d*arctan(d*x+c)*polylog(
2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2
)+3*I*b^3*d*arctan(d*x+c)*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(
c^2+1)*f^2)+3*I*b^3*d*arctan(d*x+c)*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2
*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*arctan(d*x+c)^2/(d^2*e^2-2*c*d*e*f+(c^2+1
)*f^2)-3*I*a*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*b^3*d*polylog(3,1-2/(1-I*(d*x+c)))/(d^2*e^
2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1
-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2/(1+I*(
d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 1233, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {5153, 6873, 5165, 6820, 12, 6857, 720, 31, 649, 209, 266, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964, 4968, 5114, 6745}

$$\begin{aligned}
\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx &= \frac{id\arctan(c+dx)^3b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2}+\frac{d(de-cf)\arctan(c+dx)^3b^3}{f\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)} \\
&- \frac{3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{3d\arctan(c+dx)^2\log\left(\frac{2}{i(c+dx)+1}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{3id\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&- \frac{3id\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{3id\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)b^3}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&- \frac{3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)b^3}{2\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)} \\
&+ \frac{3d\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^3}{2\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)} \\
&+ \frac{3d\operatorname{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)b^3}{2\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)} \\
&+ \frac{3iad\arctan(c+dx)^2b^2}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{3ad(de-cf)\arctan(c+dx)^2b^2}{f\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)} \\
&- \frac{6ad\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)b^2}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{6ad\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^2}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{6ad\arctan(c+dx)\log\left(\frac{2}{i(c+dx)+1}\right)b^2}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{3iad\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)b^2}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&- \frac{3iad\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^2}{d^2e^2-2cdfe+\left(c^2+1\right)f^2} \\
&+ \frac{3iad\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)b^2}{d^2e^2-2cdfe+\left(c^2+1\right)f^2}
\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] (3*a^2*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + ((3*I)*
a*b^2*d*ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a*b^2
*d*(d*e - c*f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 + (I*b^3*d*ArcTan[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^3
*d*(d*e - c*f)*ArcTan[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])^3/(f*(e + f*x)) + (3*a^2*b*d*Log[e + f*x])/(f^2
+ (d*e - c*f)^2) - (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 -
I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (6*a*b^2*d*ArcTan[c
+ d*x]*Log[(2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[(2*d*(e + f*
x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) + (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*a^2*b*d*Log[1 + (c + d*x)^2
])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcTan[c + d*x
]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2
) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I
*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcTan[
c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 -
2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*
ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
 (1 + c^2)*f^2) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2)) + (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((
d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)) + (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5165

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}+\frac{(3bd)\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)(1+(c+dx)^2)}\,dx}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}+\frac{(3bd)\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}+\frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx,x,c+dx\right)}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{(3b)\text{Subst}\left(\int \frac{d(a+b\arctan(x))^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{(3bd)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{(3bd)\text{Subst}\left(\int \left(\frac{a^2}{(de-cf+fx)(1+x^2)} + \frac{2ab\arctan(x)}{(de-cf+fx)(1+x^2)} + \frac{b^2\arctan(x)^2}{(de-cf+fx)(1+x^2)}\right)\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} + \frac{(6ab^2d)\,\text{Subst}\left(\int \frac{\arctan(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} + \frac{(3b^3d)\,\text{Subst}\left(\int \frac{\arctan(x)^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{(6ab^2d)\,\text{Subst}\left(\int \left(\frac{f^2\arctan(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} + \frac{(de-cf-fx)\arctan(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(1+x^2)}\right)\,dx, x, c+dx\right)}{f} + \frac{(3b^3d)\,\text{Subst}\left(\int \left(\frac{f^2\arctan(x)^2}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} + \frac{(de-cf-fx)\arctan(x)^2}{(d^2e^2-2cdef+(1+c^2)f^2)(1+x^2)}\right)\,dx, x, c+dx\right)}{f} + \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{de-cf-fx}{1+x^2}\,dx, x, c+dx\right)}{f\,(f^2+(de-cf)^2)} + \frac{(3a^2bdf)\,\text{Subst}\left(\int \frac{1}{de-cf+fx}\,dx, x, c+dx\right)}{f^2+(de-cf)^2}$$

$$
\begin{aligned}
&= -\frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&+ \frac{(6ab^2d)\,\text{Subst}\left(\int \frac{(de-cf-fx)\arctan(x)}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} \\
&+ \frac{(3b^3d)\,\text{Subst}\left(\int \frac{(de-cf-fx)\arctan(x)^2}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} \\
&+ \frac{(6ab^2df)\,\text{Subst}\left(\int \frac{\arctan(x)}{de-cf+fx}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{(3b^3df)\,\text{Subst}\left(\int \frac{\arctan(x)^2}{de-cf+fx}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{f^2+(de-cf)^2} \\
&+ \frac{(3a^2bd(de-cf))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{f\left(f^2+(de-cf)^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&- \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&+ \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&+ \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)} \\
&+ \frac{3ib^3d\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&- \frac{3ib^3d\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&- \frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)} + \frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)} \\
&+ \frac{(6ab^2d)\operatorname{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&- \frac{(6ab^2d)\operatorname{Subst}\left(\int \frac{\log\left(\frac{2(de-cf+fx)}{(de+if-cf)(1-ix)}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&+ \frac{(6ab^2d)\operatorname{Subst}\left(\int \left(\frac{de\left(1-\frac{cf}{de}\right)\arctan(x)}{1+x^2} - \frac{fx\arctan(x)}{1+x^2}\right)dx,x,c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} \\
&+ \frac{(3b^3d)\operatorname{Subst}\left(\int \left(\frac{de\left(1-\frac{cf}{de}\right)\arctan(x)^2}{1+x^2} - \frac{fx\arctan(x)^2}{1+x^2}\right)dx,x,c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&- \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} - \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+ \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+ \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} - \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)} \\
&+ \frac{3ib^3d\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{3iab^2d\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{3ib^3d\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} + \frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} \\
&+ \frac{\left(6iab^2d\right)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{\left(6ab^2d\right)\operatorname{Subst}\left(\int\frac{x\arctan(x)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{\left(3b^3d\right)\operatorname{Subst}\left(\int\frac{x\arctan(x)^2}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+ \frac{\left(6ab^2d(de-cf)\right)\operatorname{Subst}\left(\int\frac{\arctan(x)}{1+x^2}\,dx,x,c+dx\right)}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} \\
&+ \frac{\left(3b^3d(de-cf)\right)\operatorname{Subst}\left(\int\frac{\arctan(x)^2}{1+x^2}\,dx,x,c+dx\right)}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\arctan(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+ \frac{3ab^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} + \frac{ib^3d\arctan(c+dx)^3}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+ \frac{b^3d(de-cf)\arctan(c+dx)^3}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} - \frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&- \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} - \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+ \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+ \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+ \frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&- \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} + \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} \\
&+ \frac{\left(6ab^2d\right)\mathrm{Subst}\left(\int\frac{\arctan(x)}{i-x}\,dx,x,c+dx\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} + \frac{\left(3b^3d\right)\mathrm{Subst}\left(\int\frac{\arctan(x)^2}{i-x}\,dx,x,c+dx\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)}+\frac{3iab^2d\arctan(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+\frac{3ab^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}+\frac{ib^3d\arctan(c+dx)^3}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+\frac{b^3d(de-cf)\arctan(c+dx)^3}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}-\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}+\frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&-\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2}-\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2}+\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&-\frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)}+\frac{3iab^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&+\frac{3ib^3d\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&-\frac{3iab^2d\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&-\frac{3ib^3d\arctan(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&-\frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}+\frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)} \\
&-\frac{\left(6ab^2d\right)\operatorname{Subst}\left(\int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2} \\
&-\frac{\left(6b^3d\right)\operatorname{Subst}\left(\int\frac{\arctan(x)\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+\left(1+c^2\right)f^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)}+\frac{3iab^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+\frac{3ab^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}+\frac{ib^3d\arctan(c+dx)^3}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+\frac{b^3d(de-cf)\arctan(c+dx)^3}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}-\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}+\frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&-\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}-\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}+\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&-\frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)}+\frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+\frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&-\frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&-\frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+\frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&-\frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}+\frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} \\
&+\frac{(6iab^2d)\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&-\frac{(3ib^3d)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{3ab^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} + \frac{ib^3d\arctan(c+dx)^3}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{b^3d(de-cf)\arctan(c+dx)^3}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} - \frac{(a+b\arctan(c+dx))^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&- \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&- \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&- \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&- \frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{3ib^3d\arctan(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&- \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} + \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} \\
&+ \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.29 (sec) , antiderivative size = 3708, normalized size of antiderivative = 3.01

method	result	size
derivativedivides	Expression too large to display	3708
default	Expression too large to display	3708
parts	Expression too large to display	3834

```
[In] int((a+b*arctan(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*d^2/(c*f-d*e-f*(d*x+c))/f+b^3*d^2*(1/(c*f-d*e-f*(d*x+c))/f*arctan(
d*x+c)^3-3/f*(1/2*arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d
*x+c)^2)+arctan(d*x+c)^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c*f-arctan(d*x+c)^
3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*d*e-arctan(d*x+c)^2*f/(c^2*f^2-2*c*d*e*f+
d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))-f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan
(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+f/(c^2*f^2-2*c*d*e*f+d^2*e^
2+f^2)*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)-1/(c^2*f^
2-2*c*d*e*f+d^2*e^2+f^2)*f^2/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e
+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-I/(c^2*f^2-2*c*d*e*f+d^2
*e^2+f^2)*f^2/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2*c/(c
*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)/(d*e+I*f-c*f))-1/2*I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2/(c*f-d*e+I*f)
*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/(c^
2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2*c/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-
d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2/(c^2*f^2-2*c*d*e*
f+d^2*e^2+f^2)*f^2*c/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/3*I*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan
(d*x+c)^3+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*d*e/(c*f-d*e+I*f)*arctan(d*x+
c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/2/(c
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*d*e/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(
```

```
1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^
2)*f*d*e/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^
2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/4*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-I*Pi
*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3+2*I*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3-I*Pi*csgn(I*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3-2*I*Pi*csgn
(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*
f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+I*Pi*csgn(I*(1+(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)))^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)-I*Pi*csgn
(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2
))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2
)+2*I*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(I*f*(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1
+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))+I*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^
2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2
))^2)^2+I*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(
1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2-I*Pi*csgn(I*(1+I*(d*x+c
))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))-2*I*Pi*csgn
(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+2*I*Pi*csgn(I*(1+I*(
d*x+c))/(1+(d*x+c)^2)^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2+I*Pi*c
sgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3-2*I*Pi*csgn(I*(1+(1+I*(d*x+c))
^2/(1+(d*x+c)^2)))*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2+4*ln(2))*a
rctan(d*x+c)^2-2/3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(c*f-d*e)*arctan(d*x+c)^
3))+3*a*b^2*d^2*(1/(c*f-d*e-f*(d*x+c))/f*arctan(d*x+c)^2-2/f*(1/2*arctan(d*
x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)+1/(c^2*f^2-2*c*d*e*f
+d^2*e^2+f^2)*arctan(d*x+c)^2*c*f-1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(
d*x+c)^2*d*e-arctan(d*x+c)*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(
d*x+c))-1/2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(c*f-d*e)*arctan(d*x+c)^2-1/2*f
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*l
n(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*
I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(
d*x+c+I)*ln(1/2*I*(d*x+c-I))))-f^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*I*
ln(c*f-d*e-f*(d*x+c))*(ln((I*f+f*(d*x+c))/(c*f-d*e+I*f))-ln((I*f-f*(d*x+c))
/(d*e+I*f-c*f)))/f-1/2*I*(dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f))-dilog((I*f-f
*(d*x+c))/(d*e+I*f-c*f)))/f)))+3*a^2*b*d^2*(1/(c*f-d*e-f*(d*x+c))/f*arctan(
d*x+c)-1/f*(1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(1/2*f*ln(1+(d*x+c)^2)+(c*f-d
*e)*arctan(d*x+c))-f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c)))
))
```

Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx=\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*atan(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx=\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 3/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2
 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*
f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
- 2*arctan(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/32*(4*b^3*
arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 - 32*(f^2*x + e*f)*integrate(1/32*(28*(b^3*d^2*f*x^2 + 2*b^3*c*d*f*x + (b
^3*c^2 + b^3)*f)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*f*x^2 + b^3*d*e + (16*
a*b^2*c + b^3)*d*f*x + 8*(a*b^2*c^2 + a*b^2)*f)*arctan(d*x + c)^2 - 12*(b^3
*d^2*f*x^2 + b^3*c*d*e + (b^3*d^2*e + b^3*c*d*f)*x)*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1) - 3*(b^3*d*f*x + b^3*d*e - (b^3*d^2*f*x^2 + 2*b^3
*c*d*f*x + (b^3*c^2 + b^3)*f)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2)/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2
*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)
*x), x))/(f^2*x + e*f)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3}{{\left(e+f\,x\right)}^2}\,dx$$

```
[In] int((a + b*atan(c + d*x))^3/(e + f*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(e + f*x)^2, x)
```

3.41 $\int (e+fx)^m(a+b\arctan(c+dx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 177

$$\begin{aligned}\int (e+fx)^m(a+b\arctan(c+dx))\,dx &= \frac{(e+fx)^{1+m}(a+b\arctan(c+dx))}{f(1+m)}\\ &\quad-\frac{ibd(e+fx)^{2+m}\operatorname{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de+if-cf}\right)}{2f(de+(i-c)f)(1+m)(2+m)}\\ &\quad+\frac{ibd(e+fx)^{2+m}\operatorname{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(i+c)f}\right)}{2f(de-(i+c)f)(1+m)(2+m)}\end{aligned}$$

```
[Out] (f*x+e)^(1+m)*(a+b*arctan(d*x+c))/f/(1+m)-1/2*I*b*d*(f*x+e)^(2+m)*hypergeom
([1, 2+m],[3+m],d*(f*x+e)/(d*e+I*f-c*f))/f/(d*e+(I-c)*f)/(1+m)/(2+m)+1/2*I*
b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(d*e-(I+c)*f))/f/(d*e-
(I+c)*f)/(1+m)/(2+m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

= {5155, 4972, 726, 70}

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx$$
$$= \frac{(e+fx)^{m+1}(a+b\arctan(c+dx))}{f(m+1)}$$
$$- \frac{ibd(e+fx)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{de-cf+if}\right)}{2f(m+1)(m+2)(de+(-c+i)f)}$$
$$+ \frac{ibd(e+fx)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{de-(c+i)f}\right)}{2f(m+1)(m+2)(de-(c+i)f)}$$

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcTan[c + d*x]))/(f*(1 + m)) - ((I/2)*b*d*(e + f
*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c
*f)])/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) + ((I/2)*b*d*(e + f*x)^(2 + m)*
Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)])/(f*(d*
e - (I + c)*f)*(1 + m)*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m (a+b\arctan(x))\,dx, x, c+dx\right)}{d}\\
&= \frac{(e+fx)^{1+m}(a+b\arctan(c+dx))}{f(1+m)} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1+x^2}\,dx, x, c+dx\right)}{f(1+m)}\\
&= \frac{(e+fx)^{1+m}(a+b\arctan(c+dx))}{f(1+m)}\\
&\quad - \frac{b\text{Subst}\left(\int \left(\frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i-x)}+\frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i+x)}\right)dx, x, c+dx\right)}{f(1+m)}\\
&= \frac{(e+fx)^{1+m}(a+b\arctan(c+dx))}{f(1+m)} - \frac{(ib)\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{i-x}\,dx, x, c+dx\right)}{2f(1+m)}\\
&\quad - \frac{(ib)\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{i+x}\,dx, x, c+dx\right)}{2f(1+m)}\\
&= \frac{(e+fx)^{1+m}(a+b\arctan(c+dx))}{f(1+m)}\\
&\quad - \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de+if-cf}\right)}{2f(de+(i-c)f)(1+m)(2+m)}\\
&\quad + \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(i+c)f}\right)}{2f(de-(i+c)f)(1+m)(2+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx$$

$$= \frac{(e+fx)^{1+m}\Bigg(2(a+b\arctan(c+dx))+\frac{bd(e+fx)\left((de-(i+c)f)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(-i+c)f}\right)+(-de+(-i+c}{(ide+f-icf)(de-(-i+c)f)(2+m)}}{2f(1+m)}$$

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(2*(a + b*ArcTan[c + d*x]) + (b*d*(e + f*x)*((d*e - (I +
 c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f)]
 + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(
d*e - (I + c)*f)]))/((I*d*e + f - I*c*f)*(d*e - (-I + c)*f)*(2 + m))))/(2*f
*(1 + m))
```

Maple [F]

$$\int (fx+e)^m (a+b\arctan(dx+c))\,dx$$

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
```

Fricas [F]

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx = \int (b\arctan(dx+c)+a)(fx+e)^m\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx = \int (b\arctan(dx+c)+a)(fx+e)^m\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((3*e*m^2 + 2*e*m + (3*f*m^2 + 2*f*m + f)*x + e)*(f*x + e)^m*arctan(d*x
 + c) + (e*m + (f*m + f)*x + e)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1
) + 2*(f*m^3 + f*m^2 + f*m + f)*integrate(-1/2*(2*((c^2 + 1)*f*m^3 + 2*(c^2
```

```
 + 1)*f*m^2 + (c^2 + 1)*f*m + (d^2*f*m^3 + 2*d^2*f*m^2 + d^2*f*m)*x^2 + 2*(
c*d*f*m^3 + 2*c*d*f*m^2 + c*d*f*m)*x)*(f*x + e)^m*arctan(d*x + c) - ((c^2 +
 1)*f*m^3 - (c^2 + 1)*f*m + (d^2*f*m^3 - d^2*f*m)*x^2 + 2*(c*d*f*m^3 - c*d*
f*m)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*((c - 1)*d*e*m^2 -
 (c + 1)*d*e*m + (d^2*f*m^2 - d^2*f*m)*x^2 + ((d^2*e + (c - 1)*d*f)*m^2 - (
d^2*e + (c + 1)*d*f)*m)*x)*(f*x + e)^m)/((c^2 + 1)*f*m^3 + (c^2 + 1)*f*m^2
+ (c^2 + 1)*f*m + (d^2*f*m^3 + d^2*f*m^2 + d^2*f*m + d^2*f)*x^2 + (c^2 + 1)
*f + 2*(c*d*f*m^3 + c*d*f*m^2 + c*d*f*m + c*d*f)*x), x))*b/(f*m^3 + f*m^2 +
 f*m + f) + (f*x + e)^(m + 1)*a/(f*(m + 1))
```

Giac [F]

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx=\int (b\arctan(dx+c)+a)(fx+e)^m\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx=\int (e+f\,x)^m\,(a+b\,\mathrm{atan}(c+d\,x))\,dx$$

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x)),x)
```

```
[Out] int((e + f*x)^m*(a + b*atan(c + d*x)), x)
```

3.42 $\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\operatorname{Int}\left((e+fx)^m(a+b\arctan(c+dx))^2,x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx$$

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^2, x]
, x, c + d*x]/d
```

Rubi steps

$$\text{integral}=\frac{\operatorname{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m(a+b\arctan(x))^2\,dx,x,c+dx\right)}{d}$$

Mathematica [N/A]

Not integrable

Time = 4.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx$$

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
[Out] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(fx+e\right)^{m}\left(a+b\arctan\left(dx+c\right)\right)^{2}dx$$

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int \left(b\arctan\left(dx+c\right)+a\right)^{2}(fx+e)^{m}\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)*(f*x + e)^m,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 8.77 (sec) , antiderivative size = 504, normalized size of antiderivative = 25.20

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int (b\arctan\left(dx+c\right)+a)^2(fx+e)^m\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (f*x + e)^(m + 1)*a^2/(f*(m + 1)) + 1/16*(4*(b^2*f*x + b^2*e)*(f*x + e)^m*a
rctan(d*x + c)^2 - (b^2*f*x + b^2*e)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2 + 16*(f*m + f)*integrate(1/16*(12*((b^2*c^2 + b^2)*f*m + (b^2*d^2*
f*m + b^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*d*f*m + b^2*c*d*f)*x)*(
f*x + e)^m*arctan(d*x + c)^2 + ((b^2*c^2 + b^2)*f*m + (b^2*d^2*f*m + b^2*d^
2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*d*f*m + b^2*c*d*f)*x)*(f*x + e)^m*l
og(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 8*(b^2*d*e - 4*(a*b*c^2 + a*b)*f*m - 4*
(a*b*d^2*f*m + a*b*d^2*f)*x^2 - 4*(a*b*c^2 + a*b)*f - (8*a*b*c*d*f*m + (8*a
*b*c - b^2)*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + 4*(b^2*d^2*f*x^2 + b^2*c*
d*e + (b^2*d^2*e + b^2*c*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 +
1))/((c^2 + 1)*f*m + (d^2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d
*f)*x), x))/(f*m + f)
```

Giac [N/A]

Not integrable

Time = 111.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int (b\arctan\left(dx+c\right)+a)^2(fx+e)^m\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int {\left(e+f\,x\right)}^m\,{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^2 \,d x$$

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^m*(a + b*atan(c + d*x))^2, x)
```

3.43 $\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\operatorname{Int}\left((e+fx)^m(a+b\arctan(c+dx))^3,x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx$$

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^3, x]
, x, c + d*x]/d
```

Rubi steps

$$\text{integral}=\frac{\operatorname{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m(a+b\arctan(x))^3\,dx,x,c+dx\right)}{d}$$

Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx$$

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m\,(a+b\arctan\,(dx+c))^3\,dx$$

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\int (b\arctan\,(dx+c)+a)^3(fx+e)^m\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 11.47 (sec) , antiderivative size = 659, normalized size of antiderivative = 32.95

$$\int (e+fx)^m (a+b\arctan(c+dx))^3 \, dx = \int (b\arctan(dx+c)+a)^3 (fx+e)^m \, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (f*x + e)^(m + 1)*a^3/(f*(m + 1)) + 1/32*(4*(b^3*f*x + b^3*e)*(f*x + e)^m*a
rctan(d*x + c)^3 - 3*(b^3*f*x + b^3*e)*(f*x + e)^m*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2 + 32*(f*m + f)*integrate(1/32*(28*((b^3*c^2 + b^
3)*f*m + (b^3*d^2*f*m + b^3*d^2*f)*x^2 + (b^3*c^2 + b^3)*f + 2*(b^3*c*d*f*m
 + b^3*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c)^3 - 12*(b^3*d*e - 8*(a*b^2*c^2
 + a*b^2)*f*m - 8*(a*b^2*d^2*f*m + a*b^2*d^2*f)*x^2 - 8*(a*b^2*c^2 + a*b^2)
*f - (16*a*b^2*c*d*f*m + (16*a*b^2*c - b^3)*d*f)*x)*(f*x + e)^m*arctan(d*x
+ c)^2 + 12*(b^3*d^2*f*x^2 + b^3*c*d*e + (b^3*d^2*e + b^3*c*d*f)*x)*(f*x +
e)^m*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 96*((a^2*b*c^2 + a^
2*b)*f*m + (a^2*b*d^2*f*m + a^2*b*d^2*f)*x^2 + (a^2*b*c^2 + a^2*b)*f + 2*(a
^2*b*c*d*f*m + a^2*b*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + 3*(((b^3*c^2 +
 b^3)*f*m + (b^3*d^2*f*m + b^3*d^2*f)*x^2 + (b^3*c^2 + b^3)*f + 2*(b^3*c*d*
f*m + b^3*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + (b^3*d*f*x + b^3*d*e)*(f*
x + e)^m)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/((c^2 + 1)*f*m + (d^2*f*m + d
^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

Giac [N/A]

Not integrable

Time = 112.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int (e+fx)^m (a+b\arctan(c+dx))^3 \, dx = \int (b\arctan(dx+c)+a)^3 (fx+e)^m \, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\int (e+f\,x)^m\,(a+b\,\mathrm{atan}(c+d\,x))^3\,dx$$

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x))^3,x)
[Out] int((e + f*x)^m*(a + b*atan(c + d*x))^3, x)
```

3.44 $\int x^3 \arctan(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 106

$$\begin{aligned}\int x^3 \arctan(a+bx)\,dx = {} & \frac{(1-6a^2)\,x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4} \\ & -\frac{(1-6a^2+a^4)\arctan(a+bx)}{4b^4} \\ & +\frac{1}{4}x^4\arctan(a+bx)-\frac{a(1-a^2)\log\left(1+(a+bx)^2\right)}{2b^4}\end{aligned}$$

```
[Out] 1/4*(-6*a^2+1)*x/b^3+1/2*a*(b*x+a)^2/b^4-1/12*(b*x+a)^3/b^4-1/4*(a^4-6*a^2+
1)*arctan(b*x+a)/b^4+1/4*x^4*arctan(b*x+a)-1/2*a*(-a^2+1)*ln(1+(b*x+a)^2)/b
^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\begin{aligned}\int x^3 \arctan(a+bx)\,dx = {} & -\frac{a(1-a^2)\log\left((a+bx)^2+1\right)}{2b^4}+\frac{(1-6a^2)\,x}{4b^3} \\ & -\frac{(a^4-6a^2+1)\arctan(a+bx)}{4b^4} \\ & +\frac{1}{4}x^4\arctan(a+bx)-\frac{(a+bx)^3}{12b^4}+\frac{a(a+bx)^2}{2b^4}\end{aligned}$$

```
[In] Int[x^3*ArcTan[a + b*x],x]
```

```
[Out] ((1 - 6*a^2)*x)/(4*b^3) + (a*(a + b*x)^2)/(2*b^4) - (a + b*x)^3/(12*b^4) -
((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (x^4*ArcTan[a + b*x])/4 - (a*
(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \arctan(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4\arctan(a+bx) - \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{1+x^2}\,dx, x, a+bx\right)$$

$$
\begin{aligned}
&= \frac{1}{4}x^4\arctan(a+bx) \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1-6a^2}{b^4} - \frac{4ax}{b^4} + \frac{x^2}{b^4} + \frac{1-6a^2+a^4+4a(1-a^2)\,x}{b^4\,(1+x^2)}\right) dx, x, a \right. \\
&\qquad \left. + bx\right) \\
&= \frac{(1-6a^2)\,x}{4b^3} + \frac{a(a+bx)^2}{2b^4} - \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4\arctan(a+bx) \\
&\quad - \frac{\text{Subst}\left(\int \frac{1-6a^2+a^4+4a(1-a^2)x}{1+x^2}\,dx, x, a+bx\right)}{4b^4} \\
&= \frac{(1-6a^2)\,x}{4b^3} + \frac{a(a+bx)^2}{2b^4} - \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4\arctan(a+bx) \\
&\quad - \frac{(a(1-a^2))\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b^4} \\
&\quad - \frac{(1-6a^2+a^4)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{4b^4} \\
&= \frac{(1-6a^2)\,x}{4b^3} + \frac{a(a+bx)^2}{2b^4} - \frac{(a+bx)^3}{12b^4} - \frac{(1-6a^2+a^4)\arctan(a+bx)}{4b^4} \\
&\quad + \frac{1}{4}x^4\arctan(a+bx) - \frac{a(1-a^2)\log\left(1+(a+bx)^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$
\int x^3\arctan(a+bx)\,dx
$$

$$
= \frac{6(1-6a^2)\,bx + 12a(a+bx)^2 - 2(a+bx)^3 + 6b^4x^4\arctan(a+bx) + 3i(-i+a)^4\log(i-a-bx) - 3i(i+}{24b^4}
$$

```
[In] Integrate[x^3*ArcTan[a + b*x],x]
```

```
[Out] (6*(1 - 6*a^2)*b*x + 12*a*(a + b*x)^2 - 2*(a + b*x)^3 + 6*b^4*x^4*ArcTan[a
+ b*x] + (3*I)*(-I + a)^4*Log[I - a - b*x] - (3*I)*(I + a)^4*Log[I + a + b*
x])/(24*b^4)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{3\arctan(bx+a)x^4b^4-b^3x^3+3a\,b^2x^2-3\arctan(bx+a)a^4+6a^3\ln\left(b^2x^2+2abx+a^2+1\right)-9a^2bx+18\arctan(bx+a)a^2+15a^3-}{12b^4}$
parts	$\frac{x^4\arctan(bx+a)}{4}-\frac{b\left(\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x-x}{b^4}+\frac{\frac{\left(-4a^3b+4ab\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(-3a^4-2a^2+1-\frac{\left(-4a^3b+4ab\right)a}{b}\right)\text{arcta}}{b}}{b^4}\right.}{4}$
derivativedivides	$\frac{\frac{\arctan(bx+a)a^4}{4}-\arctan(bx+a)a^3(bx+a)+\frac{3\arctan(bx+a)a^2(bx+a)^2}{2}-\arctan(bx+a)a(bx+a)^3+\frac{\arctan(bx+a)(bx+a)^4}{4}-\frac{3a^2(bx}{2}}{b^4}$
default	$\frac{\frac{\arctan(bx+a)a^4}{4}-\arctan(bx+a)a^3(bx+a)+\frac{3\arctan(bx+a)a^2(bx+a)^2}{2}-\arctan(bx+a)a(bx+a)^3+\frac{\arctan(bx+a)(bx+a)^4}{4}-\frac{3a^2(bx}{2}}{b^4}$
risch	$-\frac{ix^4\ln(1+i(bx+a))}{8}+\frac{ix^4\ln(1-i(bx+a))}{8}-\frac{x^3}{12b}-\frac{a^4\arctan(bx+a)}{4b^4}+\frac{a\,x^2}{4b^2}+\frac{a^3\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^4}-\frac{3a^2}{4b^3}$

```
[In] int(x^3*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(3*arctan(b*x+a)*x^4*b^4-b^3*x^3+3*a*b^2*x^2-3*arctan(b*x+a)*a^4+6*a^3
*ln(b^2*x^2+2*a*b*x+a^2+1)-9*a^2*b*x+18*arctan(b*x+a)*a^2+15*a^3-6*a*ln(b^2
*x^2+2*a*b*x+a^2+1)+3*b*x-3*arctan(b*x+a)-9*a)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int x^3\arctan(a+bx)\,dx =$$

$$-\frac{b^3x^3-3\,ab^2x^2+3\,(3\,a^2-1)bx-3\,(b^4x^4-a^4+6\,a^2-1)\arctan\left(bx+a\right)-6\,(a^3-a)\log\left(b^2x^2+2\,abx\right.}{12\,b^4}$$

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*x - 3*(b^4*x^4 - a^4 + 6*a^2
 - 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int x^3 \arctan(a+bx)\,dx$$
$$= \begin{cases} -\frac{a^4 \operatorname{atan}(a+bx)}{4b^4} + \frac{a^3 \log(a^2+2abx+b^2x^2+1)}{2b^4} - \frac{3a^2x}{4b^3} + \frac{3a^2 \operatorname{atan}(a+bx)}{2b^4} + \frac{ax^2}{4b^2} - \frac{a \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{x^4 \operatorname{atan}(a+bx)}{4} - \\ \frac{x^4 \operatorname{atan}(a)}{4} \end{cases}$$

```
[In] integrate(x**3*atan(b*x+a),x)
```

```
[Out] Piecewise((-a**4*atan(a + b*x)/(4*b**4) + a**3*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(2*b**4) - 3*a**2*x/(4*b**3) + 3*a**2*atan(a + b*x)/(2*b**4) + a*x*
*2/(4*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*atan(a
+ b*x)/4 - x**3/(12*b) + x/(4*b**3) - atan(a + b*x)/(4*b**4), Ne(b, 0)), (x
**4*atan(a)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int x^3 \arctan(a+bx)\,dx = \frac{1}{4}\,x^4 \arctan(bx+a)$$
$$-\frac{1}{12}\,b\left(\frac{b^2x^3 - 3\,abx^2 + 3\,(3\,a^2-1)x}{b^4} + \frac{3\,(a^4 - 6\,a^2 + 1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5} - \frac{6\,(a^3 - a)\log(b^2x^2 + 2\,abx}{b^5}\right.$$

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(b*x + a) - 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/b
^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^2*
x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

Giac [F]

$$\int x^3 \arctan(a+bx)\,dx = \int x^3 \arctan(bx+a)\,dx$$

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int x^3 \arctan(a+bx)\,dx = \frac{x^4\,\mathrm{atan}(a+b\,x)}{4} - \frac{\mathrm{atan}(a+b\,x)}{4\,b^4} + \frac{x}{4\,b^3} - \frac{x^3}{12\,b} + \frac{a^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4} + \frac{3\,a^2\,\mathrm{atan}(a+b\,x)}{2\,b^4} - \frac{a^4\,\mathrm{atan}(a+b\,x)}{4\,b^4} + \frac{a\,x^2}{4\,b^2} - \frac{3\,a^2\,x}{4\,b^3} - \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4}$$

```
[In] int(x^3*atan(a + b*x),x)
```

```
[Out] (x^4*atan(a + b*x))/4 - atan(a + b*x)/(4*b^4) + x/(4*b^3) - x^3/(12*b) + (a
^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4) + (3*a^2*atan(a + b*x))/(2*b^4
) - (a^4*atan(a + b*x))/(4*b^4) + (a*x^2)/(4*b^2) - (3*a^2*x)/(4*b^3) - (a*
log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4)
```

3.45 $\int x^2 \arctan(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 79

$$\int x^2 \arctan(a+bx)\,dx = \frac{ax}{b^2} - \frac{(a+bx)^2}{6b^3} - \frac{a(3-a^2)\arctan(a+bx)}{3b^3} + \frac{1}{3}x^3\arctan(a+bx) + \frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}$$

```
[Out] a*x/b^2-1/6*(b*x+a)^2/b^3-1/3*a*(-a^2+3)*arctan(b*x+a)/b^3+1/3*x^3*arctan(b
*x+a)+1/6*(-3*a^2+1)*ln(1+(b*x+a)^2)/b^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\int x^2 \arctan(a+bx)\,dx = -\frac{a(3-a^2)\arctan(a+bx)}{3b^3} + \frac{(1-3a^2)\log\left((a+bx)^2+1\right)}{6b^3} + \frac{1}{3}x^3\arctan(a+bx) - \frac{(a+bx)^2}{6b^3} + \frac{ax}{b^2}$$

```
[In] Int[x^2*ArcTan[a + b*x],x]
```

```
[Out] (a*x)/b^2 - (a + b*x)^2/(6*b^3) - (a*(3 - a^2)*ArcTan[a + b*x])/(3*b^3) + (
x^3*ArcTan[a + b*x])/3 + ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(6*b^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \arctan(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\arctan(a+bx) - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{1+x^2}\,dx, x, a+bx\right)$$

$$= \frac{1}{3}x^3\arctan(a+bx) - \frac{1}{3}\text{Subst}\left(\int \left(-\frac{3a}{b^3}+\frac{x}{b^3}+\frac{a(3-a^2)-(1-3a^2)\,x}{b^3\,(1+x^2)}\right)\,dx, x, a + bx\right)$$

$$
\begin{aligned}
&= \frac{ax}{b^2} - \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\arctan(a+bx) - \frac{\text{Subst}\left(\int \frac{a(3-a^2)-(1-3a^2)x}{1+x^2}\,dx, x, a+bx\right)}{3b^3} \\
&= \frac{ax}{b^2} - \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\arctan(a+bx) + \frac{(1-3a^2)\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{3b^3} \\
&\quad - \frac{(a(3-a^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{3b^3} \\
&= \frac{ax}{b^2} - \frac{(a+bx)^2}{6b^3} - \frac{a(3-a^2)\arctan(a+bx)}{3b^3} \\
&\quad + \frac{1}{3}x^3\arctan(a+bx) + \frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.44

$$
\int x^2\arctan(a+bx)\,dx
$$

$$
= \frac{\frac{1}{3}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^3\arctan(a+bx) - \frac{1}{3}b\left(-\frac{3ax}{b^2}+\frac{(a+bx)^2}{2b^3}-\frac{(1+ia)^3\log(i-a-bx)}{2b^3}-\frac{(1-ia)^3\log(i+a+bx)}{2b^3}\right)}{b}
$$

```
[In] Integrate[x^2*ArcTan[a + b*x],x]
```

```
[Out] ((b*(-(a/b) + (a + b*x)/b)^3*ArcTan[a + b*x])/3 - (b*((-3*a*x)/b^2 + (a + b
*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log[I
 + a + b*x])/(2*b^3)))/3)/b
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

method	result
parallelrisch	$-\frac{-2\arctan(bx+a)x^3b^3+b^2x^2-2\arctan(bx+a)a^3+3a^2\ln(b^2x^2+2abx+a^2+1)-4abx+6a\arctan(bx+a)+7a^2-1-\ln(b^2x}{6b^3}$
derivativedivides	$\frac{-\frac{\arctan(bx+a)a^3}{3}+\arctan(bx+a)a^2(bx+a)-\arctan(bx+a)a(bx+a)^2+\frac{\arctan(bx+a)(bx+a)^3}{3}+(bx+a)a-\frac{(bx+a)^2}{6}+\frac{(-3a^2+1}{}}{b^3}$
default	$\frac{-\frac{\arctan(bx+a)a^3}{3}+\arctan(bx+a)a^2(bx+a)-\arctan(bx+a)a(bx+a)^2+\frac{\arctan(bx+a)(bx+a)^3}{3}+(bx+a)a-\frac{(bx+a)^2}{6}+\frac{(-3a^2+1}{}}{b^3}$
parts	$\frac{x^3\arctan(bx+a)}{3}-\frac{b\left(-\frac{-\frac{1}{2}x^2b+2ax}{b^3}+\frac{\frac{(3a^2b-b)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{\left(2a^3+2a-\frac{(3a^2b-b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^3}\right)}{3}$
risch	$-\frac{ix^3\ln(1+i(bx+a))}{6}+\frac{ix^3\ln(1-i(bx+a))}{6}+\frac{a^3\arctan(bx+a)}{3b^3}-\frac{x^2}{6b}-\frac{a^2\ln(b^2x^2+2abx+a^2+1)}{2b^3}+\frac{2ax}{3b^2}-\frac{a\,\mathrm{arc}}{}$

```
[In] int(x^2*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-2*arctan(b*x+a)*x^3*b^3+b^2*x^2-2*arctan(b*x+a)*a^3+3*a^2*ln(b^2*x^2
+2*a*b*x+a^2+1)-4*a*b*x+6*a*arctan(b*x+a)+7*a^2-1-ln(b^2*x^2+2*a*b*x+a^2+1)
)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int x^2\arctan(a+bx)\,dx = -\frac{b^2x^2-4\,abx-2\,(b^3x^3+a^3-3\,a)\arctan(bx+a)+(3\,a^2-1)\log(b^2x^2+2\,abx+a^2+1)}{6\,b^3}$$

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(b^2*x^2 - 4*a*b*x - 2*(b^3*x^3 + a^3 - 3*a)*arctan(b*x + a) + (3*a^2
- 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int x^2 \arctan(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3 \operatorname{atan}(a+bx)}{3b^3} - \frac{a^2 \log(a^2+2abx+b^2x^2+1)}{2b^3} + \frac{2ax}{3b^2} - \frac{a \operatorname{atan}(a+bx)}{b^3} + \frac{x^3 \operatorname{atan}(a+bx)}{3} - \frac{x^2}{6b} + \frac{\log(a^2+2abx+b^2x^2+1)}{6b^3} & \text{for } b \neq \\ \frac{x^3 \operatorname{atan}(a)}{3} & \text{otherw} \end{cases}$$

```
[In] integrate(x**2*atan(b*x+a),x)
```

```
[Out] Piecewise((a**3*atan(a + b*x)/(3*b**3) - a**2*log(a**2 + 2*a*b*x + b**2*x**
2 + 1)/(2*b**3) + 2*a*x/(3*b**2) - a*atan(a + b*x)/b**3 + x**3*atan(a + b*x
)/3 - x**2/(6*b) + log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0)),
 (x**3*atan(a)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int x^2 \arctan(a+bx)\,dx = \frac{1}{3}\,x^3 \arctan(bx+a)$$
$$-\frac{1}{6}\,b\left(\frac{bx^2-4\,ax}{b^3} - \frac{2\,(a^3-3\,a)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{(3\,a^2-1)\log(b^2x^2+2\,abx+a^2+1)}{b^4}\right)$$

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(b*x + a) - 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arctan
((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

Giac [F]

$$\int x^2 \arctan(a+bx)\,dx = \int x^2 \arctan(bx+a)\,dx$$

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

$$\int x^2 \arctan(a+bx)\,dx = \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b^3}+\frac{x^3\,\mathrm{atan}(a+b\,x)}{3} - \frac{x^2}{6\,b}-\frac{a^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^3} + \frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b^3}-\frac{a\,\mathrm{atan}(a+b\,x)}{b^3}+\frac{2\,a\,x}{3\,b^2}$$

```
[In] int(x^2*atan(a + b*x),x)
```

```
[Out] log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b^3) + (x^3*atan(a + b*x))/3 - x^2/(6*b
) - (a^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^3) + (a^3*atan(a + b*x))/(3
*b^3) - (a*atan(a + b*x))/b^3 + (2*a*x)/(3*b^2)
```

3.46 $\int x \arctan(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x \arctan(a+bx)\,dx = -\frac{x}{2b}+\frac{(1-a^2)\arctan(a+bx)}{2b^2} + \frac{1}{2}x^2\arctan(a+bx)+\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

```
[Out] -1/2*x/b+1/2*(-a^2+1)*arctan(b*x+a)/b^2+1/2*x^2*arctan(b*x+a)+1/2*a*ln(1+(b
*x+a)^2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\int x \arctan(a+bx)\,dx = \frac{(1-a^2)\arctan(a+bx)}{2b^2}+\frac{1}{2}x^2\arctan(a+bx) + \frac{a\log\left((a+bx)^2+1\right)}{2b^2}-\frac{x}{2b}$$

```
[In] Int[x*ArcTan[a + b*x],x]
```

```
[Out] -1/2*x/b + ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) + (x^2*ArcTan[a + b*x])/2 +
(a*Log[1 + (a + b*x)^2])/(2*b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)\arctan(x)\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{2}x^2\arctan(a+bx)-\frac{1}{2}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{1+x^2}\,dx, x, a+bx\right) \\
&= \frac{1}{2}x^2\arctan(a+bx)-\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{1-a^2+2ax}{b^2\left(1+x^2\right)}\right)\,dx, x, a+bx\right) \\
&= -\frac{x}{2b}+\frac{1}{2}x^2\arctan(a+bx)+\frac{\text{Subst}\left(\int \frac{1-a^2+2ax}{1+x^2}\,dx, x, a+bx\right)}{2b^2}
\end{aligned}$$

$$= -\frac{x}{2b} + \frac{1}{2}x^2 \arctan(a+bx) + \frac{a\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b^2} + \frac{(1-a^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{2b^2}$$

$$= -\frac{x}{2b} + \frac{(1-a^2)\arctan(a+bx)}{2b^2} + \frac{1}{2}x^2 \arctan(a+bx) + \frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int x\arctan(a+bx)\,dx$$

$$= \frac{-2bx + 2b^2x^2\arctan(a+bx) + i(-i+a)^2\log(i-a-bx) + i\log(i+a+bx) + 2a\log(i+a+bx) - ia^2 \mathrm{l}}{4b^2}$$

```
[In] Integrate[x*ArcTan[a + b*x],x]
```

```
[Out] (-2*b*x + 2*b^2*x^2*ArcTan[a + b*x] + I*(-I + a)^2*Log[I - a - b*x] + I*Log
[I + a + b*x] + 2*a*Log[I + a + b*x] - I*a^2*Log[I + a + b*x])/(4*b^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\arctan(bx+a)(bx+a)^2}{2} - \arctan(bx+a)a(bx+a) - \frac{bx}{2} - \frac{a}{2} + \frac{a\ln\left(1+(bx+a)^2\right)}{2} + \frac{\arctan(bx+a)}{2}}{b^2}$
default	$\frac{\frac{\arctan(bx+a)(bx+a)^2}{2} - \arctan(bx+a)a(bx+a) - \frac{bx}{2} - \frac{a}{2} + \frac{a\ln\left(1+(bx+a)^2\right)}{2} + \frac{\arctan(bx+a)}{2}}{b^2}$
parallelrisch	$\frac{\arctan(bx+a)b^2x^2 - \arctan(bx+a)a^2 + a\ln\left(b^2x^2+2abx+a^2+1\right) - bx + \arctan(bx+a) + 2a}{2b^2}$
parts	$\frac{x^2\arctan(bx+a)}{2} - \frac{b\left(\frac{x}{b^2} + \frac{-\frac{a\ln\left(b^2x^2+2abx+a^2+1\right)}{b} + \frac{\left(a^2-1\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^2}\right)}{2}$
risch	$-\frac{ix^2\ln(1+i(bx+a))}{4} + \frac{ix^2\ln(1-i(bx+a))}{4} - \frac{a^2\arctan(bx+a)}{2b^2} + \frac{a\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2} - \frac{x}{2b} + \frac{\arctan(bx+a}{2b^2}$

```
[In] int(x*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arctan(b*x+a)*(b*x+a)^2-arctan(b*x+a)*a*(b*x+a)-1/2*b*x-1/2*a+1/
2*a*ln(1+(b*x+a)^2)+1/2*arctan(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x\arctan(a+bx)\,dx = -\frac{bx-\left(b^2x^2-a^2+1\right)\arctan\left(bx+a\right)-a\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b^2}$$

```
[In] integrate(x*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - (b^2*x^2 - a^2 + 1)*arctan(b*x + a) - a*log(b^2*x^2 + 2*a*b*x +
 a^2 + 1))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int x\arctan(a+bx)\,dx = \begin{cases} -\frac{a^2\operatorname{atan}\left(a+bx\right)}{2b^2}+\frac{a\log\left(a^2+2abx+b^2x^2+1\right)}{2b^2}+\frac{x^2\operatorname{atan}\left(a+bx\right)}{2}-\frac{x}{2b}+\frac{\operatorname{atan}\left(a+bx\right)}{2b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{atan}\left(a\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*atan(b*x+a),x)
```

```
[Out] Piecewise((-a**2*atan(a + b*x)/(2*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2
+ 1)/(2*b**2) + x**2*atan(a + b*x)/2 - x/(2*b) + atan(a + b*x)/(2*b**2), Ne
(b, 0)), (x**2*atan(a)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int x\arctan(a+bx)\,dx = \frac{1}{2}\,x^2\arctan\left(bx+a\right) - \frac{1}{2}\,b\left(\frac{x}{b^2}+\frac{\left(a^2-1\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3}-\frac{a\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^3}\right)$$

```
[In] integrate(x*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(b*x + a) - 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)/
b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

Giac [F]

$$\int x \arctan(a+bx)\,dx = \int x \arctan(bx+a)\,dx$$

```
[In] integrate(x*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int x \arctan(a+bx)\,dx$$
$$= \frac{x^2\,\mathrm{atan}(a+b\,x)}{2} + \frac{\frac{\mathrm{atan}(a+b\,x)}{2} - \frac{b\,x}{2} - \frac{a^2\,\mathrm{atan}(a+b\,x)}{2} + \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}}{b^2}$$

```
[In] int(x*atan(a + b*x),x)
```

```
[Out] (x^2*atan(a + b*x))/2 + (atan(a + b*x)/2 - (b*x)/2 - (a^2*atan(a + b*x))/2
+ (a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2)/b^2
```

3.47 $\int \arctan(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 33

$$\int \arctan(a+bx)\,dx=\frac{(a+bx)\arctan(a+bx)}{b}-\frac{\log\left(1+(a+bx)^2\right)}{2b}$$

```
[Out] (b*x+a)*arctan(b*x+a)/b-1/2*ln(1+(b*x+a)^2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5147, 4930, 266}

$$\int \arctan(a+bx)\,dx=\frac{(a+bx)\arctan(a+bx)}{b}-\frac{\log\left((a+bx)^2+1\right)}{2b}$$

```
[In] Int[ArcTan[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcTan[a + b*x])/b - Log[1 + (a + b*x)^2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \arctan(x)\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\arctan(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\arctan(a+bx)}{b} - \frac{\log\left(1+(a+bx)^2\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \arctan(a+bx)\,dx = -\frac{-2(a+bx)\arctan(a+bx)+\log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

```
[In] Integrate[ArcTan[a + b*x],x]
[Out] -1/2*(-2*(a + b*x)*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2])/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\arctan(bx+a)-\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
default	$\frac{(bx+a)\arctan(bx+a)-\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
parallelrisch	$-\frac{-2x\arctan(bx+a)b^2-2\arctan(bx+a)ab+\ln(b^2x^2+2abx+a^2+1)b}{2b^2}$	49
parts	$x\arctan(bx+a)-b\left(\frac{\ln(b^2x^2+2abx+a^2+1)}{2b^2}-\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b^2}\right)$	60
risch	$-\frac{ix\ln(1+i(bx+a))}{2}+\frac{ix\ln(1-i(bx+a))}{2}+\frac{a\arctan(bx+a)}{b}-\frac{\ln(b^2x^2+2abx+a^2+1)}{2b}$	66

```
[In] int(arctan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((b*x+a)*arctan(b*x+a)-1/2*ln(1+(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \arctan(a+bx)\,dx = \frac{2\,(bx+a)\arctan\left(bx+a\right)-\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

```
[In] integrate(arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \arctan(a+bx)\,dx = \begin{cases} \frac{a\,\mathrm{atan}\left(a+bx\right)}{b}+x\,\mathrm{atan}\left(a+bx\right)-\frac{\log\left(a^2+2abx+b^2x^2+1\right)}{2b} & \text{for } b\neq 0 \\ x\,\mathrm{atan}\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(atan(b*x+a),x)
```

```
[Out] Piecewise((a*atan(a + b*x)/b + x*atan(a + b*x) - log(a**2 + 2*a*b*x + b**2*
x**2 + 1)/(2*b), Ne(b, 0)), (x*atan(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \arctan(a+bx)\,dx = \frac{2\,(bx+a)\arctan\left(bx+a\right)-\log\left(\left(bx+a\right)^2+1\right)}{2\,b}$$

```
[In] integrate(arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \arctan(a+bx)\,dx = \frac{2\,(bx+a)\arctan\left(bx+a\right) - \log\left((bx+a)^2+1\right)}{2\,b}$$

```
[In] integrate(arctan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \arctan(a+bx)\,dx = x\,\mathrm{atan}(a+b\,x) - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)-2\,a\,\mathrm{atan}(a+b\,x)}{2\,b}$$

```
[In] int(atan(a + b*x),x)
```

```
[Out] x*atan(a + b*x) - (log(a^2 + b^2*x^2 + 2*a*b*x + 1) - 2*a*atan(a + b*x))/(2
*b)
```

3.48 $\int \frac{\arctan(a+bx)}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 120

$$\begin{aligned}\int \frac{\arctan(a+bx)}{x}\,dx = & -\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)\\ & +\arctan(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)\\ & +\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)\\ & -\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)\end{aligned}$$

```
[Out] -arctan(b*x+a)*ln(2/(1-I*(b*x+a)))+arctan(b*x+a)*ln(2*b*x/(I-a)/(1-I*(b*x+a
)))+1/2*I*polylog(2,1-2/(1-I*(b*x+a)))-1/2*I*polylog(2,1-2*b*x/(I-a)/(1-I*(
b*x+a)))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5155, 4966, 2449, 2352, 2497}

$$\begin{aligned}\int \frac{\arctan(a+bx)}{x}\,dx = & -\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)\\ & +\arctan(a+bx)\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\\ & +\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)\\ & -\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)\end{aligned}$$

```
[In] Int[ArcTan[a + b*x]/x,x]
```

```
[Out] -(ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcTan[a + b*x]*Log[(2*b*x)/(
(I - a)*(1 - I*(a + b*x)))] + (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] - (
I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\arctan(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$
\begin{aligned}
&= -\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\arctan(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)\\
&\quad+\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx,x,a+bx\right)-\text{Subst}\left(\int\frac{\log\left(\frac{2\left(-\frac{a}{b}+\frac{x}{b}\right)}{\left(\frac{i}{b}-\frac{a}{b}\right)(1-ix)}\right)}{1+x^2}\,dx,x,a+bx\right)\\
&= -\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\arctan(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)\\
&\quad-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)+i\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-i(a+bx)}\right)\\
&= -\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\arctan(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)\\
&\quad+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

$$
\begin{aligned}
\int\frac{\arctan(a+bx)}{x}\,dx &= -\frac{1}{2}i\log(1+i(a+bx))\log\left(\frac{i\left(-\frac{a}{b}+\frac{a+bx}{b}\right)}{-\frac{1}{b}-\frac{ia}{b}}\right)\\
&\quad+\frac{1}{2}i\log(1-i(a+bx))\log\left(-\frac{i\left(-\frac{a}{b}+\frac{a+bx}{b}\right)}{-\frac{1}{b}+\frac{ia}{b}}\right)\\
&\quad+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i(1-i(a+bx))}{i+a}\right)\\
&\quad-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i(1+i(a+bx))}{-i+a}\right)
\end{aligned}
$$

```
[In] Integrate[ArcTan[a + b*x]/x,x]
```

```
[Out] (-1/2*I)*Log[1 + I*(a + b*x)]*Log[(I*(-(a/b) + (a + b*x)/b))/(-b^(-1) - (I*
a)/b)] + (I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*(-(a/b) + (a + b*x)/b))/(-b^(
-1) + (I*a)/b)] + (I/2)*PolyLog[2, (I*(1 - I*(a + b*x)))/(I + a)] - (I/2)*P
olyLog[2, ((-I)*(1 + I*(a + b*x)))/(-I + a)]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

method	result
risch	$\frac{i \operatorname{dilog}\left(-\frac{ixb}{ia-1}\right)}{2}+\frac{i \ln(-ibx-ia+1) \ln\left(-\frac{ixb}{ia-1}\right)}{2}-\frac{i \operatorname{dilog}\left(\frac{ixb}{-ia-1}\right)}{2}-\frac{i \ln(ibx+ia+1) \ln\left(\frac{ixb}{-ia-1}\right)}{2}$
parts	$\ln(x) \arctan(bx+a)-b\Bigg(-\frac{i \ln(x)\left(\ln\left(\frac{-bx-a+i}{i-a}\right)-\ln\left(\frac{bx+a+i}{i+a}\right)\right)}{2b}-\frac{i\left(\operatorname{dilog}\left(\frac{-bx-a+i}{i-a}\right)-\operatorname{dilog}\left(\frac{bx+a+i}{i+a}\right)\right.}{2b}$
derivativedivides	$\ln(-bx) \arctan(bx+a)-\frac{i \ln(-bx) \ln\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i \ln(-bx) \ln\left(\frac{-bx-a+i}{i-a}\right)}{2}-\frac{i \operatorname{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}+ i\,\mathrm{dilo}$
default	$\ln(-bx) \arctan(bx+a)-\frac{i \ln(-bx) \ln\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i \ln(-bx) \ln\left(\frac{-bx-a+i}{i-a}\right)}{2}-\frac{i \operatorname{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}+ i\,\mathrm{dilo}$

```
[In] int(arctan(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*dilog(-I*x*b/(I*a-1))+1/2*I*ln(1-I*a-I*b*x)*ln(-I*x*b/(I*a-1))-1/2*I*
dilog(I*x*b/(-I*a-1))-1/2*I*ln(1+I*a+I*b*x)*ln(I*x*b/(-I*a-1))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{x}\,dx=\int \frac{\arctan(bx+a)}{x}\,dx$$

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{x}\,dx=\text{Timed out}$$

```
[In] integrate(atan(b*x+a)/x,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{\arctan(a+bx)}{x}\,dx =& -\frac{1}{2}\arctan\left(\frac{bx}{a^2+1}, -\frac{abx}{a^2+1}\right)\log\left(b^2x^2+2\,abx+a^2+1\right)\\&+\frac{1}{2}\arctan\left(bx+a\right)\log\left(\frac{b^2x^2}{a^2+1}\right)\\&+\arctan\left(bx+a\right)\log\left(x\right)-\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(x\right)\\&-\frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a+1}{i\,a+1}\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a-1}{i\,a-1}\right)\end{aligned}$$

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="maxima")
[Out] -1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2 +
 1) + 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arctan(b*x + a)*log(x) -
 arctan((b^2*x + a*b)/b)*log(x) - 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1))
+ 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{x}\,dx = \int \frac{\arctan\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{x}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{x}\,dx$$

```
[In] int(atan(a + b*x)/x,x)
[Out] int(atan(a + b*x)/x, x)
```

3.49 $\int \frac{\arctan(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = -\frac{ab\arctan(a+bx)}{1+a^2}-\frac{\arctan(a+bx)}{x}+\frac{b\log(x)}{1+a^2}-\frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}$$

```
[Out] -a*b*arctan(b*x+a)/(a^2+1)-arctan(b*x+a)/x+b*ln(x)/(a^2+1)-1/2*b*ln(1+(b*x+
a)^2)/(a^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5153, 378, 720, 31, 649, 209, 266}

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = -\frac{ab\arctan(a+bx)}{a^2+1}+\frac{b\log(x)}{a^2+1}-\frac{b\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)}-\frac{\arctan(a+bx)}{x}$$

```
[In] Int[ArcTan[a + b*x]/x^2,x]
```

```
[Out] -((a*b*ArcTan[a + b*x])/(1 + a^2)) - ArcTan[a + b*x]/x + (b*Log[x])/(1 + a^
2) - (b*Log[1 + (a + b*x)^2])/(2*(1 + a^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\arctan(a+bx)}{x} + b\int \frac{1}{x\left(1+(a+bx)^2\right)}\,dx \\
&= -\frac{\arctan(a+bx)}{x} + b\text{Subst}\left(\int \frac{1}{(-a+x)\left(1+x^2\right)}\,dx, x, a+bx\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\arctan(a+bx)}{x}+\frac{b\text{Subst}\left(\int \frac{1}{-a+x}\,dx,x,a+bx\right)}{1+a^2}+\frac{b\text{Subst}\left(\int \frac{-a-x}{1+x^2}\,dx,x,a+bx\right)}{1+a^2} \\
&= -\frac{\arctan(a+bx)}{x}+\frac{b\log(x)}{1+a^2}-\frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx\right)}{1+a^2}-\frac{(ab)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,a+bx\right)}{1+a^2} \\
&= -\frac{ab\arctan(a+bx)}{1+a^2}-\frac{\arctan(a+bx)}{x}+\frac{b\log(x)}{1+a^2}-\frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{x^2}\,dx = &-\frac{\arctan(a+bx)}{x} \\
&+\frac{b(2\log(x)+i(i+a)\log(i-a-bx)+(-1-ia)\log(i+a+bx))}{2\left(1+a^2\right)}
\end{aligned}
$$

```
[In] Integrate[ArcTan[a + b*x]/x^2,x]
```

```
[Out] -(ArcTan[a + b*x]/x) + (b*(2*Log[x] + I*(I + a)*Log[I - a - b*x] + (-1 - I*
a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
derivativedivides	$b\left(-\frac{\arctan(bx+a)}{bx}-\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}+\frac{\ln(-bx)}{a^2+1}\right)$
default	$b\left(-\frac{\arctan(bx+a)}{bx}-\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}+\frac{\ln(-bx)}{a^2+1}\right)$
parts	$-\frac{\arctan(bx+a)}{x}+b\left(\frac{\ln(x)}{a^2+1}-\frac{b\left(\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{2b}+\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{a^2+1}\right)$
parallelrisch	$\frac{-2x\arctan(bx+a)a^2b^2+2b^2\ln(x)ax-b^2\ln\left(b^2x^2+2abx+a^2+1\right)ax-2\arctan(bx+a)a^3b-2\arctan(bx+a)ab}{2xab\left(a^2+1\right)}$
risch	$\frac{i\ln(1+i(bx+a))}{2x}-\frac{i(a^2\ln(1-i(bx+a))+\ln(1-i(bx+a))-i\ln((-a^2b+3iab)x-3a+2ia^2-a^3)bx+\ln((-a^2b+3iab)x-3}{2x}$

```
[In] int(arctan(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/b/x*arctan(b*x+a)-1/(a^2+1)*(1/2*ln(1+(b*x+a)^2)+a*arctan(b*x+a))+1/(
a^2+1)*ln(-b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = -\frac{bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log\left(x\right)+2\left(abx+a^2+1\right)\arctan\left(bx+a\right)}{2\left(a^2+1\right)x}$$

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*x*log(x) + 2*(a*b*x + a^2
+ 1)*arctan(b*x + a))/((a^2 + 1)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.71

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = \begin{cases} -\frac{ib\operatorname{atan}\left(bx-i\right)}{2}-\frac{\operatorname{atan}\left(bx-i\right)}{x}-\frac{i}{2x} & \text{for}\: a=-i \\ \frac{ib\operatorname{atan}\left(bx+i\right)}{2}-\frac{\operatorname{atan}\left(bx+i\right)}{x}+\frac{i}{2x} & \text{for}\: a=i \\ -\frac{2a^2\operatorname{atan}\left(a+bx\right)}{2a^2x+2x}-\frac{2abx\operatorname{atan}\left(a+bx\right)}{2a^2x+2x}+\frac{2bx\log\left(x\right)}{2a^2x+2x}-\frac{bx\log\left(a^2+2abx+b^2x^2+1\right)}{2a^2x+2x}-\frac{2\operatorname{atan}\left(a+bx\right)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

```
[In] integrate(atan(b*x+a)/x**2,x)
```

```
[Out] Piecewise((-I*b*atan(b*x - I)/2 - atan(b*x - I)/x - I/(2*x), Eq(a, -I)), (I
*b*atan(b*x + I)/2 - atan(b*x + I)/x + I/(2*x), Eq(a, I)), (-2*a**2*atan(a
+ b*x)/(2*a**2*x + 2*x) - 2*a*b*x*atan(a + b*x)/(2*a**2*x + 2*x) + 2*b*x*lo
g(x)/(2*a**2*x + 2*x) - b*x*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a**2*x +
 2*x) - 2*atan(a + b*x)/(2*a**2*x + 2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{\arctan(a+bx)}{x^2}\,dx$$
$$= -\frac{1}{2}\,b\left(\frac{2\,a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}+\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{2\,\log\left(x\right)}{a^2+1}\right)-\frac{\arctan\left(bx+a\right)}{x}$$

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arctan(b*x + a)/x
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = \int \frac{\arctan\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{\arctan(a+bx)}{x^2}\,dx$$
$$= -\frac{\mathrm{atan}(a+b\,x)}{x}-\frac{\frac{b\,x\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}-b\,x\,\ln\left(x\right)+a\,b\,x\,\mathrm{atan}(a+b\,x)}{x\,\left(a^2+1\right)}$$

```
[In] int(atan(a + b*x)/x^2,x)
```

```
[Out] - atan(a + b*x)/x - ((b*x*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2 - b*x*log(x)
+ a*b*x*atan(a + b*x))/(x*(a^2 + 1))
```

3.50 $\int \frac{\arctan(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 96

$$\int \frac{\arctan(a+bx)}{x^3}\,dx = -\frac{b}{2(1+a^2)x} - \frac{(1-a^2)b^2\arctan(a+bx)}{2(1+a^2)^2} - \frac{\arctan(a+bx)}{2x^2} - \frac{ab^2\log(x)}{(1+a^2)^2} + \frac{ab^2\log(1+(a+bx)^2)}{2(1+a^2)^2}$$

```
[Out] -1/2*b/(a^2+1)/x-1/2*(-a^2+1)*b^2*arctan(b*x+a)/(a^2+1)^2-1/2*arctan(b*x+a)
/x^2-a*b^2*ln(x)/(a^2+1)^2+1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5153, 378, 724, 815, 649, 209, 266}

$$\int \frac{\arctan(a+bx)}{x^3}\,dx = -\frac{(1-a^2)b^2\arctan(a+bx)}{2(a^2+1)^2} - \frac{ab^2\log(x)}{(a^2+1)^2} + \frac{ab^2\log((a+bx)^2+1)}{2(a^2+1)^2} - \frac{b}{2(a^2+1)x} - \frac{\arctan(a+bx)}{2x^2}$$

```
[In] Int[ArcTan[a + b*x]/x^3,x]
```

```
[Out] -1/2*b/((1 + a^2)*x) - ((1 - a^2)*b^2*ArcTan[a + b*x])/(2*(1 + a^2)^2) - Ar
cTan[a + b*x]/(2*x^2) - (a*b^2*Log[x])/(1 + a^2)^2 + (a*b^2*Log[1 + (a + b*
x)^2])/(2*(1 + a^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\arctan(a+bx)}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2\left(1+(a+bx)^2\right)}\,dx\\
&= -\frac{\arctan(a+bx)}{2x^2}+\frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(1+x^2\right)}\,dx,x,a+bx\right)\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\arctan(a+bx)}{2x^2}+\frac{b^2\text{Subst}\left(\int \frac{-a-x}{(-a+x)\left(1+x^2\right)}\,dx,x,a+bx\right)}{2\left(1+a^2\right)}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\arctan(a+bx)}{2x^2}+\frac{b^2\text{Subst}\left(\int \left(\frac{2a}{\left(1+a^2\right)(a-x)}+\frac{-1+a^2+2ax}{\left(1+a^2\right)\left(1+x^2\right)}\right)dx,x,a+bx\right)}{2\left(1+a^2\right)}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\arctan(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{b^2\text{Subst}\left(\int \frac{-1+a^2+2ax}{1+x^2}\,dx,x,a+bx\right)}{2\left(1+a^2\right)^2}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\arctan(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}\\
&\quad+\frac{\left(ab^2\right)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx\right)}{\left(1+a^2\right)^2}-\frac{\left(\left(1-a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,a+bx\right)}{2\left(1+a^2\right)^2}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\left(1-a^2\right)b^2\arctan(a+bx)}{2\left(1+a^2\right)^2}\\
&\quad-\frac{\arctan(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{ab^2\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$
\int \frac{\arctan(a+bx)}{x^3}\,dx = \frac{-2\arctan(a+bx)+\frac{bx\left(-4abx\log(x)-i(i+a)^2bx\log(i-a-bx)+(-i+a)(-2(i+a)+(1+ia)bx\log(i+a+bx))\right)}{\left(1+a^2\right)^2}}{4x^2}
$$

```
[In] Integrate[ArcTan[a + b*x]/x^3,x]
```

```
[Out] (-2*ArcTan[a + b*x] + (b*x*(-4*a*b*x*Log[x] - I*(I + a)^2*b*x*Log[I - a - b
*x] + (-I + a)*(-2*(I + a) + (1 + I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^2)
/(4*x^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result
derivativedivides	$b^{2} \left(-\frac{\arctan \left(b x +a \right)}{2 b^{2} x^{2}}-\frac{1}{2 \left(a^{2}+1\right) b x}-\frac{a \ln \left(-b x \right)}{\left(a^{2}+1\right)^{2}}+\frac{a \ln \left(1+\left(b x +a \right)^{2}\right)+\left(a^{2}-1\right) \arctan \left(b x +a \right)}{2 \left(a^{2}+1\right)^{2}}\right)$
default	$b^{2} \left(-\frac{\arctan \left(b x +a \right)}{2 b^{2} x^{2}}-\frac{1}{2 \left(a^{2}+1\right) b x}-\frac{a \ln \left(-b x \right)}{\left(a^{2}+1\right)^{2}}+\frac{a \ln \left(1+\left(b x +a \right)^{2}\right)+\left(a^{2}-1\right) \arctan \left(b x +a \right)}{2 \left(a^{2}+1\right)^{2}}\right)$
parts	$-\frac{\arctan \left(b x +a \right)}{2 x^{2}}+\frac{b \left(-\frac{1}{\left(a^{2}+1\right) x}-\frac{2 a b \ln \left(x \right)}{\left(a^{2}+1\right)^{2}}+\frac{b^{2} \left(\frac{a \ln \left(b^{2} x^{2}+2 a b x +a^{2}+1\right)}{b}+\frac{\left(a^{2}-1\right) \arctan \left(\frac{2 b^{2} x +2 a b}{2 b}\right)}{b}\right)}{\left(a^{2}+1\right)^{2}}\right)}{2}$
parallelrisch	$-\frac{-x^{2} \arctan \left(b x +a \right) a^{2} b^{2}+2 a \,b^{2} \ln \left(x \right) x^{2}-a \,b^{2} \ln \left(b^{2} x^{2}+2 a b x +a^{2}+1\right) x^{2}+\arctan \left(b x +a \right) b^{2} x^{2}-2 a \,b^{2} x^{2}+\arctan \left(b x +a \right) a^{4}}{2 x^{2} \left(a^{4}+2 a^{2}+1\right)}$
risch	$\frac{i \ln \left(1+i \left(b x +a \right)\right)}{4 x^{2}}-\frac{i \left(a^{4} \ln \left(1-i \left(b x +a \right)\right)+2 a^{2} \ln \left(1-i \left(b x +a \right)\right)+\ln \left(1-i \left(b x +a \right)\right)-\ln \left(\left(a^{6} b -4 i a^{5} b +9 a^{4} b +8 i a^{3} b -9 a^{2} b -4 i\right.\right.\right.}{}$

```
[In] int(arctan(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2/b^2/x^2*arctan(b*x+a)-1/2/(a^2+1)/b/x-1/(a^2+1)^2*a*ln(-b*x)+1/2/
(a^2+1)^2*(a*ln(1+(b*x+a)^2)+(a^2-1)*arctan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{\arctan(a+bx)}{x^3}\,dx$$
$$= \frac{ab^2x^2\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,ab^2x^2\log\left(x\right)-(a^2+1)bx+((a^2-1)b^2x^2-a^4-2\,a^2-1)\arctan\left(\right.}{2\left(a^4+2\,a^2+1\right)x^2}$$

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^2
 + 1)*b*x + ((a^2 - 1)*b^2*x^2 - a^4 - 2*a^2 - 1)*arctan(b*x + a))/((a^4 +
2*a^2 + 1)*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.98

$$\int \frac{\arctan(a+bx)}{x^3}\,dx$$
$$= \begin{cases} -\frac{b^2 \operatorname{atan}(bx-i)}{8} - \frac{b}{8x} - \frac{\operatorname{atan}(bx-i)}{2x^2} - \frac{i}{8x^2} \\ -\frac{b^2 \operatorname{atan}(bx+i)}{8} - \frac{b}{8x} - \frac{\operatorname{atan}(bx+i)}{2x^2} + \frac{i}{8x^2} \\ -\frac{a^4 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2b^2x^2 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} - \frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2} - \frac{2a^2 \operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} - \frac{2ab^2x^2 \log(x)}{2a^4x^2+4a^2x^2+2x^2} + \frac{ab^2x^2 \log(a^2+2abx}{2a^4x^2+4a^2x^2} \end{cases}$$

```
[In] integrate(atan(b*x+a)/x**3,x)
```

```
[Out] Piecewise((-b**2*atan(b*x - I)/8 - b/(8*x) - atan(b*x - I)/(2*x**2) - I/(8*
x**2), Eq(a, -I)), (-b**2*atan(b*x + I)/8 - b/(8*x) - atan(b*x + I)/(2*x**2
) + I/(8*x**2), Eq(a, I)), (-a**4*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2
+ 2*x**2) + a**2*b**2*x**2*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**
2) - a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*atan(a + b*x)/(
2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) + a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a
**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*atan(a + b*x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) - b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - atan(a +
b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{\arctan(a+bx)}{x^3}\,dx$$
$$= \frac{1}{2}\left(\frac{(a^2-1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2\,a^2+1} + \frac{ab\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^4+2\,a^2+1} - \frac{2\,ab\log(x)}{a^4+2\,a^2+1} - \frac{1}{(a^2+1)x}\right)b - \frac{\arctan(bx+a)}{2\,x^2}$$

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1) -
 1/((a^2 + 1)*x))*b - 1/2*arctan(b*x + a)/x^2
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{x^3}\,dx = \int \frac{\arctan\left(bx+a\right)}{x^3}\,dx$$

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.42

$$\int \frac{\arctan(a+bx)}{x^3}\,dx = \frac{a\,b^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,{\left(a^2+1\right)}^2} - \frac{\frac{b\,x}{2}+\mathrm{atan}(a+b\,x)\,\left(\frac{a^2}{2}+\frac{1}{2}\right)+\frac{b^2\,x^2\,\mathrm{atan}(a+b\,x)}{2}+\frac{x^3\,\left(b^3-3\,a^2\,b^3\right)}{2\,\left(a^4+2\,a^2+1\right)}-\frac{a\,b^4\,x^4}{{\left(a^2+1\right)}^2}+a\,b\,x\,\mathrm{atan}(a+b\,x)}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4+x^2} - \frac{\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(b^3-a^2\,b^3\right)}{\sqrt{b^2}\,\left(2\,a^4+4\,a^2+2\right)} - \frac{a\,b^2\,\ln\left(x\right)}{{\left(a^2+1\right)}^2}$$

```
[In] int(atan(a + b*x)/x^3,x)
```

```
[Out] (a*b^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*(a^2 + 1)^2) - ((b*x)/2 + atan(
a + b*x)*(a^2/2 + 1/2) + (b^2*x^2*atan(a + b*x))/2 + (x^3*(b^3 - 3*a^2*b^3)
)/(2*(2*a^2 + a^4 + 1)) - (a*b^4*x^4)/(a^2 + 1)^2 + a*b*x*atan(a + b*x))/(x
^2 + a^2*x^2 + b^2*x^4 + 2*a*b*x^3) - (atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2
+ 1) - a^2*b^2)^(1/2)))*(b^3 - a^2*b^3))/((b^2)^(1/2)*(4*a^2 + 2*a^4 + 2))
- (a*b^2*log(x))/(a^2 + 1)^2
```

3.51 $\int \frac{\arctan(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 129

$$\begin{aligned}\int \frac{\arctan(a+bx)}{x^4}\,dx = & -\frac{b}{6\left(1+a^2\right)x^2}+\frac{2ab^2}{3\left(1+a^2\right)^2x}+\frac{a(3-a^2)\,b^3\arctan(a+bx)}{3\left(1+a^2\right)^3}\\ & -\frac{\arctan(a+bx)}{3x^3}-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3}\\ & +\frac{\left(1-3a^2\right)b^3\log\left(1+(a+bx)^2\right)}{6\left(1+a^2\right)^3}\end{aligned}$$

```
[Out] -1/6*b/(a^2+1)/x^2+2/3*a*b^2/(a^2+1)^2/x+1/3*a*(-a^2+3)*b^3*arctan(b*x+a)/(
a^2+1)^3-1/3*arctan(b*x+a)/x^3-1/3*(-3*a^2+1)*b^3*ln(x)/(a^2+1)^3+1/6*(-3*a
^2+1)*b^3*ln(1+(b*x+a)^2)/(a^2+1)^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5153, 378, 724, 815, 649, 209, 266}

$$\begin{aligned}\int \frac{\arctan(a+bx)}{x^4}\,dx = & \frac{a(3-a^2)\,b^3\arctan(a+bx)}{3\left(a^2+1\right)^3}-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(a^2+1\right)^3}\\ & +\frac{\left(1-3a^2\right)b^3\log\left((a+bx)^2+1\right)}{6\left(a^2+1\right)^3}\\ & +\frac{2ab^2}{3\left(a^2+1\right)^2x}-\frac{b}{6\left(a^2+1\right)x^2}-\frac{\arctan(a+bx)}{3x^3}\end{aligned}$$

```
[In] Int[ArcTan[a + b*x]/x^4,x]
```

```
[Out] -1/6*b/((1 + a^2)*x^2) + (2*a*b^2)/(3*(1 + a^2)^2*x) + (a*(3 - a^2)*b^3*Arc
Tan[a + b*x])/(3*(1 + a^2)^3) - ArcTan[a + b*x]/(3*x^3) - ((1 - 3*a^2)*b^3*
Log[x])/(3*(1 + a^2)^3) + ((1 - 3*a^2)*b^3*Log[1 + (a + b*x)^2])/(6*(1 + a^
2)^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
```

```
+ d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan(a+bx)}{3x^3}+\frac{1}{3}b\int\frac{1}{x^3\left(1+(a+bx)^2\right)}\,dx\\
&= -\frac{\arctan(a+bx)}{3x^3}+\frac{1}{3}b^3\text{Subst}\left(\int\frac{1}{(-a+x)^3\left(1+x^2\right)}\,dx,x,a+bx\right)\\
&= -\frac{b}{6\left(1+a^2\right)x^2}-\frac{\arctan(a+bx)}{3x^3}+\frac{b^3\text{Subst}\left(\int\frac{-a-x}{(-a+x)^2\left(1+x^2\right)}\,dx,x,a+bx\right)}{3\left(1+a^2\right)}\\
&= -\frac{b}{6\left(1+a^2\right)x^2}-\frac{\arctan(a+bx)}{3x^3}\\
&\quad+\frac{b^3\text{Subst}\left(\int\left(-\frac{2a}{\left(1+a^2\right)(a-x)^2}+\frac{1-3a^2}{\left(1+a^2\right)^2(a-x)}+\frac{a\left(3-a^2\right)+\left(1-3a^2\right)x}{\left(1+a^2\right)^2\left(1+x^2\right)}\right)dx,x,a+bx\right)}{3\left(1+a^2\right)}\\
&= -\frac{b}{6\left(1+a^2\right)x^2}+\frac{2ab^2}{3\left(1+a^2\right)^2x}-\frac{\arctan(a+bx)}{3x^3}\\
&\quad-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3}+\frac{b^3\text{Subst}\left(\int\frac{a\left(3-a^2\right)+\left(1-3a^2\right)x}{1+x^2}\,dx,x,a+bx\right)}{3\left(1+a^2\right)^3}\\
&= -\frac{b}{6\left(1+a^2\right)x^2}+\frac{2ab^2}{3\left(1+a^2\right)^2x}-\frac{\arctan(a+bx)}{3x^3}\\
&\quad-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3}+\frac{\left(\left(1-3a^2\right)b^3\right)\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,a+bx\right)}{3\left(1+a^2\right)^3}\\
&\quad+\frac{\left(a\left(3-a^2\right)b^3\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,a+bx\right)}{3\left(1+a^2\right)^3}\\
&= -\frac{b}{6\left(1+a^2\right)x^2}+\frac{2ab^2}{3\left(1+a^2\right)^2x}+\frac{a\left(3-a^2\right)b^3\arctan(a+bx)}{3\left(1+a^2\right)^3}\\
&\quad-\frac{\arctan(a+bx)}{3x^3}-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3}+\frac{\left(1-3a^2\right)b^3\log\left(1+(a+bx)^2\right)}{6\left(1+a^2\right)^3}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \frac{\arctan(a+bx)}{x^4}\,dx$$

$$= \frac{-2(1+a^2)^3\arctan(a+bx)+2(-1+3a^2)\,b^3x^3\log(x)+i(i+a)^3b^3x^3\log(i-a-bx)-(-i+a)bx((i+}{6\,(1+a^2)^3\,x^3}$$

```
[In] Integrate[ArcTan[a + b*x]/x^4,x]
```

```
[Out] (-2*(1 + a^2)^3*ArcTan[a + b*x] + 2*(-1 + 3*a^2)*b^3*x^3*Log[x] + I*(I + a)
^3*b^3*x^3*Log[I - a - b*x] - (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) + I
*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.89

method	result
derivativedivides	$b^3 \Big(-\frac{\arctan(bx+a)}{3b^3x^3} - \frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3} - \frac{1}{6(a^2+1)b^2x^2} + \frac{2a}{3(a^2+1)^2bx} - \frac{\frac{(3a^2-1)\ln(1+(bx+a)^2)}{2}+(a^3-3a)}{3(a^2+1)^3}$
default	$b^3 \Big(-\frac{\arctan(bx+a)}{3b^3x^3} - \frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3} - \frac{1}{6(a^2+1)b^2x^2} + \frac{2a}{3(a^2+1)^2bx} - \frac{\frac{(3a^2-1)\ln(1+(bx+a)^2)}{2}+(a^3-3a)}{3(a^2+1)^3}$
parts	$-\frac{\arctan(bx+a)}{3x^3} + \frac{b\Big(-\frac{1}{2(a^2+1)x^2}+\frac{b^2(3a^2-1)\ln(x)}{(a^2+1)^3}+\frac{2ab}{(a^2+1)^2x}-\frac{b^3\Big(\frac{(3a^2b-b)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{(4a^3-4a-(3a}{}}{(a^2+1)^3}}{3}$
parallelrisch	$\frac{-2x^3\arctan(bx+a)a^3b^3+6\ln(x)x^3a^2b^3-3\ln(b^2x^2+2abx+a^2+1)x^3a^2b^3+6x^3\arctan(bx+a)a\,b^3-7x^3a^2b^3-2b^3\ln(x)x^3}{}$
risch	Expression too large to display

```
[In] int(arctan(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3/b^3/x^3*arctan(b*x+a)-1/3*(-3*a^2+1)/(a^2+1)^3*ln(-b*x)-1/6/(a^2+
1)/b^2/x^2+2/3/(a^2+1)^2*a/b/x-1/3/(a^2+1)^3*(1/2*(3*a^2-1)*ln(1+(b*x+a)^2)
+(a^3-3*a)*arctan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(a+bx)}{x^4}\,dx = -\frac{(3\,a^2-1)b^3x^3\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(3\,a^2-1)b^3x^3\log\left(x\right)-4\,(a^3+a)b^2x^2+(a^4+2\,a^2+1)bx}{6\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((3*a^2 - 1)*b^3*x^3*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)*
b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x^2 + (a^4 + 2*a^2 + 1)*b*x + 2*((a^3 - 3*
a)*b^3*x^3 + a^6 + 3*a^4 + 3*a^2 + 1)*arctan(b*x + a))/((a^6 + 3*a^4 + 3*a^
2 + 1)*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.89

$$\int \frac{\arctan(a+bx)}{x^4}\,dx$$

$$= \begin{cases} \frac{ib^3\operatorname{atan}\left(bx-i\right)}{24} + \frac{ib^2}{24x} - \frac{b}{24x^2} - \frac{\operatorname{atan}\left(bx-i\right)}{3x^3} - \frac{i}{18x^3} \\ -\frac{ib^3\operatorname{atan}\left(bx+i\right)}{24} - \frac{ib^2}{24x} - \frac{b}{24x^2} - \frac{\operatorname{atan}\left(bx+i\right)}{3x^3} + \frac{i}{18x^3} \\ -\frac{2a^6\operatorname{atan}\left(a+bx\right)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} - \frac{a^4bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} - \frac{6a^4\operatorname{atan}\left(a+bx\right)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} - \frac{2a^3b^3x^3\operatorname{atan}\left(a+bx\right)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} + \frac{}{6a^6} \end{cases}$$

```
[In] integrate(atan(b*x+a)/x**4,x)
```

```
[Out] Piecewise((I*b**3*atan(b*x - I)/24 + I*b**2/(24*x) - b/(24*x**2) - atan(b*x
 - I)/(3*x**3) - I/(18*x**3), Eq(a, -I)), (-I*b**3*atan(b*x + I)/24 - I*b**
2/(24*x) - b/(24*x**2) - atan(b*x + I)/(3*x**3) + I/(18*x**3), Eq(a, I)), (
-2*a**6*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
- a**4*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**4*at
an(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**3*b
**3*x**3*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
 + 4*a**3*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) +
6*a**2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3
) - 3*a**2*b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**2*b*x/(6*a**6*x**3 + 18*a**4*x**3
 + 18*a**2*x**3 + 6*x**3) - 6*a**2*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**
```

```
3 + 18*a**2*x**3 + 6*x**3) + 6*a*b**3*x**3*atan(a + b*x)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 4*a*b**2*x**2/(6*a**6*x**3 + 18*a**4*x
**3 + 18*a**2*x**3 + 6*x**3) - 2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x*
*3 + 18*a**2*x**3 + 6*x**3) + b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)
/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - b*x/(6*a**6*x**3 +
18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*atan(a + b*x)/(6*a**6*x**3 + 18*a
**4*x**3 + 18*a**2*x**3 + 6*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.28

$$\int \frac{\arctan(a+bx)}{x^4}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\,(a^3-3\,a)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1}+\frac{(3\,a^2-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{2\,(3\,a^2-1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1}-\right.$$

$$-\frac{\arctan\left(bx+a\right)}{3\,x^3}$$

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1) +
 (3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1)
 - 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 - 1)
/((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arctan(b*x + a)/x^3
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{x^4}\,dx = \int \frac{\arctan\left(bx+a\right)}{x^4}\,dx$$

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.23

$$\int \frac{\arctan(a+bx)}{x^4}\,dx =$$

$$-\frac{\frac{b\,x}{6}+\operatorname{atan}(a+b\,x)\,\left(\frac{a^2}{3}+\frac{1}{3}\right)+\frac{b^2\,x^2\,\operatorname{atan}(a+b\,x)}{3}+\frac{x^3\,\left(b^3-7\,a^2\,b^3\right)}{6\,\left(a^4+2\,a^2+1\right)}-\frac{a\,b^2\,x^2}{3\,\left(a^2+1\right)}-\frac{2\,a\,b^4\,x^4}{3\,{\left(a^2+1\right)}^2}+\frac{2\,a\,b\,x\,\operatorname{atan}(a+b\,x)}{3}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5+x^3}$$

$$-\frac{\ln\left(x\right)\,\left(\frac{b^3}{3}-a^2\,b^3\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{b^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)\,\left(3\,a^2-1\right)}{6\,\left(a^6+3\,a^4+3\,a^2+1\right)}$$

$$-\frac{a\,\operatorname{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(a^2-3\right)\,{\left(b^2\right)}^{3/2}}{3\,\left(a^6+3\,a^4+3\,a^2+1\right)}$$

```
[In] int(atan(a + b*x)/x^4,x)
```

```
[Out] - ((b*x)/6 + atan(a + b*x)*(a^2/3 + 1/3) + (b^2*x^2*atan(a + b*x))/3 + (x^3
*(b^3 - 7*a^2*b^3))/(6*(2*a^2 + a^4 + 1)) - (a*b^2*x^2)/(3*(a^2 + 1)) - (2*
a*b^4*x^4)/(3*(a^2 + 1)^2) + (2*a*b*x*atan(a + b*x))/3)/(x^3 + a^2*x^3 + b^
2*x^5 + 2*a*b*x^4) - (log(x)*(b^3/3 - a^2*b^3))/(3*a^2 + 3*a^4 + a^6 + 1) -
 (b^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1)*(3*a^2 - 1))/(6*(3*a^2 + 3*a^4 + a^6
 + 1)) - (a*atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(a^
2 - 3)*(b^2)^(3/2))/(3*(3*a^2 + 3*a^4 + a^6 + 1))
```

3.52 $\int \frac{\arctan(a+bx)}{c+dx^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 863

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = & -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{\sqrt[6]{-1}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{\sqrt[6]{-1}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[3]{-1}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{(-1)^{5/6}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(-1)^{2/3}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{(-1)^{5/6}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[6]{-1}(1-ia)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{i\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{d}(i-a-bx)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{(-1)^{5/6}\operatorname{PolyLog}\left(2,-\frac{\sqrt[6]{-1}\sqrt[3]{d}(i-a-bx)}{ib\sqrt[3]{c}-\sqrt[6]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{\sqrt[6]{-1}\operatorname{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\sqrt[3]{d}(i-a-bx)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{i\operatorname{PolyLog}\left(2,-\frac{\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{\sqrt[6]{-1}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}+\sqrt[3]{-1}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{(-1)^{5/6}\operatorname{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}-(-1)^{2/3}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}
\end{aligned}
$$

```
[Out] -1/6*I*ln(1+I*a+I*b*x)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)+(I-a)*d^(1/3)))/
c^(2/3)/d^(1/3)+1/6*I*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)-(
I+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*ln(1+I*a+I*b*x)*ln(b*(c^(1/3)
-(-1)^(1/3)*d^(1/3)*x)/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3)))/c^(2/3)/d^(1/3
)-1/6*(-1)^(1/6)*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)-(-1)^(1/3)*d^(1/3)*x)/(b*c^(
1/3)+(-1)^(1/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(5/6)*ln(1+I*a+I*b
*x)*ln(b*(c^(1/3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(2/3)*(I-a)*d^(1/3)
))/c^(2/3)/d^(1/3)-1/6*(-1)^(5/6)*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)+(-1)^(2/3)*
d^(1/3)*x)/(b*c^(1/3)+(-1)^(1/6)*(1-I*a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*I*po
lylog(2,d^(1/3)*(I-a-b*x)/(b*c^(1/3)+(I-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-
1)^(5/6)*polylog(2,-(-1)^(1/6)*d^(1/3)*(I-a-b*x)/(I*b*c^(1/3)-(-1)^(1/6)*(I
-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*polylog(2,-(-1)^(1/3)*d^(1/3)*
(I-a-b*x)/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*I*polyl
og(2,-d^(1/3)*(I+a+b*x)/(b*c^(1/3)-(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)
^(1/6)*polylog(2,(-1)^(1/3)*d^(1/3)*(I+a+b*x)/(b*c^(1/3)+(-1)^(1/3)*(I+a)*d
^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)^(5/6)*polylog(2,-(-1)^(2/3)*d^(1/3)*(I+a+
b*x)/(b*c^(1/3)-(-1)^(2/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used

$= \{5159,\ 2456,\ 2441,\ 2440,\ 2438\}$

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = {} & -\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& + \frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-(a+i)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& + \frac{\sqrt[6]{-1}\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& - \frac{\sqrt[6]{-1}\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{\sqrt[3]{-1}\sqrt[3]{d}(a+i)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& + \frac{(-1)^{5/6}\log(ia+ibx+1)\log\left(\frac{b\left((-1)^{2/3}\sqrt[3]{d}x+\sqrt[3]{c}\right)}{(-1)^{2/3}\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& - \frac{(-1)^{5/6}\log(-ia-ibx+1)\log\left(\frac{b\left((-1)^{2/3}\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[6]{-1}\sqrt[3]{d}(1-ia)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& - \frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}(-a-bx+i)}{\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& + \frac{(-1)^{5/6}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[6]{-1}\sqrt[3]{d}(-a-bx+i)}{ib\sqrt[3]{c}-\sqrt[6]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& + \frac{\sqrt[6]{-1}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\sqrt[3]{d}(-a-bx+i)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{d}(a+bx+i)}{b\sqrt[3]{c}-(a+i)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& - \frac{\sqrt[6]{-1}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{d}(a+bx+i)}{\sqrt[3]{-1}\sqrt[3]{d}(a+i)+b\sqrt[3]{c}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& - \frac{(-1)^{5/6}\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{d}(a+bx+i)}{b\sqrt[3]{c}-(-1)^{2/3}(a+i)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}
\end{aligned}
$$

```
[In] Int[ArcTan[a + b*x]/(c + d*x^3),x]
```

```
[Out] ((-1/6*I)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) + (
I - a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((I/6)*Log[1 - I*a - I*b*x]*Log[(b*(c
^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (I + a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((
-1)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*
c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(1/6)*L
og[1 - I*a - I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (
-1)^(1/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(5/6)*Log[1 + I*a
+ I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(
I - a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(5/6)*Log[1 - I*a - I*b*x]*Lo
g[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*a)*d^
(1/3))])/(6*c^(2/3)*d^(1/3)) - ((I/6)*PolyLog[2, (d^(1/3)*(I - a - b*x))/(b
*c^(1/3) + (I - a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((-1)^(5/6)*PolyLog[2, -(
((-1)^(1/6)*d^(1/3)*(I - a - b*x))/(I*b*c^(1/3) - (-1)^(1/6)*(I - a)*d^(1/3
)))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(1/6)*PolyLog[2, -(((-1)^(1/3)*d^(1/3)*(I
 - a - b*x))/(b*c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3)
) + ((I/6)*PolyLog[2, -((d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (I + a)*d^(1/3
)))])/(c^(2/3)*d^(1/3)) - ((-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(I + a
 + b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - (
(-1)^(5/6)*PolyLog[2, -(((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (-1
)^(2/3)*(I + a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
```

```
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\text{integral} = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+dx^3}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+dx^3}\,dx$$

$$= \frac{1}{2}i\int \left(-\frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}-\sqrt[3]{d}x\right)} - \frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x\right)} - \frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x\right)}\right)dx - \frac{1}{2}i\int \left(-\frac{\log(1+ia+ibx)}{3c^{2/3}\left(-\sqrt[3]{c}-\sqrt[3]{d}x\right)} - \frac{\log(1+ia+ibx)}{3c^{2/3}\left(-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x\right)} -$$

$$= -\frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-\sqrt[3]{d}x}\,dx}{6c^{2/3}} - \frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x}\,dx}{6c^{2/3}} - \frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x}\,dx}{6c^{2/3}} + \frac{i\int \frac{\log(1+ia+ibx)}{-\sqrt[3]{c}-\sqrt[3]{d}x}\,dx}{6c^{2/3}} + \frac{i\int \frac{\log(1+ia+ibx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x}\,dx}{6c^{2/3}} + \frac{i\int \frac{\log(1+ia+ibx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x}\,dx}{6c^{2/3}}$$

$$
\begin{aligned}
= & -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{\sqrt[6]{-1}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{\sqrt[6]{-1}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[3]{-1}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{(-1)^{5/6}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(-1)^{2/3}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{(-1)^{5/6}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[6]{-1}(1-ia)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{b\int\frac{\log\left(-\frac{ib\left(-\sqrt[3]{c}-\sqrt[3]{d}x\right)}{ib\sqrt[3]{c}+(1-ia)\sqrt[3]{d}}\right)}{1-ia-ibx}dx}{6c^{2/3}\sqrt[3]{d}}-\frac{b\int\frac{\log\left(\frac{ib\left(-\sqrt[3]{c}-\sqrt[3]{d}x\right)}{-ib\sqrt[3]{c}+(1+ia)\sqrt[3]{d}}\right)}{1+ia+ibx}dx}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{\left(\sqrt[3]{-1}b\right)\int\frac{\log\left(-\frac{ib\left(-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x\right)}{ib\sqrt[3]{c}+(-1)^{2/3}(1-ia)\sqrt[3]{d}}\right)}{1-ia-ibx}dx}{6c^{2/3}\sqrt[3]{d}} \\
& +\frac{\left(\sqrt[3]{-1}b\right)\int\frac{\log\left(\frac{ib\left(-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x\right)}{-ib\sqrt[3]{c}+(-1)^{2/3}(1+ia)\sqrt[3]{d}}\right)}{1+ia+ibx}dx}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{\left((-1)^{2/3}b\right)\int\frac{\log\left(-\frac{ib\left(-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x\right)}{ib\sqrt[3]{c}-\sqrt[3]{-1}(1-ia)\sqrt[3]{d}}\right)}{1-ia-ibx}dx}{6c^{2/3}\sqrt[3]{d}} \\
& -\frac{\left((-1)^{2/3}b\right)\int\frac{\log\left(\frac{ib\left(-\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{d}x\right)}{-ib\sqrt[3]{c}-\sqrt[3]{-1}(1+ia)\sqrt[3]{d}}\right)}{1+ia+ibx}dx}{6c^{2/3}\sqrt[3]{d}}
\end{aligned}
$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$+\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$+\frac{\sqrt[6]{-1}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$-\frac{\sqrt[6]{-1}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[3]{-1}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$+\frac{(-1)^{5/6}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(-1)^{2/3}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$-\frac{(-1)^{5/6}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[6]{-1}(1-ia)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$-\frac{i\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[3]{d}x}{ib\sqrt[3]{c}+(1-ia)\sqrt[3]{d}}\right)}{x}dx,x,1-ia-ibx\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$+\frac{i\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[3]{d}x}{-ib\sqrt[3]{c}+(1+ia)\sqrt[3]{d}}\right)}{x}dx,x,1+ia+ibx\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$+\frac{\sqrt[6]{-1}\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{d}x}{ib\sqrt[3]{c}-\sqrt[3]{-1}(1-ia)\sqrt[3]{d}}\right)}{x}dx,x,1-ia-ibx\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$-\frac{\sqrt[6]{-1}\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{d}x}{-ib\sqrt[3]{c}-\sqrt[3]{-1}(1+ia)\sqrt[3]{d}}\right)}{x}dx,x,1+ia+ibx\right)}{6c^{2/3}\sqrt[3]{d}}$$

$$(-1)^{5/6}\text{Subst}\left(\int\frac{\log\left(1-\frac{(-1)^{2/3}\sqrt[3]{d}x}{ib\sqrt[3]{c}+(-1)^{2/3}(1-ia)\sqrt[3]{d}}\right)}{x}dx,x,1-ia-ibx\right.$$

$$
\begin{aligned}
&= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad + \frac{\sqrt[6]{-1}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad - \frac{\sqrt[6]{-1}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[3]{-1}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad + \frac{(-1)^{5/6}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(-1)^{2/3}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad - \frac{(-1)^{5/6}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[6]{-1}(1-ia)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad - \frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}(i-a-bx)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{5/6}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[6]{-1}\sqrt[3]{d}(i-a-bx)}{ib\sqrt[3]{c}-\sqrt[6]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad + \frac{\sqrt[6]{-1}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\sqrt[3]{d}(i-a-bx)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[6]{-1}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}+\sqrt[3]{-1}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} \\
&\quad - \frac{(-1)^{5/6}\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}-(-1)^{2/3}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 701, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx$$

$$= \frac{-i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(-i+a)\sqrt[3]{d}}\right)+i\log(-i(i+a+bx))\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)+\sqrt[6]{-1}\log(1+ia}{}$$

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x^3),x]
```

```
[Out] ((-I)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (-I +
 a)*d^(1/3))] + I*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*
c^(1/3) - (I + a)*d^(1/3))] + (-1)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/
3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] - (-
1)^(1/6)*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(
b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)*Log[(-I)*(I + a + b*x
)]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*
a)*d^(1/3))] + (-1)^(5/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)
*d^(1/3)*x))/(b*c^(1/3) - (-1)^(2/3)*(-I + a)*d^(1/3))] - I*PolyLog[2, (d^(
1/3)*(-I + a + b*x))/(-(b*c^(1/3)) + (-I + a)*d^(1/3))] + (-1)^(5/6)*PolyLo
g[2, ((-1)^(1/6)*d^(1/3)*(-I + a + b*x))/(I*b*c^(1/3) + (-1)^(1/6)*(-I + a)
*d^(1/3))] + (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(-I + a + b*x))/(b*c
^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] + I*PolyLog[2, (d^(1/3)*(I + a + b*x
))/(-(b*c^(1/3)) + (I + a)*d^(1/3))] - (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^
(1/3)*(I + a + b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)
*PolyLog[2, ((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(
I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{i b^{2} \left(\sum_{\_R1=\mathrm{RootOf}\left(d\_Z^{3}+\left(3\,\mathrm{RootOf}\left(\_Z^{2}+1,\mathrm{index}=1\right)ad-3d\right)\_Z^{2}+\left(-6\,\mathrm{RootOf}\left(\_Z^{2}+1,\mathrm{index}=1\right)ad-3a^{2}d+3d\right)\_Z-\mathrm{Root}}\right.}{}$
derivativedivides	$\frac{-\frac{b^{3}\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^{3}-3ad\_Z^{2}+3a^{2}d\_Z-d\,a^{3}+b^{3}c\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^{2}+2\_Ra-a^{2}}\right)\arctan\left(bx+a\right)}{3d}+\frac{b^{3}\left(\arctan\left(bx+a\right)\left(\sum_{\_R=\mathrm{Root}}\right.\right.}{}}{}$
default	$\frac{-\frac{b^{3}\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^{3}-3ad\_Z^{2}+3a^{2}d\_Z-d\,a^{3}+b^{3}c\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^{2}+2\_Ra-a^{2}}\right)\arctan\left(bx+a\right)}{3d}+\frac{b^{3}\left(\arctan\left(bx+a\right)\left(\sum_{\_R=\mathrm{Root}}\right.\right.}{}}{}$

```
[In] int(arctan(b*x+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*b^2/d*sum(1/(1+2*I*a*_R1-2*I*a+_R1^2-a^2-2*_R1)*(ln(1-I*a-I*b*x)*ln(
(_R1+I*b*x+I*a-1)/_R1)+dilog((_R1+I*b*x+I*a-1)/_R1)),_R1=RootOf(d*_Z^3+(3*R
ootOf(_Z^2+1,index=1)*a*d-3*d)*_Z^2+(-6*RootOf(_Z^2+1,index=1)*a*d-3*a^2*d+
3*d)*_Z-RootOf(_Z^2+1,index=1)*a^3*d+RootOf(_Z^2+1,index=1)*b^3*c+3*RootOf(
_Z^2+1,index=1)*a*d+3*a^2*d-d))+1/6*I*b^2/d*sum(1/(1-2*I*a*_R1+2*I*a+_R1^2-
a^2-2*_R1)*(ln(1+I*a+I*b*x)*ln((_R1-I*b*x-I*a-1)/_R1)+dilog((_R1-I*b*x-I*a-
1)/_R1)),_R1=RootOf(d*_Z^3+(-3*RootOf(_Z^2+1,index=1)*a*d-3*d)*_Z^2+(6*Root
Of(_Z^2+1,index=1)*a*d-3*a^2*d+3*d)*_Z+RootOf(_Z^2+1,index=1)*a^3*d-RootOf(
_Z^2+1,index=1)*b^3*c-3*RootOf(_Z^2+1,index=1)*a*d+3*a^2*d-d))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \int \frac{\arctan\left(bx+a\right)}{dx^3+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="fricas")
[Out] integral(arctan(b*x + a)/(d*x^3 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate(atan(b*x+a)/(d*x**3+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \int \frac{\arctan\left(bx+a\right)}{dx^3+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(arctan(b*x + a)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \int \frac{\arctan\left(bx+a\right)}{dx^3+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{d\,x^3+c}\,dx$$

```
[In] int(atan(a + b*x)/(c + d*x^3),x)
[Out] int(atan(a + b*x)/(c + d*x^3), x)
```

3.53 $\int \frac{\arctan(a+bx)}{c+dx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 543

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = & -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\
& +\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\
& +\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\
& -\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\
& -\frac{i\,\text{PolyLog}\left(2,-\frac{\sqrt{d}(i-a-bx)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\,\text{PolyLog}\left(2,\frac{\sqrt{d}(i-a-bx)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} \\
& -\frac{i\,\text{PolyLog}\left(2,-\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\,\text{PolyLog}\left(2,\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}
\end{aligned}
$$

```
[Out] -1/4*I*ln(1+I*a+I*b*x)*ln(b*((-c)^(1/2)-x*d^(1/2))/(b*(-c)^(1/2)-(I-a)*d^(1
/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*ln(1-I*a-I*b*x)*ln(b*((-c)^(1/2)-x*d^(1/2))/
(b*(-c)^(1/2)+(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*ln(1+I*a+I*b*x)*ln(b
*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)+(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/
4*I*ln(1-I*a-I*b*x)*ln(b*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)-(I+a)*d^(1/2)
))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,-(I-a-b*x)*d^(1/2)/(b*(-c)^(1/2)-(I-a
)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*polylog(2,(I-a-b*x)*d^(1/2)/(b*(-c)^(1
```

```
/2)+(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,-(I+a+b*x)*d^(1/2)/(
b*(-c)^(1/2)-(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*polylog(2,(I+a+b*x)*d
^(1/2)/(b*(-c)^(1/2)+(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5159, 2456, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = &-\frac{i\,\text{PolyLog}\left(2,-\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\,\text{PolyLog}\left(2,\frac{\sqrt{d}(-a-bx+i)}{\sqrt{d}(i-a)+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{i\,\text{PolyLog}\left(2,-\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\,\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx+i)}{\sqrt{d}(a+i)+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &+\frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &+\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}+(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\end{aligned}$$

```
[In] Int[ArcTan[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/4*I)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] -
 (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 - I*a - I*b*x]*Log[(b
*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]
) + ((I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c]
+ (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*Log[1 - I*a - I*b*x]*Log[(
b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d
]) - ((I/4)*PolyLog[2, -((Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] - (I - a)*Sqrt
[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I - a - b*x))/(b*S
qrt[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt
[d]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + (
(I/4)*PolyLog[2, (Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(
Sqrt[-c]*Sqrt[d])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int\frac{\log(1-ia-ibx)}{c+dx^2}\,dx-\frac{1}{2}i\int\frac{\log(1+ia+ibx)}{c+dx^2}\,dx\\
&= \frac{1}{2}i\int\left(\frac{\sqrt{-c}\log(1-ia-ibx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(1-ia-ibx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx\\
&\quad-\frac{1}{2}i\int\left(\frac{\sqrt{-c}\log(1+ia+ibx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(1+ia+ibx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx\\
&= -\frac{i\int\frac{\log(1-ia-ibx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{4\sqrt{-c}}-\frac{i\int\frac{\log(1-ia-ibx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{4\sqrt{-c}}+\frac{i\int\frac{\log(1+ia+ibx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{4\sqrt{-c}}+\frac{i\int\frac{\log(1+ia+ibx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{4\sqrt{-c}}
\end{aligned}
$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$
$$+\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$
$$-\frac{b\int\frac{\log\left(-\frac{ib\left(\sqrt{-c}-\sqrt{d}x\right)}{-ib\sqrt{-c}+(1-ia)\sqrt{d}}\right)}{1-ia-ibx}dx}{4\sqrt{-c}\sqrt{d}}-\frac{b\int\frac{\log\left(\frac{ib\left(\sqrt{-c}-\sqrt{d}x\right)}{ib\sqrt{-c}+(1+ia)\sqrt{d}}\right)}{1+ia+ibx}dx}{4\sqrt{-c}\sqrt{d}}$$
$$+\frac{b\int\frac{\log\left(-\frac{ib\left(\sqrt{-c}+\sqrt{d}x\right)}{-ib\sqrt{-c}-(1-ia)\sqrt{d}}\right)}{1-ia-ibx}dx}{4\sqrt{-c}\sqrt{d}}+\frac{b\int\frac{\log\left(\frac{ib\left(\sqrt{-c}+\sqrt{d}x\right)}{ib\sqrt{-c}-(1+ia)\sqrt{d}}\right)}{1+ia+ibx}dx}{4\sqrt{-c}\sqrt{d}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$
$$+\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$
$$+\frac{i\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{d}x}{-ib\sqrt{-c}-(1-ia)\sqrt{d}}\right)}{x}dx,x,1-ia-ibx\right)}{4\sqrt{-c}\sqrt{d}}$$
$$-\frac{i\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{d}x}{-ib\sqrt{-c}+(1-ia)\sqrt{d}}\right)}{x}dx,x,1-ia-ibx\right)}{4\sqrt{-c}\sqrt{d}}$$
$$-\frac{i\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{d}x}{ib\sqrt{-c}-(1+ia)\sqrt{d}}\right)}{x}dx,x,1+ia+ibx\right)}{4\sqrt{-c}\sqrt{d}}$$
$$+\frac{i\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{d}x}{ib\sqrt{-c}+(1+ia)\sqrt{d}}\right)}{x}dx,x,1+ia+ibx\right)}{4\sqrt{-c}\sqrt{d}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{d}(i-a-bx)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt{d}(i-a-bx)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.75

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx =$$

$$-\frac{i\left(\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)-\log(-i(i+a+bx))\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)-\log(1+ia+ibx)\right.}{}$$

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/4*I)*(Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c]
+ (-I + a)*Sqrt[d])] - Log[(-I)*(I + a + b*x)]*Log[(b*(Sqrt[-c] - Sqrt[d]*x
))/(b*Sqrt[-c] + (I + a)*Sqrt[d])] - Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c]
+ Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d])] + Log[(-I)*(I + a + b*x)]*Lo
g[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])] - PolyLog[2, (
Sqrt[d]*(-I + a + b*x))/(-(b*Sqrt[-c]) + (-I + a)*Sqrt[d])] + PolyLog[2, (S
qrt[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])] + PolyLog[2, (Sqrt[
d]*(I + a + b*x))/(-(b*Sqrt[-c]) + (I + a)*Sqrt[d])] - PolyLog[2, (Sqrt[d]*
(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]))/(Sqrt[-c]*Sqrt[d])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\ln(-bxi-ia+1)\ln\left(\frac{iad-b\sqrt{cd}+(-bxi-ia+1)d-d}{iad-b\sqrt{cd}-d}\right)\sqrt{cd}}{4cd}-\frac{\ln(-bxi-ia+1)\ln\left(\frac{iad+b\sqrt{cd}+(-bxi-ia+1)d-d}{iad+b\sqrt{cd}-d}\right)\sqrt{cd}}{4cd}+\mathrm{dilog}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arctan(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(1-I*a-I*b*x)/c/d*ln((I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d-b
*(c*d)^(1/2)-d))*(c*d)^(1/2)-1/4*ln(1-I*a-I*b*x)/c/d*ln((I*a*d+b*(c*d)^(1/2
)+(1-I*a-I*b*x)*d-d)/(I*a*d+b*(c*d)^(1/2)-d))*(c*d)^(1/2)+1/4/c/d*dilog((I*
a*d-b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d-b*(c*d)^(1/2)-d))*(c*d)^(1/2)-1
/4/c/d*dilog((I*a*d+b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d+b*(c*d)^(1/2)-d
))*(c*d)^(1/2)+1/4*ln(1+I*a+I*b*x)/c/d*ln((I*a*d+b*(c*d)^(1/2)-(1+I*a+I*b*x
)*d+d)/(I*a*d+b*(c*d)^(1/2)+d))*(c*d)^(1/2)-1/4*ln(1+I*a+I*b*x)/c/d*ln((I*a
*d-b*(c*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^(1/2)+d))*(c*d)^(1/2)+1/
4/c/d*(c*d)^(1/2)*dilog((I*a*d+b*(c*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d+b*(c
*d)^(1/2)+d))-1/4/c/d*(c*d)^(1/2)*dilog((I*a*d-b*(c*d)^(1/2)-(1+I*a+I*b*x)*
d+d)/(I*a*d-b*(c*d)^(1/2)+d))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx=\int \frac{\arctan\left(bx+a\right)}{dx^2+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx=\text{Timed out}$$

```
[In] integrate(atan(b*x+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8520 vs. $2(369) = 738$.

Time = 3.85 (sec) , antiderivative size = 8520, normalized size of antiderivative = 15.69

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*b*(8*arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/b - (4*arctan(sqrt(d
)*x/sqrt(c))*arctan2((2*a*b^2*c*d + (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^
2 + 3)*b^2*d)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^
2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a^2 + 1)*b*d)
*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + (2*a*b^2*d*
x + b^3*c + 3*(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2
)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a
^2 + 1)*b*d)*sqrt(c)*sqrt(d))) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan2((2*a*b
^2*c*d - (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^2 + 3)*b^2*d)*x)*sqrt(c)*sq
rt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (
a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d)), ((a^2 +
3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - (2*a*b^2*d*x + b^3*c + 3*(a^2 + 1)*b*d
)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)
*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d
))) + log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 + 22*a^2 + 21)*b^
20*c^10*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^3 + 33*(5*a^8 +
260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(a^10 + 61*a^8 + 5
70*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21*a^12 + 1386*a^10
+ 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22
*(21*a^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169455*a^6 + 201773*a^4 +
121771*a^2 + 29393)*b^10*c^5*d^7 + 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^
10 + 9438*a^8 + 13728*a^6 + 11492*a^4 + 5168*a^2 + 969)*b^8*c^4*d^8 + 33*(5
*a^18 + 285*a^16 + 3220*a^14 + 15876*a^12 + 42966*a^10 + 70070*a^8 + 70980*
a^6 + 43860*a^4 + 15181*a^2 + 2261)*b^6*c^3*d^9 + 55*(a^20 + 46*a^18 + 465*
a^16 + 2184*a^14 + 5922*a^12 + 10164*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4
 + 1102*a^2 + 133)*b^4*c^2*d^10 + 11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16
 + 2730*a^14 + 4662*a^12 + 5502*a^10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*
a^2 + 21)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 79
2*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^1
2 + (b^24*c^11*d + 11*(a^2 + 21)*b^22*c^10*d^2 + 55*(a^4 + 38*a^2 + 133)*b^
20*c^9*d^3 + 33*(5*a^6 + 255*a^4 + 1615*a^2 + 2261)*b^18*c^8*d^4 + 330*(a^8
 + 60*a^6 + 510*a^4 + 1292*a^2 + 969)*b^16*c^7*d^5 + 22*(21*a^10 + 1365*a^8
 + 13650*a^6 + 46410*a^4 + 62985*a^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 +
1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^
```

```
5*d^7 + 330*(a^14 + 63*a^12 + 693*a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4
199*a^2 + 969)*b^10*c^4*d^8 + 33*(5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^1
0 + 30030*a^8 + 40040*a^6 + 30940*a^4 + 12920*a^2 + 2261)*b^8*c^3*d^9 + 55*
(a^18 + 45*a^16 + 420*a^14 + 1764*a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 +
3060*a^4 + 969*a^2 + 133)*b^6*c^2*d^10 + 11*(a^20 + 30*a^18 + 225*a^16 + 84
0*a^14 + 1890*a^12 + 2772*a^10 + 2730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 +
21)*b^4*c*d^11 + (a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12
 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b^2*d^12)*x^2 + 2*(1
1*(a^2 + 1)*b^21*c^10*d + 110*(a^4 + 8*a^2 + 7)*b^19*c^9*d^2 + 33*(15*a^6 +
 205*a^4 + 589*a^2 + 399)*b^17*c^8*d^3 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 64
6*a^2 + 323)*b^15*c^7*d^4 + 110*(21*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 +
6137*a^2 + 2261)*b^13*c^6*d^5 + 4*(693*a^12 + 15708*a^10 + 105105*a^8 + 308
880*a^6 + 449735*a^4 + 319124*a^2 + 88179)*b^11*c^5*d^6 + 110*(21*a^14 + 48
3*a^12 + 3465*a^10 + 11583*a^8 + 20735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*
b^9*c^4*d^7 + 264*(5*a^16 + 110*a^14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 68
90*a^6 + 4930*a^4 + 1938*a^2 + 323)*b^7*c^3*d^8 + 33*(15*a^18 + 295*a^16 +
2044*a^14 + 7308*a^12 + 15554*a^10 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 298
3*a^2 + 399)*b^5*c^2*d^9 + 110*(a^20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a
^12 + 1008*a^10 + 966*a^8 + 624*a^6 + 261*a^4 + 64*a^2 + 7)*b^3*c*d^10 + 11
*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 33
0*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11 + (11*b^23*c^10*d + 110*(a^2
+ 7)*b^21*c^9*d^2 + 33*(15*a^4 + 190*a^2 + 399)*b^19*c^8*d^3 + 264*(5*a^6 +
 85*a^4 + 323*a^2 + 323)*b^17*c^7*d^4 + 110*(21*a^8 + 420*a^6 + 2142*a^4 +
3876*a^2 + 2261)*b^15*c^6*d^5 + 4*(693*a^10 + 15015*a^8 + 90090*a^6 + 21879
0*a^4 + 230945*a^2 + 88179)*b^13*c^5*d^6 + 110*(21*a^12 + 462*a^10 + 3003*a
^8 + 8580*a^6 + 12155*a^4 + 8398*a^2 + 2261)*b^11*c^4*d^7 + 264*(5*a^14 + 1
05*a^12 + 693*a^10 + 2145*a^8 + 3575*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c
^3*d^8 + 33*(15*a^16 + 280*a^14 + 1764*a^12 + 5544*a^10 + 10010*a^8 + 10920
*a^6 + 7140*a^4 + 2584*a^2 + 399)*b^7*c^2*d^9 + 110*(a^18 + 15*a^16 + 84*a^
14 + 252*a^12 + 462*a^10 + 546*a^8 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c*
d^10 + 11*(a^20 + 10*a^18 + 45*a^16 + 120*a^14 + 210*a^12 + 252*a^10 + 210*
a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^3*d^11)*x^2 + 2*(11*a*b^22*c^10*d +
110*(a^3 + 7*a)*b^20*c^9*d^2 + 33*(15*a^5 + 190*a^3 + 399*a)*b^18*c^8*d^3 +
 264*(5*a^7 + 85*a^5 + 323*a^3 + 323*a)*b^16*c^7*d^4 + 110*(21*a^9 + 420*a^
7 + 2142*a^5 + 3876*a^3 + 2261*a)*b^14*c^6*d^5 + 4*(693*a^11 + 15015*a^9 +
90090*a^7 + 218790*a^5 + 230945*a^3 + 88179*a)*b^12*c^5*d^6 + 110*(21*a^13
+ 462*a^11 + 3003*a^9 + 8580*a^7 + 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^4*
d^7 + 264*(5*a^15 + 105*a^13 + 693*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 +
1615*a^3 + 323*a)*b^8*c^3*d^8 + 33*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*a
^11 + 10010*a^9 + 10920*a^7 + 7140*a^5 + 2584*a^3 + 399*a)*b^6*c^2*d^9 + 11
0*(a^19 + 15*a^17 + 84*a^15 + 252*a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 204
*a^5 + 57*a^3 + 7*a)*b^4*c*d^10 + 11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15 +
 210*a^13 + 252*a^11 + 210*a^9 + 120*a^7 + 45*a^5 + 10*a^3 + a)*b^2*d^11)*x
)*sqrt(c)*sqrt(d) + 2*(a*b^23*c^11*d + 11*(a^3 + 21*a)*b^21*c^10*d^2 + 55*(
a^5 + 38*a^3 + 133*a)*b^19*c^9*d^3 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*
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a)*b^17*c^8*d^4 + 330*(a^9 + 60*a^7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^7*
d^5 + 22*(21*a^11 + 1365*a^9 + 13650*a^7 + 46410*a^5 + 62985*a^3 + 29393*a)
*b^13*c^6*d^6 + 22*(21*a^13 + 1386*a^11 + 15015*a^9 + 60060*a^7 + 109395*a^
5 + 92378*a^3 + 29393*a)*b^11*c^5*d^7 + 330*(a^15 + 63*a^13 + 693*a^11 + 30
03*a^9 + 6435*a^7 + 7293*a^5 + 4199*a^3 + 969*a)*b^9*c^4*d^8 + 33*(5*a^17 +
 280*a^15 + 2940*a^13 + 12936*a^11 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 12
920*a^3 + 2261*a)*b^7*c^3*d^9 + 55*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13 +
 4158*a^11 + 6006*a^9 + 5460*a^7 + 3060*a^5 + 969*a^3 + 133*a)*b^5*c^2*d^10
 + 11*(a^21 + 30*a^19 + 225*a^17 + 840*a^15 + 1890*a^13 + 2772*a^11 + 2730*
a^9 + 1800*a^7 + 765*a^5 + 190*a^3 + 21*a)*b^3*c*d^11 + (a^23 + 11*a^21 + 5
5*a^19 + 165*a^17 + 330*a^15 + 462*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 55
*a^5 + 11*a^3 + a)*b*d^12)*x)/(b^24*c^12 + 12*(a^2 + 23)*b^22*c^11*d + 66*(
a^4 + 42*a^2 + 161)*b^20*c^10*d^2 + 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*
b^18*c^9*d^3 + 99*(5*a^8 + 340*a^6 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^8*d
^4 + 264*(3*a^10 + 225*a^8 + 2550*a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*c
^7*d^5 + 4*(231*a^12 + 18018*a^10 + 225225*a^8 + 1021020*a^6 + 2078505*a^4
+ 1939938*a^2 + 676039)*b^12*c^6*d^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10 +
 15015*a^8 + 36465*a^6 + 46189*a^4 + 29393*a^2 + 7429)*b^10*c^5*d^7 + 99*(5
*a^16 + 360*a^14 + 4620*a^12 + 24024*a^10 + 64350*a^8 + 97240*a^6 + 83980*a
^4 + 38760*a^2 + 7429)*b^8*c^4*d^8 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 19
404*a^12 + 54054*a^10 + 90090*a^8 + 92820*a^6 + 58140*a^4 + 20349*a^2 + 305
9)*b^6*c^3*d^9 + 66*(a^20 + 50*a^18 + 525*a^16 + 2520*a^14 + 6930*a^12 + 12
012*a^10 + 13650*a^8 + 10200*a^6 + 4845*a^4 + 1330*a^2 + 161)*b^4*c^2*d^10
+ 12*(a^22 + 33*a^20 + 275*a^18 + 1155*a^16 + 2970*a^14 + 5082*a^12 + 6006*
a^10 + 4950*a^8 + 2805*a^6 + 1045*a^4 + 231*a^2 + 23)*b^2*c*d^11 + (a^24 +
12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 +
495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 + 8*(3*b^23*c^11 + 11*(3*a^2
+ 23)*b^21*c^10*d + 33*(5*a^4 + 70*a^2 + 161)*b^19*c^9*d^2 + 99*(5*a^6 + 95
*a^4 + 399*a^2 + 437)*b^17*c^8*d^3 + 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 116
28*a^2 + 7429)*b^15*c^7*d^4 + 6*(231*a^10 + 5775*a^8 + 39270*a^6 + 106590*a
^4 + 124355*a^2 + 52003)*b^13*c^6*d^5 + 6*(231*a^12 + 6006*a^10 + 45045*a^8
 + 145860*a^6 + 230945*a^4 + 176358*a^2 + 52003)*b^11*c^5*d^6 + 22*(45*a^14
 + 1155*a^12 + 9009*a^10 + 32175*a^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 +
7429)*b^9*c^4*d^7 + 99*(5*a^16 + 120*a^14 + 924*a^12 + 3432*a^10 + 7150*a^8
 + 8840*a^6 + 6460*a^4 + 2584*a^2 + 437)*b^7*c^3*d^8 + 33*(5*a^18 + 105*a^1
6 + 756*a^14 + 2772*a^12 + 6006*a^10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 119
7*a^2 + 161)*b^5*c^2*d^9 + 11*(3*a^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 23
10*a^12 + 3276*a^10 + 3150*a^8 + 2040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c*d
^10 + 3*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^
10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11)*sqrt(c)*sqrt(d))) - l
og(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 + 22*a^2 + 21)*b^20*c^10
*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^3 + 33*(5*a^8 + 260*a^6
 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(a^10 + 61*a^8 + 570*a^6
+ 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21*a^12 + 1386*a^10 + 15015
*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^
```

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14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169455*a^6 + 201773*a^4 + 121771*
a^2 + 29393)*b^10*c^5*d^7 + 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 94
38*a^8 + 13728*a^6 + 11492*a^4 + 5168*a^2 + 969)*b^8*c^4*d^8 + 33*(5*a^18 +
 285*a^16 + 3220*a^14 + 15876*a^12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 4
3860*a^4 + 15181*a^2 + 2261)*b^6*c^3*d^9 + 55*(a^20 + 46*a^18 + 465*a^16 +
2184*a^14 + 5922*a^12 + 10164*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102
*a^2 + 133)*b^4*c^2*d^10 + 11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730
*a^14 + 4662*a^12 + 5502*a^10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 2
1)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14
+ 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 + (b^
24*c^11*d + 11*(a^2 + 21)*b^22*c^10*d^2 + 55*(a^4 + 38*a^2 + 133)*b^20*c^9*
d^3 + 33*(5*a^6 + 255*a^4 + 1615*a^2 + 2261)*b^18*c^8*d^4 + 330*(a^8 + 60*a
^6 + 510*a^4 + 1292*a^2 + 969)*b^16*c^7*d^5 + 22*(21*a^10 + 1365*a^8 + 1365
0*a^6 + 46410*a^4 + 62985*a^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^
10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^5*d^7 +
 330*(a^14 + 63*a^12 + 693*a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2
 + 969)*b^10*c^4*d^8 + 33*(5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^10 + 300
30*a^8 + 40040*a^6 + 30940*a^4 + 12920*a^2 + 2261)*b^8*c^3*d^9 + 55*(a^18 +
 45*a^16 + 420*a^14 + 1764*a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^
4 + 969*a^2 + 133)*b^6*c^2*d^10 + 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14
+ 1890*a^12 + 2772*a^10 + 2730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4
*c*d^11 + (a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*
a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b^2*d^12)*x^2 - 2*(11*(a^2
+ 1)*b^21*c^10*d + 110*(a^4 + 8*a^2 + 7)*b^19*c^9*d^2 + 33*(15*a^6 + 205*a^
4 + 589*a^2 + 399)*b^17*c^8*d^3 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 +
 323)*b^15*c^7*d^4 + 110*(21*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^
2 + 2261)*b^13*c^6*d^5 + 4*(693*a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6
 + 449735*a^4 + 319124*a^2 + 88179)*b^11*c^5*d^6 + 110*(21*a^14 + 483*a^12
+ 3465*a^10 + 11583*a^8 + 20735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^4
*d^7 + 264*(5*a^16 + 110*a^14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6
+ 4930*a^4 + 1938*a^2 + 323)*b^7*c^3*d^8 + 33*(15*a^18 + 295*a^16 + 2044*a^
14 + 7308*a^12 + 15554*a^10 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 +
 399)*b^5*c^2*d^9 + 110*(a^20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1
008*a^10 + 966*a^8 + 624*a^6 + 261*a^4 + 64*a^2 + 7)*b^3*c*d^10 + 11*(a^22
+ 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 +
 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11 + (11*b^23*c^10*d + 110*(a^2 + 7)*b^
21*c^9*d^2 + 33*(15*a^4 + 190*a^2 + 399)*b^19*c^8*d^3 + 264*(5*a^6 + 85*a^4
 + 323*a^2 + 323)*b^17*c^7*d^4 + 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^
2 + 2261)*b^15*c^6*d^5 + 4*(693*a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 +
 230945*a^2 + 88179)*b^13*c^5*d^6 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 85
80*a^6 + 12155*a^4 + 8398*a^2 + 2261)*b^11*c^4*d^7 + 264*(5*a^14 + 105*a^12
 + 693*a^10 + 2145*a^8 + 3575*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^3*d^8
+ 33*(15*a^16 + 280*a^14 + 1764*a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 +
7140*a^4 + 2584*a^2 + 399)*b^7*c^2*d^9 + 110*(a^18 + 15*a^16 + 84*a^14 + 25
2*a^12 + 462*a^10 + 546*a^8 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c*d^10 +
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11*(a^20 + 10*a^18 + 45*a^16 + 120*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 1
20*a^6 + 45*a^4 + 10*a^2 + 1)*b^3*d^11)*x^2 + 2*(11*a*b^22*c^10*d + 110*(a^
3 + 7*a)*b^20*c^9*d^2 + 33*(15*a^5 + 190*a^3 + 399*a)*b^18*c^8*d^3 + 264*(5
*a^7 + 85*a^5 + 323*a^3 + 323*a)*b^16*c^7*d^4 + 110*(21*a^9 + 420*a^7 + 214
2*a^5 + 3876*a^3 + 2261*a)*b^14*c^6*d^5 + 4*(693*a^11 + 15015*a^9 + 90090*a
^7 + 218790*a^5 + 230945*a^3 + 88179*a)*b^12*c^5*d^6 + 110*(21*a^13 + 462*a
^11 + 3003*a^9 + 8580*a^7 + 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^4*d^7 + 2
64*(5*a^15 + 105*a^13 + 693*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^
3 + 323*a)*b^8*c^3*d^8 + 33*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 1
0010*a^9 + 10920*a^7 + 7140*a^5 + 2584*a^3 + 399*a)*b^6*c^2*d^9 + 110*(a^19
 + 15*a^17 + 84*a^15 + 252*a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 +
57*a^3 + 7*a)*b^4*c*d^10 + 11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^
13 + 252*a^11 + 210*a^9 + 120*a^7 + 45*a^5 + 10*a^3 + a)*b^2*d^11)*x)*sqrt(
c)*sqrt(d) + 2*(a*b^23*c^11*d + 11*(a^3 + 21*a)*b^21*c^10*d^2 + 55*(a^5 + 3
8*a^3 + 133*a)*b^19*c^9*d^3 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17
*c^8*d^4 + 330*(a^9 + 60*a^7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^7*d^5 + 2
2*(21*a^11 + 1365*a^9 + 13650*a^7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c
^6*d^6 + 22*(21*a^13 + 1386*a^11 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 923
78*a^3 + 29393*a)*b^11*c^5*d^7 + 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9
+ 6435*a^7 + 7293*a^5 + 4199*a^3 + 969*a)*b^9*c^4*d^8 + 33*(5*a^17 + 280*a^
15 + 2940*a^13 + 12936*a^11 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3
 + 2261*a)*b^7*c^3*d^9 + 55*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a
^11 + 6006*a^9 + 5460*a^7 + 3060*a^5 + 969*a^3 + 133*a)*b^5*c^2*d^10 + 11*(
a^21 + 30*a^19 + 225*a^17 + 840*a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1
800*a^7 + 765*a^5 + 190*a^3 + 21*a)*b^3*c*d^11 + (a^23 + 11*a^21 + 55*a^19
+ 165*a^17 + 330*a^15 + 462*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 +
11*a^3 + a)*b*d^12)*x)/(b^24*c^12 + 12*(a^2 + 23)*b^22*c^11*d + 66*(a^4 + 4
2*a^2 + 161)*b^20*c^10*d^2 + 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^
9*d^3 + 99*(5*a^8 + 340*a^6 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^8*d^4 + 26
4*(3*a^10 + 225*a^8 + 2550*a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*c^7*d^5
+ 4*(231*a^12 + 18018*a^10 + 225225*a^8 + 1021020*a^6 + 2078505*a^4 + 19399
38*a^2 + 676039)*b^12*c^6*d^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*
a^8 + 36465*a^6 + 46189*a^4 + 29393*a^2 + 7429)*b^10*c^5*d^7 + 99*(5*a^16 +
 360*a^14 + 4620*a^12 + 24024*a^10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38
760*a^2 + 7429)*b^8*c^4*d^8 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^1
2 + 54054*a^10 + 90090*a^8 + 92820*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*
c^3*d^9 + 66*(a^20 + 50*a^18 + 525*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^1
0 + 13650*a^8 + 10200*a^6 + 4845*a^4 + 1330*a^2 + 161)*b^4*c^2*d^10 + 12*(a
^22 + 33*a^20 + 275*a^18 + 1155*a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 +
4950*a^8 + 2805*a^6 + 1045*a^4 + 231*a^2 + 23)*b^2*c*d^11 + (a^24 + 12*a^22
 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8
 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 - 8*(3*b^23*c^11 + 11*(3*a^2 + 23)*b
^21*c^10*d + 33*(5*a^4 + 70*a^2 + 161)*b^19*c^9*d^2 + 99*(5*a^6 + 95*a^4 +
399*a^2 + 437)*b^17*c^8*d^3 + 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2
+ 7429)*b^15*c^7*d^4 + 6*(231*a^10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 12
```

```
4355*a^2 + 52003)*b^13*c^6*d^5 + 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 1458
60*a^6 + 230945*a^4 + 176358*a^2 + 52003)*b^11*c^5*d^6 + 22*(45*a^14 + 1155
*a^12 + 9009*a^10 + 32175*a^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*b
^9*c^4*d^7 + 99*(5*a^16 + 120*a^14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 8840
*a^6 + 6460*a^4 + 2584*a^2 + 437)*b^7*c^3*d^8 + 33*(5*a^18 + 105*a^16 + 756
*a^14 + 2772*a^12 + 6006*a^10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2 +
 161)*b^5*c^2*d^9 + 11*(3*a^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^12
 + 3276*a^10 + 3150*a^8 + 2040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c*d^10 + 3
*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 33
0*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11)*sqrt(c)*sqrt(d)) + 2*dilog(
((a + I)*b*d*x + b^2*c + (I*b^2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(b^2*c +
 2*(-I*a + 1)*b*sqrt(c)*sqrt(d) - (a^2 + 2*I*a - 1)*d)) - 2*dilog(((a + I)*
b*d*x + b^2*c - (I*b^2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a
+ 1)*b*sqrt(c)*sqrt(d) - (a^2 + 2*I*a - 1)*d)) - 2*dilog(((a - I)*b*d*x + b
^2*c + (I*b^2*x + (-I*a - 1)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a - 1)*b*sq
rt(c)*sqrt(d) - (a^2 - 2*I*a - 1)*d)) + 2*dilog(((a - I)*b*d*x + b^2*c - (I
*b^2*x + (-I*a - 1)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a - 1)*b*sqrt(c)*sqr
t(d) - (a^2 - 2*I*a - 1)*d)))/b)/sqrt(c*d) + arctan(b*x + a)*arctan(d*x/sqr
t(c*d))/sqrt(c*d) - arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/sqrt(c*d)
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx=\int \frac{\arctan\left(bx+a\right)}{dx^2+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{d\,x^2+c}\,dx$$

```
[In] int(atan(a + b*x)/(c + d*x^2),x)
[Out] int(atan(a + b*x)/(c + d*x^2), x)
```

3.54 $\int \frac{\arctan(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 152

$$\begin{aligned}\int \frac{\arctan(a+bx)}{c+dx}\,dx = &-\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}\\&+\frac{\arctan(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}\\&+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}\\&-\frac{i\,\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}\end{aligned}$$

```
[Out] -arctan(b*x+a)*ln(2/(1-I*(b*x+a)))/d+arctan(b*x+a)*ln(2*b*(d*x+c)/(b*c+I*d-
a*d)/(1-I*(b*x+a)))/d+1/2*I*polylog(2,1-2/(1-I*(b*x+a)))/d-1/2*I*polylog(2,
1-2*b*(d*x+c)/(b*c+I*d-a*d)/(1-I*(b*x+a)))/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used

$= \{5155, 4966, 2449, 2352, 2497\}$

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \frac{\arctan(a+bx)\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d} - \frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} - \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc-ad+id)(1-i(a+bx))}\right)}{2d} + \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}$$

```
[In] Int[ArcTan[a + b*x]/(c + d*x),x]
```

```
[Out] -((ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcTan[a + b*x]*Log[(2*b
*(c + d*x))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d + ((I/2)*PolyLog[2, 1
 - 2/(1 - I*(a + b*x))])/d - ((I/2)*PolyLog[2, 1 - (2*b*(c + d*x))/((b*c +
I*d - a*d)*(1 - I*(a + b*x)))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\arctan(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{\arctan(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, a+bx\right)}{d}-\frac{\text{Subst}\left(\int \frac{\log\left(\frac{2\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}{\left(\frac{id}{b}+\frac{bc-ad}{b}\right)(1-ix)}\right)}{1+x^2}\,dx, x, a+bx\right)}{d}\\
&= -\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{\arctan(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}\\
&\quad-\frac{i\,\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}+\frac{i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-i(a+bx)}\right)}{d}\\
&= -\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{\arctan(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}\\
&\quad+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.52

$$\begin{aligned}
\int \frac{\arctan(a+bx)}{c+dx}\,dx &= \frac{i\log(1-i(a+bx))\log\left(-\frac{i\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{-\frac{d}{b}-\frac{i(bc-ad)}{b}}\right)}{2d}\\
&\quad-\frac{i\log(1+i(a+bx))\log\left(\frac{i\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{-\frac{d}{b}+\frac{i(bc-ad)}{b}}\right)}{2d}\\
&\quad+\frac{i\,\text{PolyLog}\left(2,-\frac{id(1-i(a+bx))}{bc-id-ad}\right)}{2d}-\frac{i\,\text{PolyLog}\left(2,\frac{id(1+i(a+bx))}{bc+id-ad}\right)}{2d}
\end{aligned}$$

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x),x]
```

```
[Out] ((I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*((b*c - a*d)/b + (d*(a + b*x))/b))/(-
(d/b) - (I*(b*c - a*d))/b)])/d - ((I/2)*Log[1 + I*(a + b*x)]*Log[(I*((b*c -
 a*d)/b + (d*(a + b*x))/b))/(-(d/b) + (I*(b*c - a*d))/b)])/d + ((I/2)*PolyL
og[2, ((-I)*d*(1 - I*(a + b*x)))/(b*c - I*d - a*d)])/d - ((I/2)*PolyLog[2,
(I*d*(1 + I*(a + b*x)))/(b*c + I*d - a*d)])/d
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\dfrac{\frac{b\ln(ad-bc-d(bx+a))\arctan(bx+a)}{d}+b\Bigg(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\left(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\right.}{2d}}{b}$
default	$\dfrac{\frac{b\ln(ad-bc-d(bx+a))\arctan(bx+a)}{d}+b\Bigg(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\left(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\right.}{2d}}{b}$
parts	$\frac{\ln(dx+c)\arctan(bx+a)}{d}-b\Bigg(-\frac{i\ln(dx+c)\left(\ln\left(\frac{id-ad+bc-b(dx+c)}{-ad+bc+id}\right)-\ln\left(\frac{id+ad-bc+b(dx+c)}{ad-bc+id}\right)\right)}{2db}-\frac{i\left(\operatorname{dilog}\left(\frac{id-ad+bc}{-ad+}\right.\right.}{}$
risch	$\frac{i\operatorname{dilog}\left(\frac{iad-ibc+(-bxi-ia+1)d-d}{iad-ibc-d}\right)}{2d}+\frac{i\ln(-bxi-ia+1)\ln\left(\frac{iad-ibc+(-bxi-ia+1)d-d}{iad-ibc-d}\right)}{2d}-\frac{i\operatorname{dilog}\left(\frac{-iad+ibc+(bxi+ia+1)}{-iad+ibc-d}\right.}{2d}$

```
[In] int(arctan(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b*ln(a*d-b*c-d*(b*x+a))/d*arctan(b*x+a)+b*(-1/2*I*ln(a*d-b*c-d*(b*x+a)
)*(ln((I*d+d*(b*x+a))/(a*d-b*c+I*d))-ln((I*d-d*(b*x+a))/(b*c+I*d-a*d)))/d-1
/2*I*(dilog((I*d+d*(b*x+a))/(a*d-b*c+I*d))-dilog((I*d-d*(b*x+a))/(b*c+I*d-a
*d)))/d))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx=\int \frac{\arctan\left(bx+a\right)}{dx+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \text{Timed out}$$

```
[In] integrate(atan(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(130) = 260.

Time = 0.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.87

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \frac{\arctan\left(bx+a\right)\log\left(dx+c\right)}{d} - \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+c\right)}{d} - \frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+(a^2+1)d^2}, \frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)\log\left(b^2x^2+2\,abx+a^2+1\right) - \arctan\left(bx+a\right)\log}{2\,d}$$

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] arctan(b*x + a)*log(d*x + c)/d - arctan((b^2*x + a*b)/b)*log(d*x + c)/d - 1
/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2*c
^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)
)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((I*b*d
*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a - 1)*d)
/(-I*b*c + (I*a - 1)*d)))/d
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \int \frac{\arctan\left(bx+a\right)}{dx+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{c+d\,x}\,dx$$

```
[In] int(atan(a + b*x)/(c + d*x),x)
[Out] int(atan(a + b*x)/(c + d*x), x)
```

3.55 $\int \frac{\arctan(a+bx)}{c+\frac{d}{x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 244

$$\begin{aligned}\int \frac{\arctan(a+bx)}{c+\frac{d}{x}} \, dx = & -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} \\ & -\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\ & -\frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2} \\ & +\frac{id\log(1+ia+ibx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2} \\ & +\frac{id\operatorname{PolyLog}\left(2,\frac{c(i-a-bx)}{ic-ac+bd}\right)}{2c^2}-\frac{id\operatorname{PolyLog}\left(2,\frac{c(i+a+bx)}{(i+a)c-bd}\right)}{2c^2}\end{aligned}$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c-1/2*I*d*ln(1-I*a-I*b*x)*ln(-b*(c*x+d)/((I+a)*c-b*d))/c^2+1/2*I*d*ln(1+I*
a+I*b*x)*ln(b*(c*x+d)/((I-a)*c+b*d))/c^2+1/2*I*d*polylog(2,c*(I-a-b*x)/(I*c
-a*c+b*d))/c^2-1/2*I*d*polylog(2,c*(I+a+b*x)/((I+a)*c-b*d))/c^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5159, 2456, 2436, 2332, 2441, 2440, 2438}

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx = \frac{id\,\text{PolyLog}\left(2,\frac{c(-a-bx+i)}{-ac+ic+bd}\right)}{2c^2} - \frac{id\,\text{PolyLog}\left(2,\frac{c(a+bx+i)}{(a+i)c-bd}\right)}{2c^2} + \frac{id\log(ia+ibx+1)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2} - \frac{id\log(-ia-ibx+1)\log\left(-\frac{b(cx+d)}{-bd+(a+i)c}\right)}{2c^2} - \frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc} - \frac{(-ia-ibx+1)\log(-i(a+bx+i))}{2bc}$$

```
[In] Int[ArcTan[a + b*x]/(c + d/x),x]
```

```
[Out] -1/2*((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - ((1 - I*a - I*b*x)*Lo
g[(-I)*(I + a + b*x)])/(2*b*c) - ((I/2)*d*Log[1 - I*a - I*b*x]*Log[-((b*(d
+ c*x))/((I + a)*c - b*d))])/c^2 + ((I/2)*d*Log[1 + I*a + I*b*x]*Log[(b*(d
+ c*x))/((I - a)*c + b*d)])/c^2 + ((I/2)*d*PolyLog[2, (c*(I - a - b*x))/(I*
c - a*c + b*d)])/c^2 - ((I/2)*d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b
*d)])/c^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x}}\,dx-\frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x}}\,dx\\
&= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c}-\frac{d\log(1-ia-ibx)}{c(d+cx)}\right)dx\\
&\quad -\frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c}-\frac{d\log(1+ia+ibx)}{c(d+cx)}\right)dx\\
&= \frac{i\int \log(1-ia-ibx)\,dx}{2c}-\frac{i\int \log(1+ia+ibx)\,dx}{2c}\\
&\quad -\frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx}\,dx}{2c}+\frac{(id)\int \frac{\log(1+ia+ibx)}{d+cx}\,dx}{2c}\\
&= -\frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2}+\frac{id\log(1+ia+ibx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2}\\
&\quad -\frac{\text{Subst}(\int \log(x)\,dx,x,1-ia-ibx)}{2bc}-\frac{\text{Subst}(\int \log(x)\,dx,x,1+ia+ibx)}{2bc}\\
&\quad +\frac{(bd)\int \frac{\log\left(-\frac{ib(d+cx)}{-((1-ia)c)-ibd}\right)}{1-ia-ibx}\,dx}{2c^2}+\frac{(bd)\int \frac{\log\left(\frac{ib(d+cx)}{-((1+ia)c)+ibd}\right)}{1+ia+ibx}\,dx}{2c^2}
\end{aligned}
$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2} + \frac{id\log(1+ia+ibx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2} + \frac{(id)\text{Subst}\left(\int\frac{\log\left(1+\frac{cx}{-((1-ia)c)-ibd}\right)}{x}\,dx, x, 1-ia-ibx\right)}{2c^2} - \frac{(id)\text{Subst}\left(\int\frac{\log\left(1+\frac{cx}{-((1+ia)c)+ibd}\right)}{x}\,dx, x, 1+ia+ibx\right)}{2c^2}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2} + \frac{id\log(1+ia+ibx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2} + \frac{id\,\text{PolyLog}\left(2, \frac{c(i-a-bx)}{(i-a)c+bd}\right)}{2c^2} - \frac{id\,\text{PolyLog}\left(2, \frac{c(i+a+bx)}{(i+a)c-bd}\right)}{2c^2}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 771 vs. 2(244) = 488.

Time = 8.24 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.16

$$\int\frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx$$

$$= \frac{-2a^2c^2\arctan(a+bx) + 2abcd\arctan(a+bx) + iabcd\pi\arctan(a+bx) - ib^2d^2\pi\arctan(a+bx) - 2abc^2}{}$$

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x),x]
```

```
[Out] (-2*a^2*c^2*ArcTan[a + b*x] + 2*a*b*c*d*ArcTan[a + b*x] + I*a*b*c*d*Pi*ArcT
an[a + b*x] - I*b^2*d^2*Pi*ArcTan[a + b*x] - 2*a*b*c^2*x*ArcTan[a + b*x] +
2*b^2*c*d*x*ArcTan[a + b*x] + (2*I)*a*b*c*d*ArcTan[a - (b*d)/c]*ArcTan[a +
b*x] - (2*I)*b^2*d^2*ArcTan[a - (b*d)/c]*ArcTan[a + b*x] - b*c*d*ArcTan[a +
 b*x]^2 + I*a*b*c*d*ArcTan[a + b*x]^2 - I*b^2*d^2*ArcTan[a + b*x]^2 + (b*c*
d*Sqrt[1 + a^2 - (2*a*b*d)/c + (b^2*d^2)/c^2]*ArcTan[a + b*x]^2)/E^(I*ArcTa
n[a - (b*d)/c]) + a*b*c*d*Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])] - b^2*d^2*
Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])] - 2*a*b*c*d*ArcTan[a + b*x]*Log[1 +
E^((2*I)*ArcTan[a + b*x])] + 2*b^2*d^2*ArcTan[a + b*x]*Log[1 + E^((2*I)*Arc
Tan[a + b*x])] - 2*a*b*c*d*ArcTan[a - (b*d)/c]*Log[1 - E^((2*I)*(-ArcTan[a
- (b*d)/c] + ArcTan[a + b*x]))] + 2*b^2*d^2*ArcTan[a - (b*d)/c]*Log[1 - E^(
```

```
(2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))] + 2*a*b*c*d*ArcTan[a + b*x
]*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))] - 2*b^2*d^2*A
rcTan[a + b*x]*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))]
 - 2*a*c^2*Log[1/Sqrt[1 + (a + b*x)^2]] + 2*b*c*d*Log[1/Sqrt[1 + (a + b*x)^2
]] - a*b*c*d*Pi*Log[1/Sqrt[1 + (a + b*x)^2]] + b^2*d^2*Pi*Log[1/Sqrt[1 + (a
 + b*x)^2]] + 2*a*b*c*d*ArcTan[a - (b*d)/c]*Log[Sin[ArcTan[(-(a*c) + b*d)/c
] + ArcTan[a + b*x]]] - 2*b^2*d^2*ArcTan[a - (b*d)/c]*Log[Sin[ArcTan[(-(a*c
) + b*d)/c] + ArcTan[a + b*x]]] + I*b*d*(a*c - b*d)*PolyLog[2, -E^((2*I)*Ar
cTan[a + b*x])] + I*b*d*(-(a*c) + b*d)*PolyLog[2, E^((2*I)*(-ArcTan[a - (b*
d)/c] + ArcTan[a + b*x]))])/(2*b*c^2*(-(a*c) + b*d))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{\arctan(bx+a)(bx+a)}{c}-\frac{\arctan(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}+\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))+c\right.}{2}}{}}{}$
default	$\frac{\frac{\arctan(bx+a)(bx+a)}{c}-\frac{\arctan(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}+\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))+c\right.}{2}}{}}{}$
parts	$\frac{\arctan(bx+a)x}{c}-\frac{\arctan(bx+a)d\ln(cx+d)}{c^2}-\frac{b\left(\frac{\ln\left(a^2c^2-2abcd+2abc(cx+d)+b^2d^2-2b^2d(cx+d)+b^2(cx+d)^2+c^2\right)}{2b^2}-\frac{a\arctan}{}\right.}{}$
risch	$\frac{i\ln(-bxi-ia+1)a}{2bc}+\frac{id\ln(bxi+ia+1)\ln\left(\frac{-iac+ibd+(bxi+ia+1)c-c}{-iac+ibd-c}\right)}{2c^2}-\frac{i\ln(bxi+ia+1)a}{2bc}+\frac{i\ln(-bxi-ia+1)x}{2c}-\frac{\ln(}{}$

```
[In] int(arctan(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctan(b*x+a)/c*(b*x+a)-arctan(b*x+a)*d*b/c^2*ln(a*c-b*d-c*(b*x+a))+1/
c*(-1/2*ln(a^2*c^2-2*a*b*c*d+b^2*d^2-2*a*c*(a*c-b*d-c*(b*x+a))+2*b*d*(a*c-b
*d-c*(b*x+a))+c^2+(a*c-b*d-c*(b*x+a))^2)-b*d*(-1/2*I*ln(a*c-b*d-c*(b*x+a))*
(ln((I*c+c*(b*x+a))/(a*c-b*d+I*c))-ln((I*c-c*(b*x+a))/(I*c-a*c+b*d)))/c-1/2
*I*(dilog((I*c+c*(b*x+a))/(a*c-b*d+I*c))-dilog((I*c-c*(b*x+a))/(I*c-a*c+b*d
)))/c)))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="fricas")
[Out] integral(x*arctan(b*x + a)/(c*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx = \text{Timed out}$$

```
[In] integrate(atan(b*x+a)/(c+d/x),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.16

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx = -\frac{bd\arctan\left(bx+a\right)\log\left(-\frac{b^2c^2x^2+2\,b^2cdx+b^2d^2}{2\,abcd-b^2d^2-(a^2+1)c^2}\right)+i\,bd\mathrm{Li}_2\left(-\frac{i\,bcx+(i\,a-1)c}{(-i\,a+1)c+i\,bd}\right)-i\,bd\mathrm{Li}_2\left(-\frac{i\,bcx+(i\,a+1)c}{(-i\,a-1)c+i\,bd}\right)-2\,(b}{}$$

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="maxima")
[Out] -1/2*(b*d*arctan(b*x + a)*log(-(b^2*c^2*x^2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*b
*c*d - b^2*d^2 - (a^2 + 1)*c^2)) + I*b*d*dilog(-(I*b*c*x + (I*a - 1)*c)/((-
I*a + 1)*c + I*b*d)) - I*b*d*dilog(-(I*b*c*x + (I*a + 1)*c)/((-I*a - 1)*c +
 I*b*d)) - 2*(b*c*x + a*c)*arctan(b*x + a) - (b*d*arctan2(-(b*c^2*x + b*c*d
)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*c^2 - b^
2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2 + 2*a*b*x
 + a^2 + 1))/(b*c^2)
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

```
[In] int(atan(a + b*x)/(c + d/x),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x), x)
```

3.56 $\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 668

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = &-\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} \\
&-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&+\frac{i\sqrt{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&-\frac{i\sqrt{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+\frac{i\sqrt{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&-\frac{i\sqrt{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{i\sqrt{-c}-a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+\frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i-a-bx)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&-\frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(1+ia+ibx)}{(1+ia)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+\frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i+a+bx)}{i\sqrt{-c}+a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&-\frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i+a+bx)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}
\end{aligned}
$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c+1/4*I*ln(1+I*a+I*b*x)*ln(-b*(-x*(-c)^(1/2)+d^(1/2))/(I*(-c)^(1/2)-a*(-c)
^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*I*ln(1-I*a-I*b*x)*ln(-b*(x*(-c)^(
1/2)+d^(1/2))/((I+a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*ln(1+I
*a+I*b*x)*ln(b*(x*(-c)^(1/2)+d^(1/2))/(I*(-c)^(1/2)-a*(-c)^(1/2)+b*d^(1/2))
)*d^(1/2)/(-c)^(3/2)-1/4*I*ln(1-I*a-I*b*x)*ln(b*(-x*(-c)^(1/2)+d^(1/2))/(I*
(-c)^(1/2)+a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*I*polylog(2,(I-a
-b*x)*(-c)^(1/2)/(I*(-c)^(1/2)-a*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+
1/4*I*polylog(2,(I+a+b*x)*(-c)^(1/2)/(I*(-c)^(1/2)+a*(-c)^(1/2)-b*d^(1/2)))
*d^(1/2)/(-c)^(3/2)-1/4*I*polylog(2,(1+I*a+I*b*x)*(-c)^(1/2)/((1+I*a)*(-c)^
(1/2)-I*b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*polylog(2,(I+a+b*x)*(-c)^(1/2)
/(I*(-c)^(1/2)+a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5159, 2456, 2436, 2332, 2441, 2440, 2438}

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = {} & \frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+i)}{-\sqrt{-c}a+i\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
& -\frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(ia+ibx+1)}{(ia+1)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}} \\
& +\frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+i)}{\sqrt{-c}a+i\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
& -\frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+i)}{\sqrt{-c}a+i\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
& +\frac{i\sqrt{d}\log(ia+ibx+1)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\left(-\sqrt{-c}\right)-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}} \\
& -\frac{i\sqrt{d}\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{a\left(-\sqrt{-c}\right)+b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}} \\
& -\frac{i\sqrt{d}\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{a\sqrt{-c}+b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}} \\
& +\frac{i\sqrt{d}\log(-ia-ibx+1)\log\left(-\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{-b\sqrt{d}+(a+i)\sqrt{-c}}\right)}{4(-c)^{3/2}} \\
& -\frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc} \\
& -\frac{(-ia-ibx+1)\log(-i(a+bx+i))}{2bc}
\end{aligned}
$$

```
[In] Int[ArcTan[a + b*x]/(c + d/x^2),x]
```

```
[Out] -1/2*((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - ((1 - I*a - I*b*x)*Lo
g[(-I)*(I + a + b*x)])/(2*b*c) + ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[-(
(b*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d]))])/(-c)^(3
/2) - ((I/4)*Sqrt[d]*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I
*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*Log[1 - I
*a - I*b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d])
)])/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] + Sqrt
[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d
```

```
]*PolyLog[2, (Sqrt[-c]*(I - a - b*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])
])/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + I*a + I*b*x))/((1
+ I*a)*Sqrt[-c] - I*b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sq
rt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])])/(-c)^(3/2) -
((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c]
 + b*Sqrt[d])])/(-c)^(3/2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
```

```
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^2}}\,dx-\frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^2}}\,dx\\
&= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c}-\frac{d\log(1-ia-ibx)}{c\,(d+cx^2)}\right)dx\\
&\quad -\frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c}-\frac{d\log(1+ia+ibx)}{c\,(d+cx^2)}\right)dx\\
&= \frac{i\int \log(1-ia-ibx)\,dx}{2c}-\frac{i\int \log(1+ia+ibx)\,dx}{2c}\\
&\quad -\frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx^2}\,dx}{2c}+\frac{(id)\int \frac{\log(1+ia+ibx)}{d+cx^2}\,dx}{2c}\\
&= -\frac{\operatorname{Subst}\left(\int \log(x)\,dx,x,1-ia-ibx\right)}{2bc}-\frac{\operatorname{Subst}\left(\int \log(x)\,dx,x,1+ia+ibx\right)}{2bc}\\
&\quad -\frac{(id)\int \left(\frac{\log(1-ia-ibx)}{2\sqrt{d}\left(\sqrt{d}-\sqrt{-c}x\right)}+\frac{\log(1-ia-ibx)}{2\sqrt{d}\left(\sqrt{d}+\sqrt{-c}x\right)}\right)dx}{2c}\\
&\quad +\frac{(id)\int \left(\frac{\log(1+ia+ibx)}{2\sqrt{d}\left(\sqrt{d}-\sqrt{-c}x\right)}+\frac{\log(1+ia+ibx)}{2\sqrt{d}\left(\sqrt{d}+\sqrt{-c}x\right)}\right)dx}{2c}\\
&= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc}-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}\\
&\quad -\frac{\left(i\sqrt{d}\right)\int \frac{\log(1-ia-ibx)}{\sqrt{d}-\sqrt{-c}x}\,dx}{4c}-\frac{\left(i\sqrt{d}\right)\int \frac{\log(1-ia-ibx)}{\sqrt{d}+\sqrt{-c}x}\,dx}{4c}\\
&\quad +\frac{\left(i\sqrt{d}\right)\int \frac{\log(1+ia+ibx)}{\sqrt{d}-\sqrt{-c}x}\,dx}{4c}+\frac{\left(i\sqrt{d}\right)\int \frac{\log(1+ia+ibx)}{\sqrt{d}+\sqrt{-c}x}\,dx}{4c}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&\quad + \frac{i\sqrt{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&\quad - \frac{i\sqrt{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&\quad + \frac{i\sqrt{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&\quad - \frac{i\sqrt{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{i\sqrt{-c}-a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&\quad + \frac{\left(b\sqrt{d}\right)\int\frac{\log\left(-\frac{ib\left(\sqrt{d}-\sqrt{-c}x\right)}{(1-ia)\sqrt{-c}-ib\sqrt{d}}\right)}{1-ia-ibx}dx}{4(-c)^{3/2}} + \frac{\left(b\sqrt{d}\right)\int\frac{\log\left(\frac{ib\left(\sqrt{d}-\sqrt{-c}x\right)}{(1+ia)\sqrt{-c}+ib\sqrt{d}}\right)}{1+ia+ibx}dx}{4(-c)^{3/2}} \\
&\quad - \frac{\left(b\sqrt{d}\right)\int\frac{\log\left(-\frac{ib\left(\sqrt{d}+\sqrt{-c}x\right)}{-((1-ia)\sqrt{-c})-ib\sqrt{d}}\right)}{1-ia-ibx}dx}{4(-c)^{3/2}} - \frac{\left(b\sqrt{d}\right)\int\frac{\log\left(\frac{ib\left(\sqrt{d}+\sqrt{-c}x\right)}{-((1+ia)\sqrt{-c})+ib\sqrt{d}}\right)}{1+ia+ibx}dx}{4(-c)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&+ \frac{i\sqrt{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&- \frac{i\sqrt{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+ \frac{i\sqrt{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&- \frac{i\sqrt{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{i\sqrt{-c}-a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&- \frac{\left(i\sqrt{d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-c}x}{-((1-ia)\sqrt{-c})-ib\sqrt{d}}\right)}{x}\,dx,x,1-ia-ibx\right)}{4(-c)^{3/2}} \\
&+ \frac{\left(i\sqrt{d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-c}x}{(1-ia)\sqrt{-c}-ib\sqrt{d}}\right)}{x}\,dx,x,1-ia-ibx\right)}{4(-c)^{3/2}} \\
&+ \frac{\left(i\sqrt{d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-c}x}{-((1+ia)\sqrt{-c})+ib\sqrt{d}}\right)}{x}\,dx,x,1+ia+ibx\right)}{4(-c)^{3/2}} \\
&- \frac{\left(i\sqrt{d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-c}x}{(1+ia)\sqrt{-c}+ib\sqrt{d}}\right)}{x}\,dx,x,1+ia+ibx\right)}{4(-c)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&+ \frac{i\sqrt{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&- \frac{i\sqrt{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+ \frac{i\sqrt{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&- \frac{i\sqrt{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{i\sqrt{-c}-a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+ \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i-a-bx)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(1+ia+ibx)}{(1+ia)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+ \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i+a+bx)}{i\sqrt{-c}+a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i+a+bx)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 660, normalized size of antiderivative = 0.99

$$
\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = -\frac{i\Big(2i\sqrt{-c}\log(1+ia+ibx)-2a\sqrt{-c}\log(1+ia+ibx)-2b\sqrt{-c}x\log(1+ia+ibx)+2i\sqrt{-c}\log(-i(i+}{}
$$

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x^2),x]
```

```
[Out] ((-1/4*I)*((2*I)*Sqrt[-c]*Log[1 + I*a + I*b*x] - 2*a*Sqrt[-c]*Log[1 + I*a +
 I*b*x] - 2*b*Sqrt[-c]*x*Log[1 + I*a + I*b*x] + (2*I)*Sqrt[-c]*Log[(-I)*(I
+ a + b*x)] + 2*a*Sqrt[-c]*Log[(-I)*(I + a + b*x)] + 2*b*Sqrt[-c]*x*Log[(-I
)*(I + a + b*x)] - b*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c
]*x))/((-I)*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])] + b*Sqrt[d]*Log[(-I)*(I + a
 + b*x)]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d
])] - b*Sqrt[d]*Log[(-I)*(I + a + b*x)]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((
I + a)*Sqrt[-c] - b*Sqrt[d]))] + b*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqr
t[d] + Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])] + b*Sqrt[d]*Poly
Log[2, (Sqrt[-c]*(-I + a + b*x))/((-I)*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])]
- b*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(-I + a + b*x))/((-I)*Sqrt[-c] + a*Sqrt[-c
```

```
] + b*Sqrt[d])] - b*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c]
 + a*Sqrt[-c] - b*Sqrt[d])] + b*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))
/(I*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])]))/(b*(-c)^(3/2))
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 647, normalized size of antiderivative = 0.97

method	result
risch	$\frac{i\ln(-bxi-ia+1)x}{2c}+\frac{i\ln(-bxi-ia+1)a}{2bc}-\frac{i\ln(bxi+ia+1)x}{2c}-\frac{i\ln(bxi+ia+1)a}{2bc}-\frac{\ln(-bxi-ia+1)}{2bc}+\frac{1}{bc}-$ ln(−
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arctan(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/c*ln(1-I*a-I*b*x)*x+1/2*I/b/c*ln(1-I*a-I*b*x)*a-1/2*I/c*ln(1+I*a+I*b*
x)*x-1/2*I/b/c*ln(1+I*a+I*b*x)*a-1/2/b/c*ln(1-I*a-I*b*x)+1/b/c-1/4/c^2*ln(1
-I*a-I*b*x)*(c*d)^(1/2)*ln((I*a*c-b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c-b
*(c*d)^(1/2)-c))+1/4/c^2*ln(1-I*a-I*b*x)*(c*d)^(1/2)*ln((I*a*c+b*(c*d)^(1/2
)+(1-I*a-I*b*x)*c-c)/(I*a*c+b*(c*d)^(1/2)-c))-1/4/c^2*dilog((I*a*c-b*(c*d)^
(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c-b*(c*d)^(1/2)-c))*(c*d)^(1/2)+1/4/c^2*dilog
((I*a*c+b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c+b*(c*d)^(1/2)-c))*(c*d)^(1/
2)-1/2/b/c*ln(1+I*a+I*b*x)-1/4/c^2*ln(1+I*a+I*b*x)*(c*d)^(1/2)*ln((I*a*c+b*
(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c+b*(c*d)^(1/2)+c))+1/4/c^2*ln(1+I*a+I*
b*x)*(c*d)^(1/2)*ln((I*a*c-b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c-b*(c*d)^
(1/2)+c))-1/4/c^2*(c*d)^(1/2)*dilog((I*a*c+b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)
/(I*a*c+b*(c*d)^(1/2)+c))+1/4/c^2*(c*d)^(1/2)*dilog((I*a*c-b*(c*d)^(1/2)-(1
+I*a+I*b*x)*c+c)/(I*a*c-b*(c*d)^(1/2)+c))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(b*x + a)/(c*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Timed out}$$

```
[In] integrate(atan(b*x+a)/(c+d/x**2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8518 vs. $2(466) = 932$.

Time = 0.91 (sec) , antiderivative size = 8518, normalized size of antiderivative = 12.75

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] -(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arctan(b*x + a) + 1/8*(8*a*c
*arctan(b*x + a) + (4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d + (a
*b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*
b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2
 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d
+ (a^4 + 2*a^2 + 1)*c^2 + (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*s
qrt(d) + (a*b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d +
(a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + 4*b*a
rctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d - (a*b^3*d + (a^3 + a)*b*c +
(b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*c^2
)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4*(b^3*d + (a
^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 -
 (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^
3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4
*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + b*log(c*x^2 + d)*log(((a^2 + 1
)*b^22*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(a^6 + 39*a^4 + 1
71*a^2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 22
61)*b^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969
)*b^14*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a
^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10
 + 75075*a^8 + 169455*a^6 + 201773*a^4 + 121771*a^2 + 29393)*b^10*c^7*d^5 +
 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9438*a^8 + 13728*a^6 + 11492*
a^4 + 5168*a^2 + 969)*b^8*c^8*d^4 + 33*(5*a^18 + 285*a^16 + 3220*a^14 + 158
76*a^12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 43860*a^4 + 15181*a^2 + 2261
```

```
)*b^6*c^9*d^3 + 55*(a^20 + 46*a^18 + 465*a^16 + 2184*a^14 + 5922*a^12 + 101
64*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102*a^2 + 133)*b^4*c^10*d^2 +
11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730*a^14 + 4662*a^12 + 5502*a^
10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 21)*b^2*c^11*d + (a^24 + 12*
a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495
*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*c^12 + (b^24*c*d^11 + 11*(a^2 + 21)*b
^22*c^2*d^10 + 55*(a^4 + 38*a^2 + 133)*b^20*c^3*d^9 + 33*(5*a^6 + 255*a^4 +
 1615*a^2 + 2261)*b^18*c^4*d^8 + 330*(a^8 + 60*a^6 + 510*a^4 + 1292*a^2 + 9
69)*b^16*c^5*d^7 + 22*(21*a^10 + 1365*a^8 + 13650*a^6 + 46410*a^4 + 62985*a
^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6
+ 109395*a^4 + 92378*a^2 + 29393)*b^12*c^7*d^5 + 330*(a^14 + 63*a^12 + 693*
a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2 + 969)*b^10*c^8*d^4 + 33*(
5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30030*a^8 + 40040*a^6 + 30940*
a^4 + 12920*a^2 + 2261)*b^8*c^9*d^3 + 55*(a^18 + 45*a^16 + 420*a^14 + 1764*
a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^4 + 969*a^2 + 133)*b^6*c^10
*d^2 + 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14 + 1890*a^12 + 2772*a^10 + 2
730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4*c^11*d + (a^22 + 11*a^20 +
 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 +
55*a^4 + 11*a^2 + 1)*b^2*c^12)*x^2 + 2*(11*(a^2 + 1)*b^21*c*d^10 + 110*(a^4
 + 8*a^2 + 7)*b^19*c^2*d^9 + 33*(15*a^6 + 205*a^4 + 589*a^2 + 399)*b^17*c^3
*d^8 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 + 323)*b^15*c^4*d^7 + 110*(2
1*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^2 + 2261)*b^13*c^5*d^6 + 4*
(693*a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6 + 449735*a^4 + 319124*a^2
+ 88179)*b^11*c^6*d^5 + 110*(21*a^14 + 483*a^12 + 3465*a^10 + 11583*a^8 + 2
0735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^7*d^4 + 264*(5*a^16 + 110*a^
14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6 + 4930*a^4 + 1938*a^2 + 323
)*b^7*c^8*d^3 + 33*(15*a^18 + 295*a^16 + 2044*a^14 + 7308*a^12 + 15554*a^10
 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 + 399)*b^5*c^9*d^2 + 110*(a^
20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1008*a^10 + 966*a^8 + 624*a^
6 + 261*a^4 + 64*a^2 + 7)*b^3*c^10*d + 11*(a^22 + 11*a^20 + 55*a^18 + 165*a
^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2
+ 1)*b*c^11 + (11*b^23*c*d^10 + 110*(a^2 + 7)*b^21*c^2*d^9 + 33*(15*a^4 + 1
90*a^2 + 399)*b^19*c^3*d^8 + 264*(5*a^6 + 85*a^4 + 323*a^2 + 323)*b^17*c^4*
d^7 + 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^2 + 2261)*b^15*c^5*d^6 + 4*
(693*a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 + 230945*a^2 + 88179)*b^13*c
^6*d^5 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8580*a^6 + 12155*a^4 + 8398*a
^2 + 2261)*b^11*c^7*d^4 + 264*(5*a^14 + 105*a^12 + 693*a^10 + 2145*a^8 + 35
75*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^8*d^3 + 33*(15*a^16 + 280*a^14 +
1764*a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 + 7140*a^4 + 2584*a^2 + 399)*
b^7*c^9*d^2 + 110*(a^18 + 15*a^16 + 84*a^14 + 252*a^12 + 462*a^10 + 546*a^8
 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c^10*d + 11*(a^20 + 10*a^18 + 45*a^1
6 + 120*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 +
1)*b^3*c^11)*x^2 + 2*(11*a*b^22*c*d^10 + 110*(a^3 + 7*a)*b^20*c^2*d^9 + 33*
(15*a^5 + 190*a^3 + 399*a)*b^18*c^3*d^8 + 264*(5*a^7 + 85*a^5 + 323*a^3 + 3
23*a)*b^16*c^4*d^7 + 110*(21*a^9 + 420*a^7 + 2142*a^5 + 3876*a^3 + 2261*a)*
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b^14*c^5*d^6 + 4*(693*a^11 + 15015*a^9 + 90090*a^7 + 218790*a^5 + 230945*a^
3 + 88179*a)*b^12*c^6*d^5 + 110*(21*a^13 + 462*a^11 + 3003*a^9 + 8580*a^7 +
 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^7*d^4 + 264*(5*a^15 + 105*a^13 + 693
*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^3 + 323*a)*b^8*c^8*d^3 + 33
*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 10010*a^9 + 10920*a^7 + 7140
*a^5 + 2584*a^3 + 399*a)*b^6*c^9*d^2 + 110*(a^19 + 15*a^17 + 84*a^15 + 252*
a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 + 57*a^3 + 7*a)*b^4*c^10*d +
11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^13 + 252*a^11 + 210*a^9 + 1
20*a^7 + 45*a^5 + 10*a^3 + a)*b^2*c^11)*x)*sqrt(c)*sqrt(d) + 2*(a*b^23*c*d^
11 + 11*(a^3 + 21*a)*b^21*c^2*d^10 + 55*(a^5 + 38*a^3 + 133*a)*b^19*c^3*d^9
 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17*c^4*d^8 + 330*(a^9 + 60*a^
7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^5*d^7 + 22*(21*a^11 + 1365*a^9 + 136
50*a^7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c^6*d^6 + 22*(21*a^13 + 1386
*a^11 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 92378*a^3 + 29393*a)*b^11*c^7*
d^5 + 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9 + 6435*a^7 + 7293*a^5 + 419
9*a^3 + 969*a)*b^9*c^8*d^4 + 33*(5*a^17 + 280*a^15 + 2940*a^13 + 12936*a^11
 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3 + 2261*a)*b^7*c^9*d^3 + 55
*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a^11 + 6006*a^9 + 5460*a^7 +
 3060*a^5 + 969*a^3 + 133*a)*b^5*c^10*d^2 + 11*(a^21 + 30*a^19 + 225*a^17 +
 840*a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1800*a^7 + 765*a^5 + 190*a^3
 + 21*a)*b^3*c^11*d + (a^23 + 11*a^21 + 55*a^19 + 165*a^17 + 330*a^15 + 462
*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 + 11*a^3 + a)*b*c^12)*x)/(b^2
4*d^12 + 12*(a^2 + 23)*b^22*c*d^11 + 66*(a^4 + 42*a^2 + 161)*b^20*c^2*d^10
+ 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^3*d^9 + 99*(5*a^8 + 340*a^6
 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^4*d^8 + 264*(3*a^10 + 225*a^8 + 2550*
a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*c^5*d^7 + 4*(231*a^12 + 18018*a^10
+ 225225*a^8 + 1021020*a^6 + 2078505*a^4 + 1939938*a^2 + 676039)*b^12*c^6*d
^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*a^8 + 36465*a^6 + 46189*a^4
 + 29393*a^2 + 7429)*b^10*c^7*d^5 + 99*(5*a^16 + 360*a^14 + 4620*a^12 + 240
24*a^10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38760*a^2 + 7429)*b^8*c^8*d^4
 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^12 + 54054*a^10 + 90090*a^8
+ 92820*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*c^9*d^3 + 66*(a^20 + 50*a^1
8 + 525*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^10 + 13650*a^8 + 10200*a^6 +
 4845*a^4 + 1330*a^2 + 161)*b^4*c^10*d^2 + 12*(a^22 + 33*a^20 + 275*a^18 +
1155*a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 + 4950*a^8 + 2805*a^6 + 1045*
a^4 + 231*a^2 + 23)*b^2*c^11*d + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495
*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^
2 + 1)*c^12 + 8*(3*b^23*d^11 + 11*(3*a^2 + 23)*b^21*c*d^10 + 33*(5*a^4 + 70
*a^2 + 161)*b^19*c^2*d^9 + 99*(5*a^6 + 95*a^4 + 399*a^2 + 437)*b^17*c^3*d^8
 + 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2 + 7429)*b^15*c^4*d^7 + 6*(2
31*a^10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 124355*a^2 + 52003)*b^13*c^5*
d^6 + 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 145860*a^6 + 230945*a^4 + 17635
8*a^2 + 52003)*b^11*c^6*d^5 + 22*(45*a^14 + 1155*a^12 + 9009*a^10 + 32175*a
^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*b^9*c^7*d^4 + 99*(5*a^16 + 1
20*a^14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 8840*a^6 + 6460*a^4 + 2584*a^2
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+ 437)*b^7*c^8*d^3 + 33*(5*a^18 + 105*a^16 + 756*a^14 + 2772*a^12 + 6006*a^
10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2 + 161)*b^5*c^9*d^2 + 11*(3*a
^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^12 + 3276*a^10 + 3150*a^8 + 2
040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c^10*d + 3*(a^22 + 11*a^20 + 55*a^18
+ 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 +
11*a^2 + 1)*b*c^11)*sqrt(c)*sqrt(d))) - b*log(c*x^2 + d)*log(((a^2 + 1)*b^2
2*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(a^6 + 39*a^4 + 171*a^
2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b
^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^1
4*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 +
92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10 + 75
075*a^8 + 169455*a^6 + 201773*a^4 + 121771*a^2 + 29393)*b^10*c^7*d^5 + 330*
(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9438*a^8 + 13728*a^6 + 11492*a^4 +
 5168*a^2 + 969)*b^8*c^8*d^4 + 33*(5*a^18 + 285*a^16 + 3220*a^14 + 15876*a^
12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 43860*a^4 + 15181*a^2 + 2261)*b^6
*c^9*d^3 + 55*(a^20 + 46*a^18 + 465*a^16 + 2184*a^14 + 5922*a^12 + 10164*a^
10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102*a^2 + 133)*b^4*c^10*d^2 + 11*(a
^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730*a^14 + 4662*a^12 + 5502*a^10 +
4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 21)*b^2*c^11*d + (a^24 + 12*a^22
+ 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8
+ 220*a^6 + 66*a^4 + 12*a^2 + 1)*c^12 + (b^24*c*d^11 + 11*(a^2 + 21)*b^22*c
^2*d^10 + 55*(a^4 + 38*a^2 + 133)*b^20*c^3*d^9 + 33*(5*a^6 + 255*a^4 + 1615
*a^2 + 2261)*b^18*c^4*d^8 + 330*(a^8 + 60*a^6 + 510*a^4 + 1292*a^2 + 969)*b
^16*c^5*d^7 + 22*(21*a^10 + 1365*a^8 + 13650*a^6 + 46410*a^4 + 62985*a^2 +
29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109
395*a^4 + 92378*a^2 + 29393)*b^12*c^7*d^5 + 330*(a^14 + 63*a^12 + 693*a^10
+ 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2 + 969)*b^10*c^8*d^4 + 33*(5*a^1
6 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30030*a^8 + 40040*a^6 + 30940*a^4 +
 12920*a^2 + 2261)*b^8*c^9*d^3 + 55*(a^18 + 45*a^16 + 420*a^14 + 1764*a^12
+ 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^4 + 969*a^2 + 133)*b^6*c^10*d^2
+ 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14 + 1890*a^12 + 2772*a^10 + 2730*a
^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4*c^11*d + (a^22 + 11*a^20 + 55*a
^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^
4 + 11*a^2 + 1)*b^2*c^12)*x^2 - 2*(11*(a^2 + 1)*b^21*c*d^10 + 110*(a^4 + 8*
a^2 + 7)*b^19*c^2*d^9 + 33*(15*a^6 + 205*a^4 + 589*a^2 + 399)*b^17*c^3*d^8
+ 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 + 323)*b^15*c^4*d^7 + 110*(21*a^1
0 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^2 + 2261)*b^13*c^5*d^6 + 4*(693*
a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6 + 449735*a^4 + 319124*a^2 + 881
79)*b^11*c^6*d^5 + 110*(21*a^14 + 483*a^12 + 3465*a^10 + 11583*a^8 + 20735*
a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^7*d^4 + 264*(5*a^16 + 110*a^14 +
798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6 + 4930*a^4 + 1938*a^2 + 323)*b^7
*c^8*d^3 + 33*(15*a^18 + 295*a^16 + 2044*a^14 + 7308*a^12 + 15554*a^10 + 20
930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 + 399)*b^5*c^9*d^2 + 110*(a^20 +
16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1008*a^10 + 966*a^8 + 624*a^6 + 2
61*a^4 + 64*a^2 + 7)*b^3*c^10*d + 11*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 +
```

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 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*
b*c^11 + (11*b^23*c*d^10 + 110*(a^2 + 7)*b^21*c^2*d^9 + 33*(15*a^4 + 190*a^
2 + 399)*b^19*c^3*d^8 + 264*(5*a^6 + 85*a^4 + 323*a^2 + 323)*b^17*c^4*d^7 +
 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^2 + 2261)*b^15*c^5*d^6 + 4*(693*
a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 + 230945*a^2 + 88179)*b^13*c^6*d^
5 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8580*a^6 + 12155*a^4 + 8398*a^2 +
2261)*b^11*c^7*d^4 + 264*(5*a^14 + 105*a^12 + 693*a^10 + 2145*a^8 + 3575*a^
6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^8*d^3 + 33*(15*a^16 + 280*a^14 + 1764*
a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 + 7140*a^4 + 2584*a^2 + 399)*b^7*c
^9*d^2 + 110*(a^18 + 15*a^16 + 84*a^14 + 252*a^12 + 462*a^10 + 546*a^8 + 42
0*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c^10*d + 11*(a^20 + 10*a^18 + 45*a^16 + 1
20*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^
3*c^11)*x^2 + 2*(11*a*b^22*c*d^10 + 110*(a^3 + 7*a)*b^20*c^2*d^9 + 33*(15*a
^5 + 190*a^3 + 399*a)*b^18*c^3*d^8 + 264*(5*a^7 + 85*a^5 + 323*a^3 + 323*a)
*b^16*c^4*d^7 + 110*(21*a^9 + 420*a^7 + 2142*a^5 + 3876*a^3 + 2261*a)*b^14*
c^5*d^6 + 4*(693*a^11 + 15015*a^9 + 90090*a^7 + 218790*a^5 + 230945*a^3 + 8
8179*a)*b^12*c^6*d^5 + 110*(21*a^13 + 462*a^11 + 3003*a^9 + 8580*a^7 + 1215
5*a^5 + 8398*a^3 + 2261*a)*b^10*c^7*d^4 + 264*(5*a^15 + 105*a^13 + 693*a^11
 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^3 + 323*a)*b^8*c^8*d^3 + 33*(15*
a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 10010*a^9 + 10920*a^7 + 7140*a^5
+ 2584*a^3 + 399*a)*b^6*c^9*d^2 + 110*(a^19 + 15*a^17 + 84*a^15 + 252*a^13
+ 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 + 57*a^3 + 7*a)*b^4*c^10*d + 11*(a
^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^13 + 252*a^11 + 210*a^9 + 120*a^
7 + 45*a^5 + 10*a^3 + a)*b^2*c^11)*x)*sqrt(c)*sqrt(d) + 2*(a*b^23*c*d^11 +
11*(a^3 + 21*a)*b^21*c^2*d^10 + 55*(a^5 + 38*a^3 + 133*a)*b^19*c^3*d^9 + 33
*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17*c^4*d^8 + 330*(a^9 + 60*a^7 + 5
10*a^5 + 1292*a^3 + 969*a)*b^15*c^5*d^7 + 22*(21*a^11 + 1365*a^9 + 13650*a^
7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c^6*d^6 + 22*(21*a^13 + 1386*a^11
 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 92378*a^3 + 29393*a)*b^11*c^7*d^5 +
 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9 + 6435*a^7 + 7293*a^5 + 4199*a^3
 + 969*a)*b^9*c^8*d^4 + 33*(5*a^17 + 280*a^15 + 2940*a^13 + 12936*a^11 + 30
030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3 + 2261*a)*b^7*c^9*d^3 + 55*(a^1
9 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a^11 + 6006*a^9 + 5460*a^7 + 3060
*a^5 + 969*a^3 + 133*a)*b^5*c^10*d^2 + 11*(a^21 + 30*a^19 + 225*a^17 + 840*
a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1800*a^7 + 765*a^5 + 190*a^3 + 21
*a)*b^3*c^11*d + (a^23 + 11*a^21 + 55*a^19 + 165*a^17 + 330*a^15 + 462*a^13
 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 + 11*a^3 + a)*b*c^12)*x)/(b^24*d^1
2 + 12*(a^2 + 23)*b^22*c*d^11 + 66*(a^4 + 42*a^2 + 161)*b^20*c^2*d^10 + 44*
(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^3*d^9 + 99*(5*a^8 + 340*a^6 + 32
30*a^4 + 9044*a^2 + 7429)*b^16*c^4*d^8 + 264*(3*a^10 + 225*a^8 + 2550*a^6 +
 9690*a^4 + 14535*a^2 + 7429)*b^14*c^5*d^7 + 4*(231*a^12 + 18018*a^10 + 225
225*a^8 + 1021020*a^6 + 2078505*a^4 + 1939938*a^2 + 676039)*b^12*c^6*d^6 +
264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*a^8 + 36465*a^6 + 46189*a^4 + 29
393*a^2 + 7429)*b^10*c^7*d^5 + 99*(5*a^16 + 360*a^14 + 4620*a^12 + 24024*a^
10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38760*a^2 + 7429)*b^8*c^8*d^4 + 44
```

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*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^12 + 54054*a^10 + 90090*a^8 + 928
20*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*c^9*d^3 + 66*(a^20 + 50*a^18 + 5
25*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^10 + 13650*a^8 + 10200*a^6 + 4845
*a^4 + 1330*a^2 + 161)*b^4*c^10*d^2 + 12*(a^22 + 33*a^20 + 275*a^18 + 1155*
a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 + 4950*a^8 + 2805*a^6 + 1045*a^4 +
 231*a^2 + 23)*b^2*c^11*d + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16
 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1
)*c^12 - 8*(3*b^23*d^11 + 11*(3*a^2 + 23)*b^21*c*d^10 + 33*(5*a^4 + 70*a^2
+ 161)*b^19*c^2*d^9 + 99*(5*a^6 + 95*a^4 + 399*a^2 + 437)*b^17*c^3*d^8 + 22
*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2 + 7429)*b^15*c^4*d^7 + 6*(231*a^
10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 124355*a^2 + 52003)*b^13*c^5*d^6 +
 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 145860*a^6 + 230945*a^4 + 176358*a^2
 + 52003)*b^11*c^6*d^5 + 22*(45*a^14 + 1155*a^12 + 9009*a^10 + 32175*a^8 +
60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*b^9*c^7*d^4 + 99*(5*a^16 + 120*a^
14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 8840*a^6 + 6460*a^4 + 2584*a^2 + 437
)*b^7*c^8*d^3 + 33*(5*a^18 + 105*a^16 + 756*a^14 + 2772*a^12 + 6006*a^10 +
8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2 + 161)*b^5*c^9*d^2 + 11*(3*a^20 +
 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^12 + 3276*a^10 + 3150*a^8 + 2040*a
^6 + 855*a^4 + 210*a^2 + 23)*b^3*c^10*d + 3*(a^22 + 11*a^20 + 55*a^18 + 165
*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^
2 + 1)*b*c^11)*sqrt(c)*sqrt(d))) + 2*b*dilog(((a + I)*b*c*x + b^2*d + (I*b^
2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + 1)*b*sqrt(c)*sqrt(d) + b^2*
d - (a^2 + 2*I*a - 1)*c)) - 2*b*dilog(-((a + I)*b*c*x + b^2*d - (I*b^2*x +
(-I*a + 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + 1)*b*sqrt(c)*sqrt(d) - b^2*d + (a
^2 + 2*I*a - 1)*c)) - 2*b*dilog(((a - I)*b*c*x + b^2*d + (I*b^2*x + (-I*a -
 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - 1)*b*sqrt(c)*sqrt(d) + b^2*d - (a^2 - 2*
I*a - 1)*c)) + 2*b*dilog(-((a - I)*b*c*x + b^2*d - (I*b^2*x + (-I*a - 1)*b)
*sqrt(c)*sqrt(d))/(2*(-I*a - 1)*b*sqrt(c)*sqrt(d) - b^2*d + (a^2 - 2*I*a -
1)*c)))*sqrt(c)*sqrt(d) - 4*c*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b*c^2)
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

```
[In] int(atan(a + b*x)/(c + d/x^2),x)
[Out] int(atan(a + b*x)/(c + d/x^2), x)
```

3.57 $\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 933

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = & -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} \\
& -\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
& -\frac{i\sqrt[3]{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& +\frac{i\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& -\frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& +\frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& -\frac{(-1)^{5/6}\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& +\frac{(-1)^{5/6}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{\sqrt[6]{-1}(1-ia)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& -\frac{\sqrt[6]{-1}\sqrt[3]{d}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{c}(i-a-bx)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& -\frac{(-1)^{5/6}\sqrt[3]{d}\,\text{PolyLog}\left(2,\frac{\sqrt[6]{-1}\sqrt[3]{c}(i-a-bx)}{\sqrt[6]{-1}(i-a)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& +\frac{i\sqrt[3]{d}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{c}(i-a-bx)}{(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& -\frac{i\sqrt[3]{d}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{c}(i+a+bx)}{(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& +\frac{(-1)^{5/6}\sqrt[3]{d}\,\text{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{c}(i+a+bx)}{(-1)^{2/3}(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
& +\frac{\sqrt[6]{-1}\sqrt[3]{d}\,\text{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{c}(i+a+bx)}{\sqrt[3]{-1}(i+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{}
\end{aligned}
$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c-1/6*I*d^(1/3)*ln(1-I*a-I*b*x)*ln(-b*(d^(1/3)+c^(1/3)*x)/((I+a)*c^(1/3)-b
*d^(1/3)))/c^(4/3)+1/6*I*d^(1/3)*ln(1+I*a+I*b*x)*ln(b*(d^(1/3)+c^(1/3)*x)/(
(I-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(1/6)*d^(1/3)*ln(1+I*a+I*b*x)*ln
(-b*(d^(1/3)-(-1)^(1/3)*c^(1/3)*x)/((-1)^(1/3)*(I-a)*c^(1/3)-b*d^(1/3)))/c^
(4/3)+1/6*(-1)^(1/6)*d^(1/3)*ln(1-I*a-I*b*x)*ln(b*(d^(1/3)-(-1)^(1/3)*c^(1/
3)*x)/((-1)^(1/3)*(I+a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(5/6)*d^(1/3)*
ln(1+I*a+I*b*x)*ln(b*(d^(1/3)+(-1)^(2/3)*c^(1/3)*x)/((-1)^(2/3)*(I-a)*c^(1/
3)+b*d^(1/3)))/c^(4/3)+1/6*(-1)^(5/6)*d^(1/3)*ln(1-I*a-I*b*x)*ln(b*(d^(1/3)
+(-1)^(2/3)*c^(1/3)*x)/((-1)^(1/6)*(1-I*a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*
(-1)^(1/6)*d^(1/3)*polylog(2,(-1)^(1/3)*c^(1/3)*(I-a-b*x)/((-1)^(1/3)*(I-a)
*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-1)^(5/6)*d^(1/3)*polylog(2,(-1)^(1/6)*c^
(1/3)*(I-a-b*x)/((-1)^(1/6)*(I-a)*c^(1/3)-I*b*d^(1/3)))/c^(4/3)+1/6*I*d^(1/
3)*polylog(2,c^(1/3)*(I-a-b*x)/((I-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*I*d^(
1/3)*polylog(2,c^(1/3)*(I+a+b*x)/((I+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1
)^(5/6)*d^(1/3)*polylog(2,(-1)^(2/3)*c^(1/3)*(I+a+b*x)/((-1)^(2/3)*(I+a)*c^
(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1)^(1/6)*d^(1/3)*polylog(2,(-1)^(1/3)*c^(1/
3)*(I+a+b*x)/((-1)^(1/3)*(I+a)*c^(1/3)+b*d^(1/3)))/c^(4/3)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used

$= \{5159,\ 2456,\ 2436,\ 2332,\ 2441,\ 2440,\ 2438\}$

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = &-\frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc} \\
&+\frac{i\sqrt[3]{d}\log\left(\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{\sqrt[3]{c}(i-a)+b\sqrt[3]{d}}\right)\log(ia+ibx+1)}{6c^{4/3}} \\
&-\frac{\sqrt[6]{-1}\sqrt[3]{d}\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)\log(ia+ibx+1)}{6c^{4/3}} \\
&-\frac{(-1)^{5/6}\sqrt[3]{d}\log\left(\frac{b\left((-1)^{2/3}\sqrt[3]{c}x+\sqrt[3]{d}\right)}{(-1)^{2/3}\sqrt[3]{c}(i-a)+b\sqrt[3]{d}}\right)\log(ia+ibx+1)}{6c^{4/3}} \\
&-\frac{(-ia-ibx+1)\log(-i(a+bx+i))}{2bc} \\
&-\frac{i\sqrt[3]{d}\log(-ia-ibx+1)\log\left(-\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{(a+i)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{\sqrt[6]{-1}\sqrt[3]{d}\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}\sqrt[3]{c}(a+i)+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{(-1)^{5/6}\sqrt[3]{d}\log(-ia-ibx+1)\log\left(\frac{b\left((-1)^{2/3}\sqrt[3]{c}x+\sqrt[3]{d}\right)}{\sqrt[6]{-1}\sqrt[3]{c}(1-ia)+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&-\frac{\sqrt[6]{-1}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{c}(-a-bx+i)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&-\frac{(-1)^{5/6}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[6]{-1}\sqrt[3]{c}(-a-bx+i)}{\sqrt[6]{-1}(i-a)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{i\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{c}(-a-bx+i)}{\sqrt[3]{c}(i-a)+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&-\frac{i\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{c}(a+bx+i)}{(a+i)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{(-1)^{5/6}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{c}(a+bx+i)}{(-1)^{2/3}(a+i)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{\sqrt[6]{-1}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{c}(a+bx+i)}{\sqrt[3]{-1}\sqrt[3]{c}(a+i)+b\sqrt[3]{d}}\right)}{6c^{4/3}}
\end{aligned}
$$

```
[In] Int[ArcTan[a + b*x]/(c + d/x^3),x]
```

```
[Out] -1/2*((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - ((1 - I*a - I*b*x)*Lo
g[(-I)*(I + a + b*x)])/(2*b*c) - ((I/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[-(
(b*(d^(1/3) + c^(1/3)*x))/((I + a)*c^(1/3) - b*d^(1/3)))])/c^(4/3) + ((I/6)
*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/((I - a)*c^(1/3
) + b*d^(1/3))])/c^(4/3) - ((-1)^(1/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[-((
b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1/3)
))])/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3)
 - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(
4/3)) - ((-1)^(5/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/
3)*c^(1/3)*x))/((-1)^(2/3)*(I - a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) + ((-
1)^(5/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*
x))/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(1/6)*
d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I - a - b*x))/((-1)^(1/3)*(I - a)*c
^(1/3) - b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(5/6)*d^(1/3)*PolyLog[2, ((-1)^(1
/6)*c^(1/3)*(I - a - b*x))/((-1)^(1/6)*(I - a)*c^(1/3) - I*b*d^(1/3))])/(6*
c^(4/3)) + ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I - a - b*x))/((I - a)*c^(1/
3) + b*d^(1/3))])/c^(4/3) - ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x
))/((I + a)*c^(1/3) - b*d^(1/3))])/c^(4/3) + ((-1)^(5/6)*d^(1/3)*PolyLog[2,
 ((-1)^(2/3)*c^(1/3)*(I + a + b*x))/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3)
)])/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I + a
 + b*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^3}}\,dx-\frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^3}}\,dx\\
&= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c}-\frac{d\log(1-ia-ibx)}{c\,(d+cx^3)}\right)dx\\
&\quad -\frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c}-\frac{d\log(1+ia+ibx)}{c\,(d+cx^3)}\right)dx\\
&= \frac{i\int \log(1-ia-ibx)\,dx}{2c}-\frac{i\int \log(1+ia+ibx)\,dx}{2c}\\
&\quad -\frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx^3}\,dx}{2c}+\frac{(id)\int \frac{\log(1+ia+ibx)}{d+cx^3}\,dx}{2c}\\
&= -\frac{\text{Subst}(\int \log(x)\,dx,x,1-ia-ibx)}{2bc}-\frac{\text{Subst}(\int \log(x)\,dx,x,1+ia+ibx)}{2bc}\\
&\quad -\frac{(id)\int \left(-\frac{\log(1-ia-ibx)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{c}x\right)}-\frac{\log(1-ia-ibx)}{3d^{2/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{c}x\right)}-\frac{\log(1-ia-ibx)}{3d^{2/3}\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{c}x\right)}\right)dx}{2c}\\
&\quad +\frac{(id)\int \left(-\frac{\log(1+ia+ibx)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{c}x\right)}-\frac{\log(1+ia+ibx)}{3d^{2/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{c}x\right)}-\frac{\log(1+ia+ibx)}{3d^{2/3}\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{c}x\right)}\right)dx}{2c}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&\quad + \frac{\left(i\sqrt[3]{d}\right)\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{d}-\sqrt[3]{c}x}\,dx}{6c} + \frac{\left(i\sqrt[3]{d}\right)\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{c}x}\,dx}{6c} \\
&\quad + \frac{\left(i\sqrt[3]{d}\right)\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{c}x}\,dx}{6c} - \frac{\left(i\sqrt[3]{d}\right)\int \frac{\log(1+ia+ibx)}{-\sqrt[3]{d}-\sqrt[3]{c}x}\,dx}{6c} \\
&\quad - \frac{\left(i\sqrt[3]{d}\right)\int \frac{\log(1+ia+ibx)}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{c}x}\,dx}{6c} - \frac{\left(i\sqrt[3]{d}\right)\int \frac{\log(1+ia+ibx)}{-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{c}x}\,dx}{6c}
\end{aligned}
$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}$$

$$- \frac{i\sqrt[3]{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}$$

$$+ \frac{i\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$

$$- \frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}$$

$$+ \frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$

$$- \frac{(-1)^{5/6}\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$

$$+ \frac{(-1)^{5/6}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{\sqrt[6]{-1}(1-ia)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$

$$+ \frac{\left(b\sqrt[3]{d}\right)\int\frac{\log\left(\frac{ib\left(-\sqrt[3]{d}-\sqrt[3]{c}x\right)}{(1+ia)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{1+ia+ibx}dx}{6c^{4/3}} + \frac{\left(b\sqrt[3]{d}\right)\int\frac{\log\left(-\frac{ib\left(-\sqrt[3]{d}-\sqrt[3]{c}x\right)}{(1-ia)\sqrt[3]{c}+ib\sqrt[3]{d}}\right)}{1-ia-ibx}dx}{6c^{4/3}}$$

$$- \frac{\left(\sqrt[3]{-1}b\sqrt[3]{d}\right)\int\frac{\log\left(\frac{ib\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(1+ia)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{1+ia+ibx}dx}{6c^{4/3}}$$

$$- \frac{\left(\sqrt[3]{-1}b\sqrt[3]{d}\right)\int\frac{\log\left(-\frac{ib\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(1-ia)\sqrt[3]{c}+ib\sqrt[3]{d}}\right)}{1-ia-ibx}dx}{6c^{4/3}}$$

$$+ \frac{\left((-1)^{2/3}b\sqrt[3]{d}\right)\int\frac{\log\left(\frac{ib\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{c}x\right)}{-\sqrt[3]{-1}(1+ia)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{1+ia+ibx}dx}{6c^{4/3}}$$

$$\left((-1)^{2/3}b\sqrt[3]{d}\right)\int\frac{\log\left(-\frac{ib\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{c}x\right)}{-\sqrt[3]{-1}(1-ia)\sqrt[3]{c}+ib\sqrt[3]{d}}\right)}{1-ia-ibx}dx$$

$$
\begin{aligned}
&= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&- \frac{i\sqrt[3]{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{i\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&- \frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&- \frac{(-1)^{5/6}\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{(-1)^{5/6}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{\sqrt[6]{-1}(1-ia)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&- \frac{\left(i\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[3]{c}x}{(1+ia)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{x}\,dx,x,1+ia+ibx\right)}{6c^{4/3}} \\
&+ \frac{\left(i\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt[3]{c}x}{(1-ia)\sqrt[3]{c}+ib\sqrt[3]{d}}\right)}{x}\,dx,x,1-ia-ibx\right)}{6c^{4/3}} \\
&+ \frac{\left(\sqrt[6]{-1}\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{c}x}{-\sqrt[3]{-1}(1+ia)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{x}\,dx,x,1+ia+ibx\right)}{6c^{4/3}} \\
&- \frac{\left(\sqrt[6]{-1}\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{c}x}{-\sqrt[3]{-1}(1-ia)\sqrt[3]{c}+ib\sqrt[3]{d}}\right)}{x}\,dx,x,1-ia-ibx\right)}{6c^{4/3}} \\
&\left((-1)^{5/6}\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{(-1)^{2/3}\sqrt[3]{c}x}{(-1)^{2/3}(1+ia)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{x}\,dx,x,1+ia+ibx\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&- \frac{i\sqrt[3]{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{i\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&- \frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&- \frac{(-1)^{5/6}\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{(-1)^{5/6}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{\sqrt[6]{-1}(1-ia)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&- \frac{\sqrt[6]{-1}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2, \frac{\sqrt[3]{-1}\sqrt[3]{c}(i-a-bx)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&- \frac{(-1)^{5/6}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2, \frac{\sqrt[6]{-1}\sqrt[3]{c}(i-a-bx)}{\sqrt[6]{-1}(i-a)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{i\sqrt[3]{d}\,\mathrm{PolyLog}\left(2, \frac{\sqrt[3]{c}(i-a-bx)}{(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} - \frac{i\sqrt[3]{d}\,\mathrm{PolyLog}\left(2, \frac{\sqrt[3]{c}(i+a+bx)}{(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{(-1)^{5/6}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2, \frac{(-1)^{2/3}\sqrt[3]{c}(i+a+bx)}{(-1)^{2/3}(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+ \frac{\sqrt[6]{-1}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2, \frac{\sqrt[3]{-1}\sqrt[3]{c}(i+a+bx)}{\sqrt[3]{-1}(i+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 896, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx$$

$$= \frac{i\left(3i\sqrt[3]{c}\log(1+ia+ibx) - 3a\sqrt[3]{c}\log(1+ia+ibx) - 3b\sqrt[3]{c}x\log(1+ia+ibx) + 3i\sqrt[3]{c}\log(-i(i+a+bx)\right.}{}$$

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x^3),x]
```

```
[Out] ((I/6)*((3*I)*c^(1/3)*Log[1 + I*a + I*b*x] - 3*a*c^(1/3)*Log[1 + I*a + I*b*
x] - 3*b*c^(1/3)*x*Log[1 + I*a + I*b*x] + (3*I)*c^(1/3)*Log[(-I)*(I + a + b
*x)] + 3*a*c^(1/3)*Log[(-I)*(I + a + b*x)] + 3*b*c^(1/3)*x*Log[(-I)*(I + a
+ b*x)] + b*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/(-((
-I + a)*c^(1/3)) + b*d^(1/3))] - b*d^(1/3)*Log[(-I)*(I + a + b*x)]*Log[(b*(
d^(1/3) + c^(1/3)*x))/(-((I + a)*c^(1/3)) + b*d^(1/3))] + (-1)^(2/3)*b*d^(1
/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/
3)*(-I + a)*c^(1/3) + b*d^(1/3))] - (-1)^(2/3)*b*d^(1/3)*Log[(-I)*(I + a +
b*x)]*Log[(b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3)
+ b*d^(1/3))] + (-1)^(1/3)*b*d^(1/3)*Log[(-I)*(I + a + b*x)]*Log[(b*(d^(1/3
) + (-1)^(2/3)*c^(1/3)*x))/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))] - (-
1)^(1/3)*b*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3
)*x))/(-((-1)^(2/3)*(-I + a)*c^(1/3)) + b*d^(1/3))] + b*d^(1/3)*PolyLog[2,
(c^(1/3)*(-I + a + b*x))/((-I + a)*c^(1/3) - b*d^(1/3))] - (-1)^(1/3)*b*d^(
1/3)*PolyLog[2, ((-1)^(1/6)*c^(1/3)*(-I + a + b*x))/((-1)^(1/6)*(-I + a)*c^
(1/3) + I*b*d^(1/3))] + (-1)^(2/3)*b*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)
*(-I + a + b*x))/((-1)^(1/3)*(-I + a)*c^(1/3) + b*d^(1/3))] - b*d^(1/3)*Pol
yLog[2, (c^(1/3)*(I + a + b*x))/((I + a)*c^(1/3) - b*d^(1/3))] + (-1)^(1/3)
*b*d^(1/3)*PolyLog[2, ((-1)^(2/3)*c^(1/3)*(I + a + b*x))/((-1)^(2/3)*(I + a
)*c^(1/3) - b*d^(1/3))] - (-1)^(2/3)*b*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/
3)*(I + a + b*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))]))/(b*c^(4/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.09 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.55

method	result
risch	$\frac{i\ln(-bxi-ia+1)x}{2c}+\frac{i\ln(-bxi-ia+1)a}{2bc}-\frac{\ln(-bxi-ia+1)}{2bc}+\frac{1}{bc}+\frac{ib^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3+\left(3\,\mathrm{RootOf}\left(\_Z^2+1,\mathrm{ind}\right.\right.\right.}\right.}{}$
derivativedivides	$\frac{\frac{\arctan(bx+a)(bx+a)}{c}+\frac{\arctan(bx+a)\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^3-3ac\_Z^2+3a^2c\_Z-a^3c+b^3d\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)db^3}{3c^2}-\frac{db^3\left(\arctan\right.}{}}{}$
default	$\frac{\frac{\arctan(bx+a)(bx+a)}{c}+\frac{\arctan(bx+a)\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^3-3ac\_Z^2+3a^2c\_Z-a^3c+b^3d\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)db^3}{3c^2}-\frac{db^3\left(\arctan\right.}{}}{}$

```
[In] int(arctan(b*x+a)/(c+d/x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/c*ln(1-I*a-I*b*x)*x+1/2*I/b/c*ln(1-I*a-I*b*x)*a-1/2/b/c*ln(1-I*a-I*b*
x)+1/b/c+1/6*I*b^2*d/c^2*sum(1/(1+2*I*a*_R1-2*I*a+_R1^2-a^2-2*_R1)*(ln(1-I*
a-I*b*x)*ln((_R1+I*b*x+I*a-1)/_R1)+dilog((_R1+I*b*x+I*a-1)/_R1)),_R1=RootOf
(c*_Z^3+(3*RootOf(_Z^2+1,index=1)*a*c-3*c)*_Z^2+(-6*RootOf(_Z^2+1,index=1)*
a*c-3*a^2*c+3*c)*_Z-RootOf(_Z^2+1,index=1)*a^3*c+RootOf(_Z^2+1,index=1)*b^3
*d+3*RootOf(_Z^2+1,index=1)*a*c+3*a^2*c-c))-1/2*I/c*ln(1+I*a+I*b*x)*x-1/2*I
/b/c*ln(1+I*a+I*b*x)*a-1/2/b/c*ln(1+I*a+I*b*x)-1/6*I*b^2*d/c^2*sum(1/(1-2*I
*a*_R1+2*I*a+_R1^2-a^2-2*_R1)*(ln(1+I*a+I*b*x)*ln((_R1-I*b*x-I*a-1)/_R1)+di
log((_R1-I*b*x-I*a-1)/_R1)),_R1=RootOf(c*_Z^3+(-3*RootOf(_Z^2+1,index=1)*a*
c-3*c)*_Z^2+(6*RootOf(_Z^2+1,index=1)*a*c-3*a^2*c+3*c)*_Z+RootOf(_Z^2+1,ind
ex=1)*a^3*c-RootOf(_Z^2+1,index=1)*b^3*d-3*RootOf(_Z^2+1,index=1)*a*c+3*a^2
*c-c))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x^3}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(b*x + a)/(c*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \text{Timed out}$$

```
[In] integrate(atan(b*x+a)/(c+d/x**3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x^3}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/x^3), x)
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x^3}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{c+\frac{d}{x^3}}\,dx$$

```
[In] int(atan(a + b*x)/(c + d/x^3),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x^3), x)
```

3.58 $\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 673

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx &= \frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} - \frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} \\
&+ \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&- \frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&+ \frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&- \frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&+ \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} \\
&- \frac{i\sqrt{x}\log(1+ia+ibx)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx)}{d^2} \\
&+ \frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)}{d^2} + \frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{d^2} \\
&- \frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2} - \frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}
\end{aligned}
$$

```
[Out] -I*c*ln(1-I*a-I*b*x)*ln(c+d*x^(1/2))/d^2+I*c*ln(1+I*a+I*b*x)*ln(c+d*x^(1/2)
)/d^2+I*c*ln(c+d*x^(1/2))*ln(d*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(-I-a)^(1/
```

```
2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(d*((I-a)^(1/2)-b^(1/2)*x^(1/2))/(
d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2))*ln(-d*((-I-a)^(1/2)+b^(1/
2)*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(-d*((I-
a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(
1/2)*(c+d*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(1/2)*(
c+d*x^(1/2))/(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x^(
1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(
d*(I-a)^(1/2)+c*b^(1/2)))/d^2-2*I*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))*(I-a
)^(1/2)/d/b^(1/2)+2*I*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a)^(1/2)/d/b^(
1/2)+I*ln(1-I*a-I*b*x)*x^(1/2)/d-I*ln(1+I*a+I*b*x)*x^(1/2)/d
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {5159, 2455, 2516, 2498, 327, 211, 2512, 266, 2463, 2441, 2440, 2438, 214}

$$\begin{aligned}
\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx &= \frac{2i\sqrt{a+i}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}d}-\frac{2i\sqrt{-a+i}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a+i}}\right)}{\sqrt{b}d}\\
&+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-i}d}\right)}{d^2}+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-i}d}\right)}{d^2}\\
&-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2}-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}\\
&+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(-\sqrt{b}\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}c+\sqrt{-a-i}d}\right)}{d^2}\\
&-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(-\sqrt{b}\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}c+\sqrt{-a+i}d}\right)}{d^2}\\
&+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{b}\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}c-\sqrt{-a-i}d}\right)}{d^2}\\
&-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{b}\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}c-\sqrt{-a+i}d}\right)}{d^2}\\
&-\frac{ic\log(-ia-ibx+1)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{ic\log(ia+ibx+1)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{i\sqrt{x}\log(-ia-ibx+1)}{d}-\frac{i\sqrt{x}\log(ia+ibx+1)}{d}
\end{aligned}$$

```
[In] Int[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((2*I)*Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) - ((2
*I)*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*d) + (I*c*
Log[(d*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[
c + d*Sqrt[x]])/d^2 - (I*c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]
*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (I*c*Log[-((d*(Sqrt[-I - a]
+ Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2
- (I*c*Log[-((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d
))]*Log[c + d*Sqrt[x]])/d^2 + (I*Sqrt[x]*Log[1 - I*a - I*b*x])/d - (I*c*Log
[c + d*Sqrt[x]]*Log[1 - I*a - I*b*x])/d^2 - (I*Sqrt[x]*Log[1 + I*a + I*b*x]
)/d + (I*c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x])/d^2 + (I*c*PolyLog[2, (
Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)])/d^2 + (I*c*PolyLog[
2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)])/d^2 - (I*c*Poly
Log[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)])/d^2 - (I*c*P
olyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)])/d^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+d\sqrt{x}}\,dx-\frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+d\sqrt{x}}\,dx\\
&= i\text{Subst}\left(\int \frac{x\log\left(1-ia-ibx^2\right)}{c+dx}\,dx,x,\sqrt{x}\right)-i\text{Subst}\left(\int \frac{x\log\left(1+ia+ibx^2\right)}{c+dx}\,dx,x,\sqrt{x}\right)\\
&= i\text{Subst}\left(\int \left(\frac{\log\left(1-ia-ibx^2\right)}{d}-\frac{c\log\left(1-ia-ibx^2\right)}{d(c+dx)}\right)\,dx,x,\sqrt{x}\right)\\
&\quad -i\text{Subst}\left(\int \left(\frac{\log\left(1+ia+ibx^2\right)}{d}-\frac{c\log\left(1+ia+ibx^2\right)}{d(c+dx)}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{i\text{Subst}\left(\int \log\left(1-ia-ibx^2\right)\,dx,x,\sqrt{x}\right)}{d}-\frac{i\text{Subst}\left(\int \log\left(1+ia+ibx^2\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad -\frac{(ic)\text{Subst}\left(\int \frac{\log\left(1-ia-ibx^2\right)}{c+dx}\,dx,x,\sqrt{x}\right)}{d}+\frac{(ic)\text{Subst}\left(\int \frac{\log\left(1+ia+ibx^2\right)}{c+dx}\,dx,x,\sqrt{x}\right)}{d}\\
&= \frac{i\sqrt{x}\log(1-ia-ibx)}{d}-\frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2}\\
&\quad -\frac{i\sqrt{x}\log(1+ia+ibx)}{d}+\frac{ic\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx)}{d^2}\\
&\quad +\frac{(2bc)\text{Subst}\left(\int \frac{x\log(c+dx)}{1-ia-ibx^2}\,dx,x,\sqrt{x}\right)}{d^2}+\frac{(2bc)\text{Subst}\left(\int \frac{x\log(c+dx)}{1+ia+ibx^2}\,dx,x,\sqrt{x}\right)}{d^2}\\
&\quad -\frac{(2b)\text{Subst}\left(\int \frac{x^2}{1-ia-ibx^2}\,dx,x,\sqrt{x}\right)}{d}-\frac{(2b)\text{Subst}\left(\int \frac{x^2}{1+ia+ibx^2}\,dx,x,\sqrt{x}\right)}{d}\\
&= \frac{i\sqrt{x}\log(1-ia-ibx)}{d}-\frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2}\\
&\quad -\frac{i\sqrt{x}\log(1+ia+ibx)}{d}+\frac{ic\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx)}{d^2}\\
&\quad +\frac{(2bc)\text{Subst}\left(\int \left(-\frac{i\log(c+dx)}{2\sqrt{b}\left(\sqrt{-i-a}-\sqrt{b}x\right)}+\frac{i\log(c+dx)}{2\sqrt{b}\left(\sqrt{-i-a}+\sqrt{b}x\right)}\right)\,dx,x,\sqrt{x}\right)}{d^2}\\
&\quad +\frac{(2bc)\text{Subst}\left(\int \left(\frac{i\log(c+dx)}{2\sqrt{b}\left(\sqrt{i-a}-\sqrt{b}x\right)}-\frac{i\log(c+dx)}{2\sqrt{b}\left(\sqrt{i-a}+\sqrt{b}x\right)}\right)\,dx,x,\sqrt{x}\right)}{d^2}\\
&\quad -\frac{(2(i-a))\text{Subst}\left(\int \frac{1}{1+ia+ibx^2}\,dx,x,\sqrt{x}\right)}{d}+\frac{(2(i+a))\text{Subst}\left(\int \frac{1}{1-ia-ibx^2}\,dx,x,\sqrt{x}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d}-\frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}\\
&+\frac{i\sqrt{x}\log(1-ia-ibx)}{d}-\frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2}\\
&-\frac{i\sqrt{x}\log(1+ia+ibx)}{d}+\frac{ic\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx)}{d^2}\\
&-\frac{\left(i\sqrt{b}c\right)\text{Subst}\left(\int\frac{\log(c+dx)}{\sqrt{-i-a}-\sqrt{b}x}\,dx,x,\sqrt{x}\right)}{d^2}+\frac{\left(i\sqrt{b}c\right)\text{Subst}\left(\int\frac{\log(c+dx)}{\sqrt{i-a}-\sqrt{b}x}\,dx,x,\sqrt{x}\right)}{d^2}\\
&+\frac{\left(i\sqrt{b}c\right)\text{Subst}\left(\int\frac{\log(c+dx)}{\sqrt{-i-a}+\sqrt{b}x}\,dx,x,\sqrt{x}\right)}{d^2}-\frac{\left(i\sqrt{b}c\right)\text{Subst}\left(\int\frac{\log(c+dx)}{\sqrt{i-a}+\sqrt{b}x}\,dx,x,\sqrt{x}\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d}-\frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}\\
&+\frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&-\frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}+\frac{i\sqrt{x}\log(1-ia-ibx)}{d}\\
&-\frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2}-\frac{i\sqrt{x}\log(1+ia+ibx)}{d}\\
&+\frac{ic\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx)}{d^2}-\frac{(ic)\text{Subst}\left(\int\frac{\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{c+dx}\,dx,x,\sqrt{x}\right)}{d}\\
&+\frac{(ic)\text{Subst}\left(\int\frac{\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)}{c+dx}\,dx,x,\sqrt{x}\right)}{d}\\
&-\frac{(ic)\text{Subst}\left(\int\frac{\log\left(\frac{d\left(\sqrt{-i-a}+\sqrt{b}x\right)}{-\sqrt{b}c+\sqrt{-i-a}d}\right)}{c+dx}\,dx,x,\sqrt{x}\right)}{d}\\
&+\frac{(ic)\text{Subst}\left(\int\frac{\log\left(\frac{d\left(\sqrt{i-a}+\sqrt{b}x\right)}{-\sqrt{b}c+\sqrt{i-a}d}\right)}{c+dx}\,dx,x,\sqrt{x}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} - \frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} \\
&+ \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} - \frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&+ \frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&- \frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&+ \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} \\
&- \frac{i\sqrt{x}\log(1+ia+ibx)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx)}{d^2} \\
&- \frac{(ic)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}c+\sqrt{-i-a}d}\right)}{x}\,dx, x, c+d\sqrt{x}\right)}{d^2} \\
&- \frac{(ic)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{x}\,dx, x, c+d\sqrt{x}\right)}{d^2} \\
&+ \frac{(ic)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}c+\sqrt{i-a}d}\right)}{x}\,dx, x, c+d\sqrt{x}\right)}{d^2} \\
&+ \frac{(ic)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}c+\sqrt{i-a}d}\right)}{x}\,dx, x, c+d\sqrt{x}\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} - \frac{2i\sqrt{i-a}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} \\
&\quad + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} - \frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&\quad + \frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&\quad - \frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&\quad + \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} \\
&\quad - \frac{i\sqrt{x}\log(1+ia+ibx)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx)}{d^2} \\
&\quad + \frac{ic\operatorname{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)}{d^2} + \frac{ic\operatorname{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{d^2} \\
&\quad - \frac{ic\operatorname{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2} - \frac{ic\operatorname{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.90

$$
\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx
$$

$$
= \frac{i\Bigg(\frac{2\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}} - \frac{2\sqrt{i-a}d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}} + c\log\Bigg(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\Bigg)\log\left(c+d\sqrt{x}\right) - c\log\Bigg(\frac{d\Big(\sqrt{i-a}-\sqrt{b}\sqrt{x}}{\sqrt{b}c+\sqrt{i-a}d}}{}
$$

```
[In] Integrate[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] (I*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (2*Sq
rt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d*(Sqr
t[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[
x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)
]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b
]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] + Sqrt[b
]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[x]*
Log[1 + I*a + I*b*x] + c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x] + d*Sqrt[x
]*Log[(-I)*(I + a + b*x)] - c*Log[c + d*Sqrt[x]]*Log[(-I)*(I + a + b*x)] +
c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)] + c*Po
```

```
lyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)] - c*PolyLo
g[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)] - c*PolyLog[2,
(Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)]))/d^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{2\arctan(bx+a)\sqrt{x}}{d}-\frac{2\arctan(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}-\frac{4b\left(\frac{d^2\left(\sum_{\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\left(-\right.\right.}\right.}{}\right.}{}$
default	$\frac{2\arctan(bx+a)\sqrt{x}}{d}-\frac{2\arctan(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}-\frac{4b\left(\frac{d^2\left(\sum_{\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\left(-\right.\right.}\right.}{}\right.}{}$

```
[In] int(arctan(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan(b*x+a)/d*x^(1/2)-2*arctan(b*x+a)*c/d^2*ln(c+d*x^(1/2))-4*b/d^2*(1/
4*d^2/b*sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*d^
2-b*c^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*b^2*c*_Z^3+(2*a*b*d^2+6*b^
2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d^4))
-1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln((-d*
x^(1/2)+_R1-c)/_R1)+dilog((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^
2*c*_Z^3+(2*a*b*d^2+6*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a
*b*c^2*d^2+b^2*c^4+d^4)))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\arctan\left(bx+a\right)}{d\sqrt{x}+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((d*sqrt(x)*arctan(b*x + a) - c*arctan(b*x + a))/(d^2*x - c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \text{Timed out}$$

```
[In] integrate(atan(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\arctan\left(bx+a\right)}{d\sqrt{x}+c}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(d*sqrt(x) + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]War
ning, replacing 0 by -24, a substitution variable should perhaps be purged.
Warnin
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

```
[In] int(atan(a + b*x)/(c + d*x^(1/2)),x)
[Out] int(atan(a + b*x)/(c + d*x^(1/2)), x)
```

3.59 $\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 770

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = & -\frac{2i\sqrt{i+a}\,d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2}+\frac{2i\sqrt{i-a}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} \\
& -\frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
& +\frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
& -\frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
& +\frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
& -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2}+\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} \\
& +\frac{id\sqrt{x}\log(1+ia+ibx)}{c^2}-\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} \\
& -\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)}{c^3} \\
& -\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
& -\frac{id^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{c^3}+\frac{id^2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3} \\
& -\frac{id^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{c^3}+\frac{id^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}
\end{aligned}
$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c-I*d^2*ln(1+I*a+I*b*x)*ln(d+c*x^(1/2))/c^3+I*d^2*polylog(2,b^(1/2)*(d+c*x
^(1/2))/(c*(I-a)^(1/2)+d*b^(1/2)))/c^3+I*d*ln(1+I*a+I*b*x)*x^(1/2)/c^2-I*d^
2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-I-a)^(1/2)-d*b^(
1/2)))/c^3+2*I*d*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))*(I-a)^(1/2)/c^2/b^(1/
2)-I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(-I-a)^(1/2)-d*b^(1/2)))/c^3-2
*I*d*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a)^(1/2)/c^2/b^(1/2)+I*d^2*ln(d
+c*x^(1/2))*ln(c*((I-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(I-a)^(1/2)+d*b^(1/2)))/c
^3+I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(I-a)^(1/2)-d*b^(1/2)))/c^3+I*
d^2*ln(1-I*a-I*b*x)*ln(d+c*x^(1/2))/c^3-I*d*ln(1-I*a-I*b*x)*x^(1/2)/c^2-I*d
^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-I-a)^(1/2)+d*b^(1/2)))/c^3-I*d^2*ln
```

```
(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(-I-a)^(1/2)+d*b^(1/2)
))/c^3+I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(I-a)^(1
/2)-d*b^(1/2)))/c^3
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {5159, 2455, 2526, 2498, 327, 211, 2504, 2436, 2332, 2512, 266, 2463, 2441, 2440, 2438, 214}

$$\begin{aligned}\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = & -\frac{2i\sqrt{a+i}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}c^2}+\frac{2i\sqrt{-a+i}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a+i}}\right)}{\sqrt{b}c^2} \\ & -\frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-i}c-\sqrt{b}d}\right)}{c^3}+\frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3} \\ & -\frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-i}c+\sqrt{b}d}\right)}{c^3}+\frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3} \\ & -\frac{id^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(-\sqrt{b}\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}d+\sqrt{-a-i}c}\right)}{c^3} \\ & +\frac{id^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(-\sqrt{b}\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}d+\sqrt{-a+i}c}\right)}{c^3} \\ & -\frac{id^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{b}\sqrt{x}+\sqrt{-a-i}\right)}{-\sqrt{b}d+\sqrt{-a-i}c}\right)}{c^3} \\ & +\frac{id^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{b}\sqrt{x}+\sqrt{-a+i}\right)}{-\sqrt{b}d+\sqrt{-a+i}c}\right)}{c^3} \\ & +\frac{id^2\log(-ia-ibx+1)\log\left(c\sqrt{x}+d\right)}{c^3} \\ & -\frac{id^2\log(ia+ibx+1)\log\left(c\sqrt{x}+d\right)}{c^3}-\frac{id\sqrt{x}\log(-ia-ibx+1)}{c^2} \\ & +\frac{id\sqrt{x}\log(ia+ibx+1)}{c^2}-\frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc} \\ & -\frac{(-ia-ibx+1)\log(-i(a+bx+i))}{2bc}\end{aligned}$$

```
[In] Int[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((-2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2)
+ ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2
```

```
) - (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[
b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[
x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d^2*Log[(c*(
Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sq
rt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
 - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d*Sqrt[x]*Log[1 - I*a - I*b*x])
/c^2 + (I*d^2*Log[d + c*Sqrt[x]]*Log[1 - I*a - I*b*x])/c^3 + (I*d*Sqrt[x]*L
og[1 + I*a + I*b*x])/c^2 - ((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c)
 - (I*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x])/c^3 - ((1 - I*a - I*b*x)
*Log[(-I)*(I + a + b*x)])/(2*b*c) - (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqr
t[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3 + (I*d^2*PolyLog[2, -((Sqrt[b]*(
d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3 - (I*d^2*PolyLog[2, (Sqr
t[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)])/c^3 + (I*d^2*PolyLog[2
, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/c^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2} i \int \frac{\log(1-ia-ibx)}{c+\frac{d}{\sqrt{x}}}\,dx - \frac{1}{2} i \int \frac{\log(1+ia+ibx)}{c+\frac{d}{\sqrt{x}}}\,dx \\
&= i\text{Subst}\left(\int \frac{x\log\left(1-ia-ibx^2\right)}{c+\frac{d}{x}}\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{x\log\left(1+ia+ibx^2\right)}{c+\frac{d}{x}}\,dx, x, \sqrt{x}\right) \\
&= i\text{Subst}\left(\int \left(-\frac{d\log\left(1-ia-ibx^2\right)}{c^2} + \frac{x\log\left(1-ia-ibx^2\right)}{c}\right.\right. \\
&\quad \left.\left. + \frac{d^2\log\left(1-ia-ibx^2\right)}{c^2(d+cx)}\right)\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \left(-\frac{d\log\left(1+ia+ibx^2\right)}{c^2}\right.\right. \\
&\quad \left.\left. + \frac{x\log\left(1+ia+ibx^2\right)}{c} + \frac{d^2\log\left(1+ia+ibx^2\right)}{c^2(d+cx)}\right)\,dx, x, \sqrt{x}\right) \\
&= \frac{i\text{Subst}\left(\int x\log\left(1-ia-ibx^2\right)\,dx, x, \sqrt{x}\right)}{c} - \frac{i\text{Subst}\left(\int x\log\left(1+ia+ibx^2\right)\,dx, x, \sqrt{x}\right)}{c} \\
&\quad - \frac{(id)\text{Subst}\left(\int \log\left(1-ia-ibx^2\right)\,dx, x, \sqrt{x}\right)}{c^2} + \frac{(id)\text{Subst}\left(\int \log\left(1+ia+ibx^2\right)\,dx, x, \sqrt{x}\right)}{c^2} \\
&\quad + \frac{\left(id^2\right)\text{Subst}\left(\int \frac{\log\left(1-ia-ibx^2\right)}{d+cx}\,dx, x, \sqrt{x}\right)}{c^2} - \frac{\left(id^2\right)\text{Subst}\left(\int \frac{\log\left(1+ia+ibx^2\right)}{d+cx}\,dx, x, \sqrt{x}\right)}{c^2}
\end{aligned}
$$

$$\begin{aligned}
&= -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} \\
&\quad + \frac{id\sqrt{x}\log(1+ia+ibx)}{c^2} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)}{c^3} \\
&\quad + \frac{i\text{Subst}(\int \log(1-ia-ibx)\,dx, x, x)}{2c} - \frac{i\text{Subst}(\int \log(1+ia+ibx)\,dx, x, x)}{2c} \\
&\quad + \frac{(2bd)\text{Subst}\left(\int \frac{x^2}{1-ia-ibx^2}\,dx, x, \sqrt{x}\right)}{c^2} + \frac{(2bd)\text{Subst}\left(\int \frac{x^2}{1+ia+ibx^2}\,dx, x, \sqrt{x}\right)}{c^2} \\
&\quad - \frac{(2bd^2)\,\text{Subst}\left(\int \frac{x\log(d+cx)}{1-ia-ibx^2}\,dx, x, \sqrt{x}\right)}{c^3} - \frac{(2bd^2)\,\text{Subst}\left(\int \frac{x\log(d+cx)}{1+ia+ibx^2}\,dx, x, \sqrt{x}\right)}{c^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} \\
&\quad + \frac{id\sqrt{x}\log(1+ia+ibx)}{c^2} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)}{c^3} \\
&\quad - \frac{\text{Subst}(\int \log(x)\,dx, x, 1-ia-ibx)}{2bc} - \frac{\text{Subst}(\int \log(x)\,dx, x, 1+ia+ibx)}{2bc} \\
&\quad + \frac{(2(i-a)d)\text{Subst}(\int \frac{1}{1+ia+ibx^2}\,dx, x, \sqrt{x})}{c^2} \\
&\quad - \frac{(2(i+a)d)\text{Subst}(\int \frac{1}{1-ia-ibx^2}\,dx, x, \sqrt{x})}{c^2} \\
&\quad - \frac{(2bd^2)\,\text{Subst}\left(\int \left(-\frac{i\log(d+cx)}{2\sqrt{b}\left(\sqrt{-i-a}-\sqrt{b}x\right)} + \frac{i\log(d+cx)}{2\sqrt{b}\left(\sqrt{-i-a}+\sqrt{b}x\right)}\right)\,dx, x, \sqrt{x}\right)}{c^3} \\
&\quad - \frac{(2bd^2)\,\text{Subst}\left(\int \left(\frac{i\log(d+cx)}{2\sqrt{b}\left(\sqrt{i-a}-\sqrt{b}x\right)} - \frac{i\log(d+cx)}{2\sqrt{b}\left(\sqrt{i-a}+\sqrt{b}x\right)}\right)\,dx, x, \sqrt{x}\right)}{c^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2}+\frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2}-\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2}\\
&\quad+\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3}+\frac{id\sqrt{x}\log(1+ia+ibx)}{c^2}\\
&\quad-\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc}-\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)}{c^3}\\
&\quad-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}+\frac{\left(i\sqrt{b}d^2\right)\text{Subst}\left(\int\frac{\log(d+cx)}{\sqrt{-i-a}-\sqrt{b}x}\,dx,x,\sqrt{x}\right)}{c^3}\\
&\quad-\frac{\left(i\sqrt{b}d^2\right)\text{Subst}\left(\int\frac{\log(d+cx)}{\sqrt{i-a}-\sqrt{b}x}\,dx,x,\sqrt{x}\right)}{c^3}\\
&\quad-\frac{\left(i\sqrt{b}d^2\right)\text{Subst}\left(\int\frac{\log(d+cx)}{\sqrt{-i-a}+\sqrt{b}x}\,dx,x,\sqrt{x}\right)}{c^3}\\
&\quad+\frac{\left(i\sqrt{b}d^2\right)\text{Subst}\left(\int\frac{\log(d+cx)}{\sqrt{i-a}+\sqrt{b}x}\,dx,x,\sqrt{x}\right)}{c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2}+\frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} \\
&\quad -\frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad +\frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad -\frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad +\frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2}+\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} \\
&\quad +\frac{id\sqrt{x}\log(1+ia+ibx)}{c^2}-\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} \\
&\quad -\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)}{c^3}-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&\quad +\frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}x\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{d+cx}dx,x,\sqrt{x}\right)}{c^2} \\
&\quad -\frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}x\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)}{d+cx}dx,x,\sqrt{x}\right)}{c^2} \\
&\quad +\frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}x\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{d+cx}dx,x,\sqrt{x}\right)}{c^2} \\
&\quad -\frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}x\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)}{d+cx}dx,x,\sqrt{x}\right)}{c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} + \frac{2i\sqrt{i-a}d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} \\
&\quad - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad + \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad + \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad - \frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} \\
&\quad + \frac{id\sqrt{x}\log(1+ia+ibx)}{c^2} - \frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} \\
&\quad - \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)}{c^3} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&\quad + \frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{x}\,dx, x, d+c\sqrt{x}\right)}{c^3} \\
&\quad - \frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{\sqrt{i-a}c-\sqrt{b}d}\right)}{x}\,dx, x, d+c\sqrt{x}\right)}{c^3} \\
&\quad + \frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{x}\,dx, x, d+c\sqrt{x}\right)}{c^3} \\
&\quad - \frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{i-a}c+\sqrt{b}d}\right)}{x}\,dx, x, d+c\sqrt{x}\right)}{c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} + \frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} \\
&\quad - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad + \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad - \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad + \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&\quad - \frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3} \\
&\quad + \frac{id\sqrt{x}\log(1+ia+ibx)}{c^2} - \frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} \\
&\quad - \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)}{c^3} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&\quad - \frac{id^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{c^3} + \frac{id^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3} \\
&\quad - \frac{id^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{c^3} + \frac{id^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.00

$$
\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx =
$$

$$
-\frac{i\left(4\sqrt{i+a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right) - 4\sqrt{i-a}\sqrt{b}cd\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right) + 2bd^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\right.\right.}{}
$$

```
[In] Integrate[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((-1/2*I)*(4*Sqrt[I + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]]
- 4*Sqrt[I - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]] + 2*b*d^
2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]*Lo
g[d + c*Sqrt[x]] - 2*b*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I
- a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + 2*b*d^2*Log[(c*(Sqrt[-I - a] + Sq
```

```
rt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]] - 2*b*d^2*
Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d)]*Log[d
+ c*Sqrt[x]] - I*c^2*Log[1 + I*a + I*b*x] + a*c^2*Log[1 + I*a + I*b*x] - 2*
b*c*d*Sqrt[x]*Log[1 + I*a + I*b*x] + b*c^2*x*Log[1 + I*a + I*b*x] + 2*b*d^2
*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x] - I*c^2*Log[(-I)*(I + a + b*x)] -
a*c^2*Log[(-I)*(I + a + b*x)] + 2*b*c*d*Sqrt[x]*Log[(-I)*(I + a + b*x)] - b
*c^2*x*Log[(-I)*(I + a + b*x)] - 2*b*d^2*Log[d + c*Sqrt[x]]*Log[(-I)*(I + a
 + b*x)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-I - a]*c)
+ Sqrt[b]*d)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*
c + Sqrt[b]*d)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[I -
a]*c) + Sqrt[b]*d)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I
- a]*c + Sqrt[b]*d)]))/(b*c^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{\arctan(bx+a)x}{c}-\frac{2\arctan(bx+a)d\sqrt{x}}{c^2}+\frac{2\arctan(bx+a)d^2\ln(d+c\sqrt{x})}{c^3}-\frac{4b\left(-\frac{c\left(\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+\right.\right.}{}\right.}{}$
default	$\frac{\arctan(bx+a)x}{c}-\frac{2\arctan(bx+a)d\sqrt{x}}{c^2}+\frac{2\arctan(bx+a)d^2\ln(d+c\sqrt{x})}{c^3}-\frac{4b\left(-\frac{c\left(\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+\right.\right.}{}\right.}{}$

```
[In] int(arctan(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(b*x+a)*x/c-2*arctan(b*x+a)/c^2*d*x^(1/2)+2*arctan(b*x+a)*d^2/c^3*ln(
d+c*x^(1/2))-4*b/c^2*(-1/8*c/b*sum((-_R^3+5*_R^2*d-7*_R*d^2+3*d^3)/(_R^3*b-
3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=RootOf(
b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_
Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))+1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*d+
a*c^2+b*d^2)*(ln(d+c*x^(1/2))*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^(1/2)+
_R1-d)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-
4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
[Out] integral((c*x*arctan(b*x + a) - d*sqrt(x)*arctan(b*x + a))/(c^2*x - d^2), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Timed out}$$

```
[In] integrate(atan(b*x+a)/(c+d/x**(1/2)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
[Out] integrate(arctan(b*x + a)/(c + d/sqrt(x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]War
ning, replacing 0 by -24, a substitution variable should perhaps be purged.
Warnin
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

```
[In] int(atan(a + b*x)/(c + d/x^(1/2)),x)
[Out] int(atan(a + b*x)/(c + d/x^(1/2)), x)
```

3.60 $\int \frac{\arctan(a+bx)}{1+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 274

$$\begin{aligned}\int \frac{\arctan(a+bx)}{1+x^2}\,dx = {} & \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) \\ & -\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) \\ & -\frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+ia+ibx) \\ & +\frac{1}{4}\log\left(-\frac{b(i+x)}{a-i(1+b)}\right)\log(1+ia+ibx) \\ & -\frac{1}{4}\operatorname{PolyLog}\left(2,-\frac{i-a-bx}{a-i(1-b)}\right)+\frac{1}{4}\operatorname{PolyLog}\left(2,-\frac{i-a-bx}{a-i(1+b)}\right) \\ & -\frac{1}{4}\operatorname{PolyLog}\left(2,\frac{i+a+bx}{i+a-ib}\right)+\frac{1}{4}\operatorname{PolyLog}\left(2,\frac{i+a+bx}{a+i(1+b)}\right)\end{aligned}$$

```
[Out] 1/4*ln(b*(I-x)/(a+I*(1+b)))*ln(1-I*a-I*b*x)-1/4*ln(-b*(I+x)/(a+I*(1-b)))*ln
(1-I*a-I*b*x)-1/4*ln(b*(I-x)/(a-I*(1-b)))*ln(1+I*a+I*b*x)+1/4*ln(-b*(I+x)/(
a-I*(1+b)))*ln(1+I*a+I*b*x)-1/4*polylog(2,(-I+a+b*x)/(a-I*(1-b)))+1/4*polyl
og(2,(-I+a+b*x)/(a-I*(1+b)))-1/4*polylog(2,(I+a+b*x)/(I+a-I*b))+1/4*polylog
(2,(I+a+b*x)/(a+I*(1+b)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5159, 2456, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{\arctan(a+bx)}{1+x^2}\,dx = & -\frac{1}{4}\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(1-b)}\right)+\frac{1}{4}\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(b+1)}\right)\\ & -\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+i}{a-ib+i}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+i}{a+i(b+1)}\right)\\ & +\frac{1}{4}\log\left(\frac{b(-x+i)}{a+i(b+1)}\right)\log(-ia-ibx+1)\\ & -\frac{1}{4}\log\left(-\frac{b(x+i)}{a+i(1-b)}\right)\log(-ia-ibx+1)\\ & -\frac{1}{4}\log\left(\frac{b(-x+i)}{a-i(1-b)}\right)\log(ia+ibx+1)\\ & +\frac{1}{4}\log\left(-\frac{b(x+i)}{a-i(b+1)}\right)\log(ia+ibx+1)\end{aligned}$$

```
[In] Int[ArcTan[a + b*x]/(1 + x^2),x]
```

```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, -((I - a - b*x)/(a - I*(1 - b)))]/4 + PolyL
og[2, -((I - a - b*x)/(a - I*(1 + b)))]/4 - PolyLog[2, (I + a + b*x)/(I + a
 - I*b)]/4 + PolyLog[2, (I + a + b*x)/(a + I*(1 + b))]/4
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{1+x^2}\,dx-\frac{1}{2}i\int \frac{\log(1+ia+ibx)}{1+x^2}\,dx\\
&= \frac{1}{2}i\int \left(\frac{i\log(1-ia-ibx)}{2(i-x)}+\frac{i\log(1-ia-ibx)}{2(i+x)}\right)dx\\
&\quad -\frac{1}{2}i\int \left(\frac{i\log(1+ia+ibx)}{2(i-x)}+\frac{i\log(1+ia+ibx)}{2(i+x)}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\log(1-ia-ibx)}{i-x}\,dx\right)-\frac{1}{4}\int \frac{\log(1-ia-ibx)}{i+x}\,dx\\
&\quad +\frac{1}{4}\int \frac{\log(1+ia+ibx)}{i-x}\,dx+\frac{1}{4}\int \frac{\log(1+ia+ibx)}{i+x}\,dx\\
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)\\
&\quad -\frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+ia+ibx)+\frac{1}{4}\log\left(-\frac{b(i+x)}{a-i(1+b)}\right)\log(1+ia+ibx)\\
&\quad +\frac{1}{4}(ib)\int \frac{\log\left(\frac{ib(i-x)}{1+ia-b}\right)}{1+ia+ibx}\,dx+\frac{1}{4}(ib)\int \frac{\log\left(-\frac{ib(i-x)}{1-ia+b}\right)}{1-ia-ibx}\,dx\\
&\quad -\frac{1}{4}(ib)\int \frac{\log\left(\frac{ib(i+x)}{-1-ia-b}\right)}{1+ia+ibx}\,dx-\frac{1}{4}(ib)\int \frac{\log\left(-\frac{ib(i+x)}{-1+ia+b}\right)}{1-ia-ibx}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) \\
&\quad - \frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+ia+ibx) + \frac{1}{4}\log\left(-\frac{b(i+x)}{a-i(1+b)}\right)\log(1+ia+ibx) \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{\log\left(1+\frac{x}{-1-ia-b}\right)}{x}\,dx, x, 1+ia+ibx\right) \\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{\log\left(1-\frac{x}{1+ia-b}\right)}{x}\,dx, x, 1+ia+ibx\right) \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{\log\left(1-\frac{x}{1-ia+b}\right)}{x}\,dx, x, 1-ia-ibx\right) \\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{\log\left(1+\frac{x}{-1+ia+b}\right)}{x}\,dx, x, 1-ia-ibx\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) \\
&\quad - \frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+ia+ibx) + \frac{1}{4}\log\left(-\frac{b(i+x)}{a-i(1+b)}\right)\log(1+ia+ibx) \\
&\quad - \frac{1}{4}\,\text{PolyLog}\left(2, -\frac{i-a-bx}{a-i(1-b)}\right) + \frac{1}{4}\,\text{PolyLog}\left(2, -\frac{i-a-bx}{a-i(1+b)}\right) \\
&\quad - \frac{1}{4}\,\text{PolyLog}\left(2, \frac{i+a+bx}{i+a-ib}\right) + \frac{1}{4}\,\text{PolyLog}\left(2, \frac{i+a+bx}{a+i(1+b)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{1+x^2}\,dx &= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) \\
&\quad - \frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) \\
&\quad - \frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+ia+ibx) \\
&\quad + \frac{1}{4}\log\left(-\frac{b(i+x)}{a-i(1+b)}\right)\log(1+ia+ibx) \\
&\quad - \frac{1}{4}\,\text{PolyLog}\left(2, \frac{1-ia-ibx}{1-ia-b}\right) + \frac{1}{4}\,\text{PolyLog}\left(2, \frac{1-ia-ibx}{1-ia+b}\right) \\
&\quad - \frac{1}{4}\,\text{PolyLog}\left(2, \frac{1+ia+ibx}{1+ia-b}\right) + \frac{1}{4}\,\text{PolyLog}\left(2, \frac{1+ia+ibx}{1+ia+b}\right)
\end{aligned}
$$

```
[In] Integrate[ArcTan[a + b*x]/(1 + x^2),x]
```

```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, (1 - I*a - I*b*x)/(1 - I*a - b)]/4 + PolyLo
g[2, (1 - I*a - I*b*x)/(1 - I*a + b)]/4 - PolyLog[2, (1 + I*a + I*b*x)/(1 +
 I*a - b)]/4 + PolyLog[2, (1 + I*a + I*b*x)/(1 + I*a + b)]/4
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\ln(-bxi-ia+1)\ln\left(\frac{-bxi+b}{ia+b-1}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{-bxi+b}{ia+b-1}\right)}{4}+\frac{\ln(-bxi-ia+1)\ln\left(\frac{-bxi-b}{ia-b-1}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{-bxi-b}{ia-b-1}\right)}{4}+\ln(bxi+ia+$
default	$\arctan(x)\arctan(bx+a)-b\Bigg(-\frac{i\arctan(x)\ln\left(1-\frac{(-ib+a-i)(ix+1)^2}{\left(x^2+1\right)(-ib-a+i)}\right)}{2b}-\frac{\arctan(x)^2}{2b}-\frac{\operatorname{polylog}\Big(2,\frac{(-ib-}{(x^2+}}{4b}$
parts	$\arctan(x)\arctan(bx+a)-b\Bigg(-\frac{i\arctan(x)\ln\left(1-\frac{(-ib+a-i)(ix+1)^2}{\left(x^2+1\right)(-ib-a+i)}\right)}{2b}-\frac{\arctan(x)^2}{2b}-\frac{\operatorname{polylog}\Big(2,\frac{(-ib-}{(x^2+}}{4b}$
derivativedivides	$\frac{b\arctan(x)\arctan(bx+a)-b^2\Bigg(-\frac{\arctan\left(b\left(\frac{bx+a}{b}-\frac{a}{b}\right)+a\right)\arctan\left(-\frac{bx+a}{b}+\frac{a}{b}\right)}{b}-\frac{-\arctan\left(-\frac{bx+a}{b}+\frac{a}{b}\right)\arctan\left(b\left(\frac{bx+a}{b}-\frac{a}{b}\right)+a\right)}{}}{}$

```
[In] int(arctan(b*x+a)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(1-I*a-I*b*x)*ln((-I*b*x+b)/(I*a+b-1))-1/4*dilog((-I*b*x+b)/(I*a+b-1
))+1/4*ln(1-I*a-I*b*x)*ln((-I*b*x-b)/(I*a-b-1))+1/4*dilog((-I*b*x-b)/(I*a-b
-1))+1/4*ln(1+I*a+I*b*x)*ln((I*b*x-b)/(-I*a-b-1))+1/4*dilog((I*b*x-b)/(-I*a
-b-1))-1/4*ln(1+I*a+I*b*x)*ln((I*b*x+b)/(-I*a+b-1))-1/4*dilog((I*b*x+b)/(-I
*a+b-1))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \int \frac{\arctan(bx+a)}{x^2+1}\,dx$$

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \int \frac{\operatorname{atan}(a+bx)}{x^2+1}\,dx$$

```
[In] integrate(atan(b*x+a)/(x**2+1),x)
```

```
[Out] Integral(atan(a + b*x)/(x**2 + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.20

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx$$
$$= \frac{1}{8}b\Bigg(\frac{8\arctan(x)\arctan\left(\frac{b^2x+ab}{b}\right)}{b} - \frac{4\arctan(x)\arctan\left(\frac{ab+(b^2+b)x}{a^2+b^2+2b+1}, \frac{abx+a^2+b+1}{a^2+b^2+2b+1}\right) - 4\arctan(x)\arctan\Big(}{}$$
$$+ \arctan(bx+a)\arctan(x) - \arctan(x)\arctan\left(\frac{b^2x+ab}{b}\right)$$

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*b*(8*arctan(x)*arctan((b^2*x + a*b)/b)/b - (4*arctan(x)*arctan2((a*b +
(b^2 + b)*x)/(a^2 + b^2 + 2*b + 1), (a*b*x + a^2 + b + 1)/(a^2 + b^2 + 2*b
+ 1)) - 4*arctan(x)*arctan2((a*b + (b^2 - b)*x)/(a^2 + b^2 - 2*b + 1), (a*b
*x + a^2 - b + 1)/(a^2 + b^2 - 2*b + 1)) + log(x^2 + 1)*log((b^2*x^2 + 2*a*
b*x + a^2 + 1)/(a^2 + b^2 + 2*b + 1)) - log(x^2 + 1)*log((b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(a^2 + b^2 - 2*b + 1)) + 2*dilog(-(I*b*x - b)/(I*a + b + 1)) -
2*dilog(-(I*b*x - b)/(I*a + b - 1)) + 2*dilog((I*b*x + b)/(-I*a + b + 1)) -
 2*dilog((I*b*x + b)/(-I*a + b - 1)))/b) + arctan(b*x + a)*arctan(x) - arct
an(x)*arctan((b^2*x + a*b)/b)
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \int \frac{\arctan\left(bx+a\right)}{x^2+1}\,dx$$

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{x^2+1}\,dx$$

```
[In] int(atan(a + b*x)/(x^2 + 1),x)
```

```
[Out] int(atan(a + b*x)/(x^2 + 1), x)
```

3.61 $\int \frac{\arctan(d+ex)}{a+bx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 543

$$
\begin{aligned}
\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = {} & \frac{i \log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} \\
& - \frac{i \log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} \\
& - \frac{i \log\left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right)\log(1+id+iex)}{4\sqrt{-a}\sqrt{b}} \\
& + \frac{i \log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right)\log(1+id+iex)}{4\sqrt{-a}\sqrt{b}} \\
& - \frac{i\,\mathrm{PolyLog}\left(2, \frac{\sqrt{b}(i-d-ex)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i\,\mathrm{PolyLog}\left(2, \frac{\sqrt{b}(i-d-ex)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} \\
& - \frac{i\,\mathrm{PolyLog}\left(2, \frac{\sqrt{b}(i+d+ex)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i\,\mathrm{PolyLog}\left(2, \frac{\sqrt{b}(i+d+ex)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}
\end{aligned}
$$

```
[Out] -1/4*I*ln(1+I*d+I*e*x)*ln(-e*((-a)^(1/2)-x*b^(1/2))/(-e*(-a)^(1/2)+(I-d)*b^
(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*ln(1-I*d-I*e*x)*ln(e*((-a)^(1/2)-x*b^(1/2)
)/(e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*ln(1+I*d+I*e*x)*ln
(e*((-a)^(1/2)+x*b^(1/2))/(e*(-a)^(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)-
1/4*I*ln(1-I*d-I*e*x)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+(I+d)*b^(
1/2)))/(-a)^(1/2)/b^(1/2)-1/4*I*polylog(2,(I-d-e*x)*b^(1/2)/(-e*(-a)^(1/2)+
(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*polylog(2,(I-d-e*x)*b^(1/2)/(e*(-a
```

```
)^(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)-1/4*I*polylog(2,(I+d+e*x)*b^(1/2
)/(-e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*polylog(2,(I+d+e*
x)*b^(1/2)/(e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5159, 2456, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = &-\frac{i\,\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\,\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}\\
&-\frac{i\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{\sqrt{b}(d+i)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{i\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{\sqrt{b}(d+i)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}\\
&+\frac{i\log(-id-iex+1)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}(d+i)}\right)}{4\sqrt{-a}\sqrt{b}}\\
&-\frac{i\log(-id-iex+1)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{-a}e+\sqrt{b}(d+i)}\right)}{4\sqrt{-a}\sqrt{b}}\\
&-\frac{i\log(id+iex+1)\log\left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{-\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{4\sqrt{-a}\sqrt{b}}\\
&+\frac{i\log(id+iex+1)\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{4\sqrt{-a}\sqrt{b}}\end{aligned}$$

```
[In] Int[ArcTan[d + e*x]/(a + b*x^2),x]
```

```
[Out] ((I/4)*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]*Log[1
 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] + Sqrt[b]*x
))/(Sqrt[b]*(I + d) - Sqrt[-a]*e))]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]
) - ((I/4)*Log[-((e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e))
]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqr
t[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) - Sqrt[-
a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqr
t[b]*(I - d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b
]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/
4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)])/(Sqr
t[-a]*Sqrt[b])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2} i \int \frac{\log(1-id-iex)}{a+bx^2}\,dx - \frac{1}{2} i \int \frac{\log(1+id+iex)}{a+bx^2}\,dx \\
&= \frac{1}{2} i \int \left(\frac{\sqrt{-a}\log(1-id-iex)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\log(1-id-iex)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)} \right) dx \\
&\quad - \frac{1}{2} i \int \left(\frac{\sqrt{-a}\log(1+id+iex)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\log(1+id+iex)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)} \right) dx \\
&= -\frac{i\int \frac{\log(1-id-iex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}} - \frac{i\int \frac{\log(1-id-iex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}} + \frac{i\int \frac{\log(1+id+iex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}} + \frac{i\int \frac{\log(1+id+iex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}}
\end{aligned}
$$

$$= \frac{i \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right) \log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log \left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right) \log(1-id-iex)}{4\sqrt{-a}\sqrt{b}}$$
$$- \frac{i \log \left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right) \log(1+id+iex)}{4\sqrt{-a}\sqrt{b}}$$
$$+ \frac{i \log \left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right) \log(1+id+iex)}{4\sqrt{-a}\sqrt{b}}$$
$$- \frac{e \int \frac{\log\left(-\frac{ie\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(1-id)-i\sqrt{-a}e}\right)}{1-id-iex}\, dx}{4\sqrt{-a}\sqrt{b}} - \frac{e \int \frac{\log\left(\frac{ie\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(1+id)+i\sqrt{-a}e}\right)}{1+id+iex}\, dx}{4\sqrt{-a}\sqrt{b}}$$
$$+ \frac{e \int \frac{\log\left(-\frac{ie\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}(1-id)-i\sqrt{-a}e}\right)}{1-id-iex}\, dx}{4\sqrt{-a}\sqrt{b}} + \frac{e \int \frac{\log\left(\frac{ie\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}(1+id)+i\sqrt{-a}e}\right)}{1+id+iex}\, dx}{4\sqrt{-a}\sqrt{b}}$$

$$= \frac{i \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right) \log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log \left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right) \log(1-id-iex)}{4\sqrt{-a}\sqrt{b}}$$
$$- \frac{i \log \left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right) \log(1+id+iex)}{4\sqrt{-a}\sqrt{b}}$$
$$+ \frac{i \log \left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right) \log(1+id+iex)}{4\sqrt{-a}\sqrt{b}}$$
$$+ \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}(1-id)-i\sqrt{-a}e}\right)}{x}\, dx, x, 1-id-iex\right)}{4\sqrt{-a}\sqrt{b}}$$
$$- \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}(1-id)-i\sqrt{-a}e}\right)}{x}\, dx, x, 1-id-iex\right)}{4\sqrt{-a}\sqrt{b}}$$
$$- \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}(1+id)+i\sqrt{-a}e}\right)}{x}\, dx, x, 1+id+iex\right)}{4\sqrt{-a}\sqrt{b}}$$
$$+ \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}(1+id)+i\sqrt{-a}e}\right)}{x}\, dx, x, 1+id+iex\right)}{4\sqrt{-a}\sqrt{b}}$$

$$
\begin{aligned}
&= \frac{i \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right) \log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} - \frac{i \log \left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right) \log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} \\
&\quad - \frac{i \log \left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right) \log(1+id+iex)}{4\sqrt{-a}\sqrt{b}} \\
&\quad + \frac{i \log \left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right) \log(1+id+iex)}{4\sqrt{-a}\sqrt{b}} \\
&\quad - \frac{i \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(i-d-ex)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(i-d-ex)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} \\
&\quad - \frac{i \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(i+d+ex)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(i+d+ex)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.75

$$
\int \frac{\arctan(d+ex)}{a+bx^2}\,dx
$$

$$
= \frac{i\left(-\log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(-i+d)+\sqrt{-a}e}\right)\log(1+id+iex)+\log \left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}(-i+d)+\sqrt{-a}e}\right)\log(1+id+iex)+\log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right.}{\sqrt{b}(i+d)+\sqrt{-}}\right.\right.}{}
$$

```
[In] Integrate[ArcTan[d + e*x]/(a + b*x^2),x]
```

```
[Out] ((I/4)*(-(Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)]*L
og[1 + I*d + I*e*x]) + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*(-I + d))
+ Sqrt[-a]*e)]*Log[1 + I*d + I*e*x] + Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[
b]*(I + d) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] - Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(-(Sqrt[b]*(I + d)) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] + PolyLo
g[2, (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) - Sqrt[-a]*e)] - PolyLog[2,
 (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)] - PolyLog[2, (Sq
rt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*
(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]))/(Sqrt[-a]*Sqrt[b])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\ln(-iex-id+1)\ln\left(\frac{ibd-e\sqrt{ab}+b(-iex-id+1)-b}{ibd-e\sqrt{ab}-b}\right)\sqrt{ab}}{4ab} - \frac{\ln(-iex-id+1)\ln\left(\frac{ibd+e\sqrt{ab}+b(-iex-id+1)-b}{ibd+e\sqrt{ab}-b}\right)\sqrt{ab}}{4ab} + \operatorname{dilog}\Big($
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arctan(e*x+d)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(1-I*d-I*e*x)/a/b*ln((I*b*d-e*(a*b)^(1/2)+b*(1-I*d-I*e*x)-b)/(I*b*d-e
*(a*b)^(1/2)-b))*(a*b)^(1/2)-1/4*ln(1-I*d-I*e*x)/a/b*ln((I*b*d+e*(a*b)^(1/2
)+b*(1-I*d-I*e*x)-b)/(I*b*d+e*(a*b)^(1/2)-b))*(a*b)^(1/2)+1/4/a/b*dilog((I*
b*d-e*(a*b)^(1/2)+b*(1-I*d-I*e*x)-b)/(I*b*d-e*(a*b)^(1/2)-b))*(a*b)^(1/2)-1
/4/a/b*dilog((I*b*d+e*(a*b)^(1/2)+b*(1-I*d-I*e*x)-b)/(I*b*d+e*(a*b)^(1/2)-b
))*(a*b)^(1/2)+1/4*ln(1+I*d+I*e*x)/a/b*ln((I*b*d+e*(a*b)^(1/2)-b*(1+I*d+I*e
*x)+b)/(I*b*d+e*(a*b)^(1/2)+b))*(a*b)^(1/2)-1/4*ln(1+I*d+I*e*x)/a/b*ln((I*b
*d-e*(a*b)^(1/2)-b*(1+I*d+I*e*x)+b)/(I*b*d-e*(a*b)^(1/2)+b))*(a*b)^(1/2)+1/
4/a/b*dilog((I*b*d+e*(a*b)^(1/2)-b*(1+I*d+I*e*x)+b)/(I*b*d+e*(a*b)^(1/2)+b)
)*(a*b)^(1/2)-1/4/a/b*dilog((I*b*d-e*(a*b)^(1/2)-b*(1+I*d+I*e*x)+b)/(I*b*d-
e*(a*b)^(1/2)+b))*(a*b)^(1/2)
```

Fricas [F]

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \int \frac{\arctan(ex+d)}{bx^2+a}\,dx$$

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(e*x + d)/(b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \text{Timed out}$$

```
[In] integrate(atan(e*x+d)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14300 vs. $2(369) = 738$.

Time = 2.00 (sec) , antiderivative size = 14300, normalized size of antiderivative = 26.34

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*e*(8*arctan(b*x/sqrt(a*b))*arctan((e^2*x + d*e)/e)/e - (4*arctan(sqrt(b
)*x/sqrt(a))*arctan2((2*a*b*d*e^2 + (a*d*e^3 + (b*d^3 + b*d)*e + (a*e^4 + (
b*d^2 + 3*b)*e^2)*x)*sqrt(a)*sqrt(b) + (3*a*b*e^3 + (b^2*d^2 + b^2)*e)*x)/(
b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a*b*d^2 + 3*a*b)*e^2 + 4*(a*e^3 + (b*d^2
 + b)*e)*sqrt(a)*sqrt(b) + b^2), (b^2*d^4 + 2*b^2*d^2 + (a*b*d^2 + 3*a*b)*e
^2 + (2*b*d*e^2*x + a*e^3 + 3*(b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2 + (a*b*d
*e^3 + (b^2*d^3 + b^2*d)*e)*x)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a*b*d^2
+ 3*a*b)*e^2 + 4*(a*e^3 + (b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2)) + 4*arctan
(sqrt(b)*x/sqrt(a))*arctan2((2*a*b*d*e^2 - (a*d*e^3 + (b*d^3 + b*d)*e + (a*
e^4 + (b*d^2 + 3*b)*e^2)*x)*sqrt(a)*sqrt(b) + (3*a*b*e^3 + (b^2*d^2 + b^2)*
e)*x)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a*b*d^2 + 3*a*b)*e^2 - 4*(a*e^3 +
 (b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2), (b^2*d^4 + 2*b^2*d^2 + (a*b*d^2 + 3
*a*b)*e^2 - (2*b*d*e^2*x + a*e^3 + 3*(b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2 +
 (a*b*d*e^3 + (b^2*d^3 + b^2*d)*e)*x)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a
*b*d^2 + 3*a*b)*e^2 - 4*(a*e^3 + (b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2)) + l
og(b*x^2 + a)*log((b^12*d^24 + 12*b^12*d^22 + 66*b^12*d^20 + 220*b^12*d^18
+ 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 + (a^11*b*d^2 + a^11*b)*e^2
2 + 792*b^12*d^10 + 11*(a^10*b^2*d^4 + 22*a^10*b^2*d^2 + 21*a^10*b^2)*e^20
+ 495*b^12*d^8 + 55*(a^9*b^3*d^6 + 39*a^9*b^3*d^4 + 171*a^9*b^3*d^2 + 133*a
^9*b^3)*e^18 + 220*b^12*d^6 + 33*(5*a^8*b^4*d^8 + 260*a^8*b^4*d^6 + 1870*a^
8*b^4*d^4 + 3876*a^8*b^4*d^2 + 2261*a^8*b^4)*e^16 + 66*b^12*d^4 + 330*(a^7*
b^5*d^10 + 61*a^7*b^5*d^8 + 570*a^7*b^5*d^6 + 1802*a^7*b^5*d^4 + 2261*a^7*b
^5*d^2 + 969*a^7*b^5)*e^14 + 12*b^12*d^2 + 22*(21*a^6*b^6*d^12 + 1386*a^6*b
^6*d^10 + 15015*a^6*b^6*d^8 + 60060*a^6*b^6*d^6 + 109395*a^6*b^6*d^4 + 9237
8*a^6*b^6*d^2 + 29393*a^6*b^6)*e^12 + b^12 + 22*(21*a^5*b^7*d^14 + 1407*a^5
*b^7*d^12 + 16401*a^5*b^7*d^10 + 75075*a^5*b^7*d^8 + 169455*a^5*b^7*d^6 + 2
01773*a^5*b^7*d^4 + 121771*a^5*b^7*d^2 + 29393*a^5*b^7)*e^10 + 330*(a^4*b^8
*d^16 + 64*a^4*b^8*d^14 + 756*a^4*b^8*d^12 + 3696*a^4*b^8*d^10 + 9438*a^4*b
^8*d^8 + 13728*a^4*b^8*d^6 + 11492*a^4*b^8*d^4 + 5168*a^4*b^8*d^2 + 969*a^4
*b^8)*e^8 + 33*(5*a^3*b^9*d^18 + 285*a^3*b^9*d^16 + 3220*a^3*b^9*d^14 + 158
76*a^3*b^9*d^12 + 42966*a^3*b^9*d^10 + 70070*a^3*b^9*d^8 + 70980*a^3*b^9*d^
6 + 43860*a^3*b^9*d^4 + 15181*a^3*b^9*d^2 + 2261*a^3*b^9)*e^6 + 55*(a^2*b^1
0*d^20 + 46*a^2*b^10*d^18 + 465*a^2*b^10*d^16 + 2184*a^2*b^10*d^14 + 5922*a
^2*b^10*d^12 + 10164*a^2*b^10*d^10 + 11466*a^2*b^10*d^8 + 8520*a^2*b^10*d^6
```

```
 + 4029*a^2*b^10*d^4 + 1102*a^2*b^10*d^2 + 133*a^2*b^10)*e^4 + 11*(a*b^11*d
^22 + 31*a*b^11*d^20 + 255*a*b^11*d^18 + 1065*a*b^11*d^16 + 2730*a*b^11*d^1
4 + 4662*a*b^11*d^12 + 5502*a*b^11*d^10 + 4530*a*b^11*d^8 + 2565*a*b^11*d^6
 + 955*a*b^11*d^4 + 211*a*b^11*d^2 + 21*a*b^11)*e^2 + (a^11*b*e^24 + 11*(a^
10*b^2*d^2 + 21*a^10*b^2)*e^22 + 55*(a^9*b^3*d^4 + 38*a^9*b^3*d^2 + 133*a^9
*b^3)*e^20 + 33*(5*a^8*b^4*d^6 + 255*a^8*b^4*d^4 + 1615*a^8*b^4*d^2 + 2261*
a^8*b^4)*e^18 + 330*(a^7*b^5*d^8 + 60*a^7*b^5*d^6 + 510*a^7*b^5*d^4 + 1292*
a^7*b^5*d^2 + 969*a^7*b^5)*e^16 + 22*(21*a^6*b^6*d^10 + 1365*a^6*b^6*d^8 +
13650*a^6*b^6*d^6 + 46410*a^6*b^6*d^4 + 62985*a^6*b^6*d^2 + 29393*a^6*b^6)*
e^14 + 22*(21*a^5*b^7*d^12 + 1386*a^5*b^7*d^10 + 15015*a^5*b^7*d^8 + 60060*
a^5*b^7*d^6 + 109395*a^5*b^7*d^4 + 92378*a^5*b^7*d^2 + 29393*a^5*b^7)*e^12
+ 330*(a^4*b^8*d^14 + 63*a^4*b^8*d^12 + 693*a^4*b^8*d^10 + 3003*a^4*b^8*d^8
 + 6435*a^4*b^8*d^6 + 7293*a^4*b^8*d^4 + 4199*a^4*b^8*d^2 + 969*a^4*b^8)*e^
10 + 33*(5*a^3*b^9*d^16 + 280*a^3*b^9*d^14 + 2940*a^3*b^9*d^12 + 12936*a^3*
b^9*d^10 + 30030*a^3*b^9*d^8 + 40040*a^3*b^9*d^6 + 30940*a^3*b^9*d^4 + 1292
0*a^3*b^9*d^2 + 2261*a^3*b^9)*e^8 + 55*(a^2*b^10*d^18 + 45*a^2*b^10*d^16 +
420*a^2*b^10*d^14 + 1764*a^2*b^10*d^12 + 4158*a^2*b^10*d^10 + 6006*a^2*b^10
*d^8 + 5460*a^2*b^10*d^6 + 3060*a^2*b^10*d^4 + 969*a^2*b^10*d^2 + 133*a^2*b
^10)*e^6 + 11*(a*b^11*d^20 + 30*a*b^11*d^18 + 225*a*b^11*d^16 + 840*a*b^11*
d^14 + 1890*a*b^11*d^12 + 2772*a*b^11*d^10 + 2730*a*b^11*d^8 + 1800*a*b^11*
d^6 + 765*a*b^11*d^4 + 190*a*b^11*d^2 + 21*a*b^11)*e^4 + (b^12*d^22 + 11*b^
12*d^20 + 55*b^12*d^18 + 165*b^12*d^16 + 330*b^12*d^14 + 462*b^12*d^12 + 46
2*b^12*d^10 + 330*b^12*d^8 + 165*b^12*d^6 + 55*b^12*d^4 + 11*b^12*d^2 + b^1
2)*e^2)*x^2 + 2*(11*(a^10*b*d^2 + a^10*b)*e^21 + 110*(a^9*b^2*d^4 + 8*a^9*b
^2*d^2 + 7*a^9*b^2)*e^19 + 33*(15*a^8*b^3*d^6 + 205*a^8*b^3*d^4 + 589*a^8*b
^3*d^2 + 399*a^8*b^3)*e^17 + 264*(5*a^7*b^4*d^8 + 90*a^7*b^4*d^6 + 408*a^7*
b^4*d^4 + 646*a^7*b^4*d^2 + 323*a^7*b^4)*e^15 + 110*(21*a^6*b^5*d^10 + 441*
a^6*b^5*d^8 + 2562*a^6*b^5*d^6 + 6018*a^6*b^5*d^4 + 6137*a^6*b^5*d^2 + 2261
*a^6*b^5)*e^13 + 4*(693*a^5*b^6*d^12 + 15708*a^5*b^6*d^10 + 105105*a^5*b^6*
d^8 + 308880*a^5*b^6*d^6 + 449735*a^5*b^6*d^4 + 319124*a^5*b^6*d^2 + 88179*
a^5*b^6)*e^11 + 110*(21*a^4*b^7*d^14 + 483*a^4*b^7*d^12 + 3465*a^4*b^7*d^10
 + 11583*a^4*b^7*d^8 + 20735*a^4*b^7*d^6 + 20553*a^4*b^7*d^4 + 10659*a^4*b^
7*d^2 + 2261*a^4*b^7)*e^9 + 264*(5*a^3*b^8*d^16 + 110*a^3*b^8*d^14 + 798*a^
3*b^8*d^12 + 2838*a^3*b^8*d^10 + 5720*a^3*b^8*d^8 + 6890*a^3*b^8*d^6 + 4930
*a^3*b^8*d^4 + 1938*a^3*b^8*d^2 + 323*a^3*b^8)*e^7 + 33*(15*a^2*b^9*d^18 +
295*a^2*b^9*d^16 + 2044*a^2*b^9*d^14 + 7308*a^2*b^9*d^12 + 15554*a^2*b^9*d^
10 + 20930*a^2*b^9*d^8 + 18060*a^2*b^9*d^6 + 9724*a^2*b^9*d^4 + 2983*a^2*b^
9*d^2 + 399*a^2*b^9)*e^5 + 110*(a*b^10*d^20 + 16*a*b^10*d^18 + 99*a*b^10*d^
16 + 336*a*b^10*d^14 + 714*a*b^10*d^12 + 1008*a*b^10*d^10 + 966*a*b^10*d^8
+ 624*a*b^10*d^6 + 261*a*b^10*d^4 + 64*a*b^10*d^2 + 7*a*b^10)*e^3 + (11*a^1
0*b*e^23 + 110*(a^9*b^2*d^2 + 7*a^9*b^2)*e^21 + 33*(15*a^8*b^3*d^4 + 190*a^
8*b^3*d^2 + 399*a^8*b^3)*e^19 + 264*(5*a^7*b^4*d^6 + 85*a^7*b^4*d^4 + 323*a
^7*b^4*d^2 + 323*a^7*b^4)*e^17 + 110*(21*a^6*b^5*d^8 + 420*a^6*b^5*d^6 + 21
42*a^6*b^5*d^4 + 3876*a^6*b^5*d^2 + 2261*a^6*b^5)*e^15 + 4*(693*a^5*b^6*d^1
0 + 15015*a^5*b^6*d^8 + 90090*a^5*b^6*d^6 + 218790*a^5*b^6*d^4 + 230945*a^5
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*b^6*d^2 + 88179*a^5*b^6)*e^13 + 110*(21*a^4*b^7*d^12 + 462*a^4*b^7*d^10 +
3003*a^4*b^7*d^8 + 8580*a^4*b^7*d^6 + 12155*a^4*b^7*d^4 + 8398*a^4*b^7*d^2
+ 2261*a^4*b^7)*e^11 + 264*(5*a^3*b^8*d^14 + 105*a^3*b^8*d^12 + 693*a^3*b^8
*d^10 + 2145*a^3*b^8*d^8 + 3575*a^3*b^8*d^6 + 3315*a^3*b^8*d^4 + 1615*a^3*b
^8*d^2 + 323*a^3*b^8)*e^9 + 33*(15*a^2*b^9*d^16 + 280*a^2*b^9*d^14 + 1764*a
^2*b^9*d^12 + 5544*a^2*b^9*d^10 + 10010*a^2*b^9*d^8 + 10920*a^2*b^9*d^6 + 7
140*a^2*b^9*d^4 + 2584*a^2*b^9*d^2 + 399*a^2*b^9)*e^7 + 110*(a*b^10*d^18 +
15*a*b^10*d^16 + 84*a*b^10*d^14 + 252*a*b^10*d^12 + 462*a*b^10*d^10 + 546*a
*b^10*d^8 + 420*a*b^10*d^6 + 204*a*b^10*d^4 + 57*a*b^10*d^2 + 7*a*b^10)*e^5
 + 11*(b^11*d^20 + 10*b^11*d^18 + 45*b^11*d^16 + 120*b^11*d^14 + 210*b^11*d
^12 + 252*b^11*d^10 + 210*b^11*d^8 + 120*b^11*d^6 + 45*b^11*d^4 + 10*b^11*d
^2 + b^11)*e^3)*x^2 + 11*(b^11*d^22 + 11*b^11*d^20 + 55*b^11*d^18 + 165*b^1
1*d^16 + 330*b^11*d^14 + 462*b^11*d^12 + 462*b^11*d^10 + 330*b^11*d^8 + 165
*b^11*d^6 + 55*b^11*d^4 + 11*b^11*d^2 + b^11)*e + 2*(11*a^10*b*d*e^22 + 110
*(a^9*b^2*d^3 + 7*a^9*b^2*d)*e^20 + 33*(15*a^8*b^3*d^5 + 190*a^8*b^3*d^3 +
399*a^8*b^3*d)*e^18 + 264*(5*a^7*b^4*d^7 + 85*a^7*b^4*d^5 + 323*a^7*b^4*d^3
 + 323*a^7*b^4*d)*e^16 + 110*(21*a^6*b^5*d^9 + 420*a^6*b^5*d^7 + 2142*a^6*b
^5*d^5 + 3876*a^6*b^5*d^3 + 2261*a^6*b^5*d)*e^14 + 4*(693*a^5*b^6*d^11 + 15
015*a^5*b^6*d^9 + 90090*a^5*b^6*d^7 + 218790*a^5*b^6*d^5 + 230945*a^5*b^6*d
^3 + 88179*a^5*b^6*d)*e^12 + 110*(21*a^4*b^7*d^13 + 462*a^4*b^7*d^11 + 3003
*a^4*b^7*d^9 + 8580*a^4*b^7*d^7 + 12155*a^4*b^7*d^5 + 8398*a^4*b^7*d^3 + 22
61*a^4*b^7*d)*e^10 + 264*(5*a^3*b^8*d^15 + 105*a^3*b^8*d^13 + 693*a^3*b^8*d
^11 + 2145*a^3*b^8*d^9 + 3575*a^3*b^8*d^7 + 3315*a^3*b^8*d^5 + 1615*a^3*b^8
*d^3 + 323*a^3*b^8*d)*e^8 + 33*(15*a^2*b^9*d^17 + 280*a^2*b^9*d^15 + 1764*a
^2*b^9*d^13 + 5544*a^2*b^9*d^11 + 10010*a^2*b^9*d^9 + 10920*a^2*b^9*d^7 + 7
140*a^2*b^9*d^5 + 2584*a^2*b^9*d^3 + 399*a^2*b^9*d)*e^6 + 110*(a*b^10*d^19
+ 15*a*b^10*d^17 + 84*a*b^10*d^15 + 252*a*b^10*d^13 + 462*a*b^10*d^11 + 546
*a*b^10*d^9 + 420*a*b^10*d^7 + 204*a*b^10*d^5 + 57*a*b^10*d^3 + 7*a*b^10*d)
*e^4 + 11*(b^11*d^21 + 10*b^11*d^19 + 45*b^11*d^17 + 120*b^11*d^15 + 210*b^
11*d^13 + 252*b^11*d^11 + 210*b^11*d^9 + 120*b^11*d^7 + 45*b^11*d^5 + 10*b^
11*d^3 + b^11*d)*e^2)*x)*sqrt(a)*sqrt(b) + 2*(a^11*b*d*e^23 + 11*(a^10*b^2*
d^3 + 21*a^10*b^2*d)*e^21 + 55*(a^9*b^3*d^5 + 38*a^9*b^3*d^3 + 133*a^9*b^3*
d)*e^19 + 33*(5*a^8*b^4*d^7 + 255*a^8*b^4*d^5 + 1615*a^8*b^4*d^3 + 2261*a^8
*b^4*d)*e^17 + 330*(a^7*b^5*d^9 + 60*a^7*b^5*d^7 + 510*a^7*b^5*d^5 + 1292*a
^7*b^5*d^3 + 969*a^7*b^5*d)*e^15 + 22*(21*a^6*b^6*d^11 + 1365*a^6*b^6*d^9 +
 13650*a^6*b^6*d^7 + 46410*a^6*b^6*d^5 + 62985*a^6*b^6*d^3 + 29393*a^6*b^6*
d)*e^13 + 22*(21*a^5*b^7*d^13 + 1386*a^5*b^7*d^11 + 15015*a^5*b^7*d^9 + 600
60*a^5*b^7*d^7 + 109395*a^5*b^7*d^5 + 92378*a^5*b^7*d^3 + 29393*a^5*b^7*d)*
e^11 + 330*(a^4*b^8*d^15 + 63*a^4*b^8*d^13 + 693*a^4*b^8*d^11 + 3003*a^4*b^
8*d^9 + 6435*a^4*b^8*d^7 + 7293*a^4*b^8*d^5 + 4199*a^4*b^8*d^3 + 969*a^4*b^
8*d)*e^9 + 33*(5*a^3*b^9*d^17 + 280*a^3*b^9*d^15 + 2940*a^3*b^9*d^13 + 1293
6*a^3*b^9*d^11 + 30030*a^3*b^9*d^9 + 40040*a^3*b^9*d^7 + 30940*a^3*b^9*d^5
+ 12920*a^3*b^9*d^3 + 2261*a^3*b^9*d)*e^7 + 55*(a^2*b^10*d^19 + 45*a^2*b^10
*d^17 + 420*a^2*b^10*d^15 + 1764*a^2*b^10*d^13 + 4158*a^2*b^10*d^11 + 6006*
a^2*b^10*d^9 + 5460*a^2*b^10*d^7 + 3060*a^2*b^10*d^5 + 969*a^2*b^10*d^3 + 1
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33*a^2*b^10*d)*e^5 + 11*(a*b^11*d^21 + 30*a*b^11*d^19 + 225*a*b^11*d^17 + 8
40*a*b^11*d^15 + 1890*a*b^11*d^13 + 2772*a*b^11*d^11 + 2730*a*b^11*d^9 + 18
00*a*b^11*d^7 + 765*a*b^11*d^5 + 190*a*b^11*d^3 + 21*a*b^11*d)*e^3 + (b^12*
d^23 + 11*b^12*d^21 + 55*b^12*d^19 + 165*b^12*d^17 + 330*b^12*d^15 + 462*b^
12*d^13 + 462*b^12*d^11 + 330*b^12*d^9 + 165*b^12*d^7 + 55*b^12*d^5 + 11*b^
12*d^3 + b^12*d)*e)*x)/(b^12*d^24 + a^12*e^24 + 12*b^12*d^22 + 66*b^12*d^20
 + 220*b^12*d^18 + 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 + 12*(a^11
*b*d^2 + 23*a^11*b)*e^22 + 792*b^12*d^10 + 66*(a^10*b^2*d^4 + 42*a^10*b^2*d
^2 + 161*a^10*b^2)*e^20 + 495*b^12*d^8 + 44*(5*a^9*b^3*d^6 + 285*a^9*b^3*d^
4 + 1995*a^9*b^3*d^2 + 3059*a^9*b^3)*e^18 + 220*b^12*d^6 + 99*(5*a^8*b^4*d^
8 + 340*a^8*b^4*d^6 + 3230*a^8*b^4*d^4 + 9044*a^8*b^4*d^2 + 7429*a^8*b^4)*e
^16 + 66*b^12*d^4 + 264*(3*a^7*b^5*d^10 + 225*a^7*b^5*d^8 + 2550*a^7*b^5*d^
6 + 9690*a^7*b^5*d^4 + 14535*a^7*b^5*d^2 + 7429*a^7*b^5)*e^14 + 12*b^12*d^2
 + 4*(231*a^6*b^6*d^12 + 18018*a^6*b^6*d^10 + 225225*a^6*b^6*d^8 + 1021020*
a^6*b^6*d^6 + 2078505*a^6*b^6*d^4 + 1939938*a^6*b^6*d^2 + 676039*a^6*b^6)*e
^12 + b^12 + 264*(3*a^5*b^7*d^14 + 231*a^5*b^7*d^12 + 3003*a^5*b^7*d^10 + 1
5015*a^5*b^7*d^8 + 36465*a^5*b^7*d^6 + 46189*a^5*b^7*d^4 + 29393*a^5*b^7*d^
2 + 7429*a^5*b^7)*e^10 + 99*(5*a^4*b^8*d^16 + 360*a^4*b^8*d^14 + 4620*a^4*b
^8*d^12 + 24024*a^4*b^8*d^10 + 64350*a^4*b^8*d^8 + 97240*a^4*b^8*d^6 + 8398
0*a^4*b^8*d^4 + 38760*a^4*b^8*d^2 + 7429*a^4*b^8)*e^8 + 44*(5*a^3*b^9*d^18
 + 315*a^3*b^9*d^16 + 3780*a^3*b^9*d^14 + 19404*a^3*b^9*d^12 + 54054*a^3*b^9
*d^10 + 90090*a^3*b^9*d^8 + 92820*a^3*b^9*d^6 + 58140*a^3*b^9*d^4 + 20349*a
^3*b^9*d^2 + 3059*a^3*b^9)*e^6 + 66*(a^2*b^10*d^20 + 50*a^2*b^10*d^18 + 525
*a^2*b^10*d^16 + 2520*a^2*b^10*d^14 + 6930*a^2*b^10*d^12 + 12012*a^2*b^10*d
^10 + 13650*a^2*b^10*d^8 + 10200*a^2*b^10*d^6 + 4845*a^2*b^10*d^4 + 1330*a^
2*b^10*d^2 + 161*a^2*b^10)*e^4 + 12*(a*b^11*d^22 + 33*a*b^11*d^20 + 275*a*b
^11*d^18 + 1155*a*b^11*d^16 + 2970*a*b^11*d^14 + 5082*a*b^11*d^12 + 6006*a*
b^11*d^10 + 4950*a*b^11*d^8 + 2805*a*b^11*d^6 + 1045*a*b^11*d^4 + 231*a*b^1
1*d^2 + 23*a*b^11)*e^2 + 8*(3*a^11*e^23 + 11*(3*a^10*b*d^2 + 23*a^10*b)*e^2
1 + 33*(5*a^9*b^2*d^4 + 70*a^9*b^2*d^2 + 161*a^9*b^2)*e^19 + 99*(5*a^8*b^3*
d^6 + 95*a^8*b^3*d^4 + 399*a^8*b^3*d^2 + 437*a^8*b^3)*e^17 + 22*(45*a^7*b^4
*d^8 + 1020*a^7*b^4*d^6 + 5814*a^7*b^4*d^4 + 11628*a^7*b^4*d^2 + 7429*a^7*b
^4)*e^15 + 6*(231*a^6*b^5*d^10 + 5775*a^6*b^5*d^8 + 39270*a^6*b^5*d^6 + 106
590*a^6*b^5*d^4 + 124355*a^6*b^5*d^2 + 52003*a^6*b^5)*e^13 + 6*(231*a^5*b^6
*d^12 + 6006*a^5*b^6*d^10 + 45045*a^5*b^6*d^8 + 145860*a^5*b^6*d^6 + 230945
*a^5*b^6*d^4 + 176358*a^5*b^6*d^2 + 52003*a^5*b^6)*e^11 + 22*(45*a^4*b^7*d^
14 + 1155*a^4*b^7*d^12 + 9009*a^4*b^7*d^10 + 32175*a^4*b^7*d^8 + 60775*a^4*
b^7*d^6 + 62985*a^4*b^7*d^4 + 33915*a^4*b^7*d^2 + 7429*a^4*b^7)*e^9 + 99*(5
*a^3*b^8*d^16 + 120*a^3*b^8*d^14 + 924*a^3*b^8*d^12 + 3432*a^3*b^8*d^10 + 7
150*a^3*b^8*d^8 + 8840*a^3*b^8*d^6 + 6460*a^3*b^8*d^4 + 2584*a^3*b^8*d^2 +
437*a^3*b^8)*e^7 + 33*(5*a^2*b^9*d^18 + 105*a^2*b^9*d^16 + 756*a^2*b^9*d^14
 + 2772*a^2*b^9*d^12 + 6006*a^2*b^9*d^10 + 8190*a^2*b^9*d^8 + 7140*a^2*b^9*
d^6 + 3876*a^2*b^9*d^4 + 1197*a^2*b^9*d^2 + 161*a^2*b^9)*e^5 + 11*(3*a*b^10
*d^20 + 50*a*b^10*d^18 + 315*a*b^10*d^16 + 1080*a*b^10*d^14 + 2310*a*b^10*d
^12 + 3276*a*b^10*d^10 + 3150*a*b^10*d^8 + 2040*a*b^10*d^6 + 855*a*b^10*d^4
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 + 210*a*b^10*d^2 + 23*a*b^10)*e^3 + 3*(b^11*d^22 + 11*b^11*d^20 + 55*b^11*
d^18 + 165*b^11*d^16 + 330*b^11*d^14 + 462*b^11*d^12 + 462*b^11*d^10 + 330*
b^11*d^8 + 165*b^11*d^6 + 55*b^11*d^4 + 11*b^11*d^2 + b^11)*e)*sqrt(a)*sqrt
(b))) - log(b*x^2 + a)*log((b^12*d^24 + 12*b^12*d^22 + 66*b^12*d^20 + 220*b
^12*d^18 + 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 + (a^11*b*d^2 + a^
11*b)*e^22 + 792*b^12*d^10 + 11*(a^10*b^2*d^4 + 22*a^10*b^2*d^2 + 21*a^10*b
^2)*e^20 + 495*b^12*d^8 + 55*(a^9*b^3*d^6 + 39*a^9*b^3*d^4 + 171*a^9*b^3*d^
2 + 133*a^9*b^3)*e^18 + 220*b^12*d^6 + 33*(5*a^8*b^4*d^8 + 260*a^8*b^4*d^6
 + 1870*a^8*b^4*d^4 + 3876*a^8*b^4*d^2 + 2261*a^8*b^4)*e^16 + 66*b^12*d^4 +
330*(a^7*b^5*d^10 + 61*a^7*b^5*d^8 + 570*a^7*b^5*d^6 + 1802*a^7*b^5*d^4 + 2
261*a^7*b^5*d^2 + 969*a^7*b^5)*e^14 + 12*b^12*d^2 + 22*(21*a^6*b^6*d^12 + 1
386*a^6*b^6*d^10 + 15015*a^6*b^6*d^8 + 60060*a^6*b^6*d^6 + 109395*a^6*b^6*d
^4 + 92378*a^6*b^6*d^2 + 29393*a^6*b^6)*e^12 + b^12 + 22*(21*a^5*b^7*d^14 +
 1407*a^5*b^7*d^12 + 16401*a^5*b^7*d^10 + 75075*a^5*b^7*d^8 + 169455*a^5*b^
7*d^6 + 201773*a^5*b^7*d^4 + 121771*a^5*b^7*d^2 + 29393*a^5*b^7)*e^10 + 330
*(a^4*b^8*d^16 + 64*a^4*b^8*d^14 + 756*a^4*b^8*d^12 + 3696*a^4*b^8*d^10 + 9
438*a^4*b^8*d^8 + 13728*a^4*b^8*d^6 + 11492*a^4*b^8*d^4 + 5168*a^4*b^8*d^2
 + 969*a^4*b^8)*e^8 + 33*(5*a^3*b^9*d^18 + 285*a^3*b^9*d^16 + 3220*a^3*b^9*d
^14 + 15876*a^3*b^9*d^12 + 42966*a^3*b^9*d^10 + 70070*a^3*b^9*d^8 + 70980*a
^3*b^9*d^6 + 43860*a^3*b^9*d^4 + 15181*a^3*b^9*d^2 + 2261*a^3*b^9)*e^6 + 55
*(a^2*b^10*d^20 + 46*a^2*b^10*d^18 + 465*a^2*b^10*d^16 + 2184*a^2*b^10*d^14
 + 5922*a^2*b^10*d^12 + 10164*a^2*b^10*d^10 + 11466*a^2*b^10*d^8 + 8520*a^2
*b^10*d^6 + 4029*a^2*b^10*d^4 + 1102*a^2*b^10*d^2 + 133*a^2*b^10)*e^4 + 11*
(a*b^11*d^22 + 31*a*b^11*d^20 + 255*a*b^11*d^18 + 1065*a*b^11*d^16 + 2730*a
*b^11*d^14 + 4662*a*b^11*d^12 + 5502*a*b^11*d^10 + 4530*a*b^11*d^8 + 2565*a
*b^11*d^6 + 955*a*b^11*d^4 + 211*a*b^11*d^2 + 21*a*b^11)*e^2 + (a^11*b*e^24
 + 11*(a^10*b^2*d^2 + 21*a^10*b^2)*e^22 + 55*(a^9*b^3*d^4 + 38*a^9*b^3*d^2
 + 133*a^9*b^3)*e^20 + 33*(5*a^8*b^4*d^6 + 255*a^8*b^4*d^4 + 1615*a^8*b^4*d^
2 + 2261*a^8*b^4)*e^18 + 330*(a^7*b^5*d^8 + 60*a^7*b^5*d^6 + 510*a^7*b^5*d^
4 + 1292*a^7*b^5*d^2 + 969*a^7*b^5)*e^16 + 22*(21*a^6*b^6*d^10 + 1365*a^6*b
^6*d^8 + 13650*a^6*b^6*d^6 + 46410*a^6*b^6*d^4 + 62985*a^6*b^6*d^2 + 29393*
a^6*b^6)*e^14 + 22*(21*a^5*b^7*d^12 + 1386*a^5*b^7*d^10 + 15015*a^5*b^7*d^8
 + 60060*a^5*b^7*d^6 + 109395*a^5*b^7*d^4 + 92378*a^5*b^7*d^2 + 29393*a^5*b
^7)*e^12 + 330*(a^4*b^8*d^14 + 63*a^4*b^8*d^12 + 693*a^4*b^8*d^10 + 3003*a^
4*b^8*d^8 + 6435*a^4*b^8*d^6 + 7293*a^4*b^8*d^4 + 4199*a^4*b^8*d^2 + 969*a^
4*b^8)*e^10 + 33*(5*a^3*b^9*d^16 + 280*a^3*b^9*d^14 + 2940*a^3*b^9*d^12 + 1
2936*a^3*b^9*d^10 + 30030*a^3*b^9*d^8 + 40040*a^3*b^9*d^6 + 30940*a^3*b^9*d
^4 + 12920*a^3*b^9*d^2 + 2261*a^3*b^9)*e^8 + 55*(a^2*b^10*d^18 + 45*a^2*b^1
0*d^16 + 420*a^2*b^10*d^14 + 1764*a^2*b^10*d^12 + 4158*a^2*b^10*d^10 + 6006
*a^2*b^10*d^8 + 5460*a^2*b^10*d^6 + 3060*a^2*b^10*d^4 + 969*a^2*b^10*d^2 +
133*a^2*b^10)*e^6 + 11*(a*b^11*d^20 + 30*a*b^11*d^18 + 225*a*b^11*d^16 + 84
0*a*b^11*d^14 + 1890*a*b^11*d^12 + 2772*a*b^11*d^10 + 2730*a*b^11*d^8 + 180
0*a*b^11*d^6 + 765*a*b^11*d^4 + 190*a*b^11*d^2 + 21*a*b^11)*e^4 + (b^12*d^2
2 + 11*b^12*d^20 + 55*b^12*d^18 + 165*b^12*d^16 + 330*b^12*d^14 + 462*b^12*
d^12 + 462*b^12*d^10 + 330*b^12*d^8 + 165*b^12*d^6 + 55*b^12*d^4 + 11*b^12*
```

```
d^2 + b^12)*e^2)*x^2 - 2*(11*(a^10*b*d^2 + a^10*b)*e^21 + 110*(a^9*b^2*d^4
+ 8*a^9*b^2*d^2 + 7*a^9*b^2)*e^19 + 33*(15*a^8*b^3*d^6 + 205*a^8*b^3*d^4 +
589*a^8*b^3*d^2 + 399*a^8*b^3)*e^17 + 264*(5*a^7*b^4*d^8 + 90*a^7*b^4*d^6 +
 408*a^7*b^4*d^4 + 646*a^7*b^4*d^2 + 323*a^7*b^4)*e^15 + 110*(21*a^6*b^5*d^
10 + 441*a^6*b^5*d^8 + 2562*a^6*b^5*d^6 + 6018*a^6*b^5*d^4 + 6137*a^6*b^5*d
^2 + 2261*a^6*b^5)*e^13 + 4*(693*a^5*b^6*d^12 + 15708*a^5*b^6*d^10 + 105105
*a^5*b^6*d^8 + 308880*a^5*b^6*d^6 + 449735*a^5*b^6*d^4 + 319124*a^5*b^6*d^2
 + 88179*a^5*b^6)*e^11 + 110*(21*a^4*b^7*d^14 + 483*a^4*b^7*d^12 + 3465*a^4
*b^7*d^10 + 11583*a^4*b^7*d^8 + 20735*a^4*b^7*d^6 + 20553*a^4*b^7*d^4 + 106
59*a^4*b^7*d^2 + 2261*a^4*b^7)*e^9 + 264*(5*a^3*b^8*d^16 + 110*a^3*b^8*d^14
 + 798*a^3*b^8*d^12 + 2838*a^3*b^8*d^10 + 5720*a^3*b^8*d^8 + 6890*a^3*b^8*d
^6 + 4930*a^3*b^8*d^4 + 1938*a^3*b^8*d^2 + 323*a^3*b^8)*e^7 + 33*(15*a^2*b^
9*d^18 + 295*a^2*b^9*d^16 + 2044*a^2*b^9*d^14 + 7308*a^2*b^9*d^12 + 15554*a
^2*b^9*d^10 + 20930*a^2*b^9*d^8 + 18060*a^2*b^9*d^6 + 9724*a^2*b^9*d^4 + 29
83*a^2*b^9*d^2 + 399*a^2*b^9)*e^5 + 110*(a*b^10*d^20 + 16*a*b^10*d^18 + 99*
a*b^10*d^16 + 336*a*b^10*d^14 + 714*a*b^10*d^12 + 1008*a*b^10*d^10 + 966*a*
b^10*d^8 + 624*a*b^10*d^6 + 261*a*b^10*d^4 + 64*a*b^10*d^2 + 7*a*b^10)*e^3
+ (11*a^10*b*e^23 + 110*(a^9*b^2*d^2 + 7*a^9*b^2)*e^21 + 33*(15*a^8*b^3*d^4
 + 190*a^8*b^3*d^2 + 399*a^8*b^3)*e^19 + 264*(5*a^7*b^4*d^6 + 85*a^7*b^4*d^
4 + 323*a^7*b^4*d^2 + 323*a^7*b^4)*e^17 + 110*(21*a^6*b^5*d^8 + 420*a^6*b^5
*d^6 + 2142*a^6*b^5*d^4 + 3876*a^6*b^5*d^2 + 2261*a^6*b^5)*e^15 + 4*(693*a^
5*b^6*d^10 + 15015*a^5*b^6*d^8 + 90090*a^5*b^6*d^6 + 218790*a^5*b^6*d^4 + 2
30945*a^5*b^6*d^2 + 88179*a^5*b^6)*e^13 + 110*(21*a^4*b^7*d^12 + 462*a^4*b^
7*d^10 + 3003*a^4*b^7*d^8 + 8580*a^4*b^7*d^6 + 12155*a^4*b^7*d^4 + 8398*a^4
*b^7*d^2 + 2261*a^4*b^7)*e^11 + 264*(5*a^3*b^8*d^14 + 105*a^3*b^8*d^12 + 69
3*a^3*b^8*d^10 + 2145*a^3*b^8*d^8 + 3575*a^3*b^8*d^6 + 3315*a^3*b^8*d^4 + 1
615*a^3*b^8*d^2 + 323*a^3*b^8)*e^9 + 33*(15*a^2*b^9*d^16 + 280*a^2*b^9*d^14
 + 1764*a^2*b^9*d^12 + 5544*a^2*b^9*d^10 + 10010*a^2*b^9*d^8 + 10920*a^2*b^
9*d^6 + 7140*a^2*b^9*d^4 + 2584*a^2*b^9*d^2 + 399*a^2*b^9)*e^7 + 110*(a*b^1
0*d^18 + 15*a*b^10*d^16 + 84*a*b^10*d^14 + 252*a*b^10*d^12 + 462*a*b^10*d^1
0 + 546*a*b^10*d^8 + 420*a*b^10*d^6 + 204*a*b^10*d^4 + 57*a*b^10*d^2 + 7*a*
b^10)*e^5 + 11*(b^11*d^20 + 10*b^11*d^18 + 45*b^11*d^16 + 120*b^11*d^14 + 2
10*b^11*d^12 + 252*b^11*d^10 + 210*b^11*d^8 + 120*b^11*d^6 + 45*b^11*d^4 +
10*b^11*d^2 + b^11)*e^3)*x^2 + 11*(b^11*d^22 + 11*b^11*d^20 + 55*b^11*d^18
+ 165*b^11*d^16 + 330*b^11*d^14 + 462*b^11*d^12 + 462*b^11*d^10 + 330*b^11*
d^8 + 165*b^11*d^6 + 55*b^11*d^4 + 11*b^11*d^2 + b^11)*e + 2*(11*a^10*b*d*e
^22 + 110*(a^9*b^2*d^3 + 7*a^9*b^2*d)*e^20 + 33*(15*a^8*b^3*d^5 + 190*a^8*b
^3*d^3 + 399*a^8*b^3*d)*e^18 + 264*(5*a^7*b^4*d^7 + 85*a^7*b^4*d^5 + 323*a^
7*b^4*d^3 + 323*a^7*b^4*d)*e^16 + 110*(21*a^6*b^5*d^9 + 420*a^6*b^5*d^7 + 2
142*a^6*b^5*d^5 + 3876*a^6*b^5*d^3 + 2261*a^6*b^5*d)*e^14 + 4*(693*a^5*b^6*
d^11 + 15015*a^5*b^6*d^9 + 90090*a^5*b^6*d^7 + 218790*a^5*b^6*d^5 + 230945*
a^5*b^6*d^3 + 88179*a^5*b^6*d)*e^12 + 110*(21*a^4*b^7*d^13 + 462*a^4*b^7*d^
11 + 3003*a^4*b^7*d^9 + 8580*a^4*b^7*d^7 + 12155*a^4*b^7*d^5 + 8398*a^4*b^7
*d^3 + 2261*a^4*b^7*d)*e^10 + 264*(5*a^3*b^8*d^15 + 105*a^3*b^8*d^13 + 693*
a^3*b^8*d^11 + 2145*a^3*b^8*d^9 + 3575*a^3*b^8*d^7 + 3315*a^3*b^8*d^5 + 161
```

```
5*a^3*b^8*d^3 + 323*a^3*b^8*d)*e^8 + 33*(15*a^2*b^9*d^17 + 280*a^2*b^9*d^15
 + 1764*a^2*b^9*d^13 + 5544*a^2*b^9*d^11 + 10010*a^2*b^9*d^9 + 10920*a^2*b^
9*d^7 + 7140*a^2*b^9*d^5 + 2584*a^2*b^9*d^3 + 399*a^2*b^9*d)*e^6 + 110*(a*b
^10*d^19 + 15*a*b^10*d^17 + 84*a*b^10*d^15 + 252*a*b^10*d^13 + 462*a*b^10*d
^11 + 546*a*b^10*d^9 + 420*a*b^10*d^7 + 204*a*b^10*d^5 + 57*a*b^10*d^3 + 7*
a*b^10*d)*e^4 + 11*(b^11*d^21 + 10*b^11*d^19 + 45*b^11*d^17 + 120*b^11*d^15
 + 210*b^11*d^13 + 252*b^11*d^11 + 210*b^11*d^9 + 120*b^11*d^7 + 45*b^11*d^
5 + 10*b^11*d^3 + b^11*d)*e^2)*x)*sqrt(a)*sqrt(b) + 2*(a^11*b*d*e^23 + 11*(
a^10*b^2*d^3 + 21*a^10*b^2*d)*e^21 + 55*(a^9*b^3*d^5 + 38*a^9*b^3*d^3 + 133
*a^9*b^3*d)*e^19 + 33*(5*a^8*b^4*d^7 + 255*a^8*b^4*d^5 + 1615*a^8*b^4*d^3 +
 2261*a^8*b^4*d)*e^17 + 330*(a^7*b^5*d^9 + 60*a^7*b^5*d^7 + 510*a^7*b^5*d^5
 + 1292*a^7*b^5*d^3 + 969*a^7*b^5*d)*e^15 + 22*(21*a^6*b^6*d^11 + 1365*a^6*
b^6*d^9 + 13650*a^6*b^6*d^7 + 46410*a^6*b^6*d^5 + 62985*a^6*b^6*d^3 + 29393
*a^6*b^6*d)*e^13 + 22*(21*a^5*b^7*d^13 + 1386*a^5*b^7*d^11 + 15015*a^5*b^7*
d^9 + 60060*a^5*b^7*d^7 + 109395*a^5*b^7*d^5 + 92378*a^5*b^7*d^3 + 29393*a^
5*b^7*d)*e^11 + 330*(a^4*b^8*d^15 + 63*a^4*b^8*d^13 + 693*a^4*b^8*d^11 + 30
03*a^4*b^8*d^9 + 6435*a^4*b^8*d^7 + 7293*a^4*b^8*d^5 + 4199*a^4*b^8*d^3 + 9
69*a^4*b^8*d)*e^9 + 33*(5*a^3*b^9*d^17 + 280*a^3*b^9*d^15 + 2940*a^3*b^9*d^
13 + 12936*a^3*b^9*d^11 + 30030*a^3*b^9*d^9 + 40040*a^3*b^9*d^7 + 30940*a^3
*b^9*d^5 + 12920*a^3*b^9*d^3 + 2261*a^3*b^9*d)*e^7 + 55*(a^2*b^10*d^19 + 45
*a^2*b^10*d^17 + 420*a^2*b^10*d^15 + 1764*a^2*b^10*d^13 + 4158*a^2*b^10*d^1
1 + 6006*a^2*b^10*d^9 + 5460*a^2*b^10*d^7 + 3060*a^2*b^10*d^5 + 969*a^2*b^1
0*d^3 + 133*a^2*b^10*d)*e^5 + 11*(a*b^11*d^21 + 30*a*b^11*d^19 + 225*a*b^11
*d^17 + 840*a*b^11*d^15 + 1890*a*b^11*d^13 + 2772*a*b^11*d^11 + 2730*a*b^11
*d^9 + 1800*a*b^11*d^7 + 765*a*b^11*d^5 + 190*a*b^11*d^3 + 21*a*b^11*d)*e^3
 + (b^12*d^23 + 11*b^12*d^21 + 55*b^12*d^19 + 165*b^12*d^17 + 330*b^12*d^15
 + 462*b^12*d^13 + 462*b^12*d^11 + 330*b^12*d^9 + 165*b^12*d^7 + 55*b^12*d^
5 + 11*b^12*d^3 + b^12*d)*e)*x)/(b^12*d^24 + a^12*e^24 + 12*b^12*d^22 + 66*
b^12*d^20 + 220*b^12*d^18 + 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 +
 12*(a^11*b*d^2 + 23*a^11*b)*e^22 + 792*b^12*d^10 + 66*(a^10*b^2*d^4 + 42*a
^10*b^2*d^2 + 161*a^10*b^2)*e^20 + 495*b^12*d^8 + 44*(5*a^9*b^3*d^6 + 285*a
^9*b^3*d^4 + 1995*a^9*b^3*d^2 + 3059*a^9*b^3)*e^18 + 220*b^12*d^6 + 99*(5*a
^8*b^4*d^8 + 340*a^8*b^4*d^6 + 3230*a^8*b^4*d^4 + 9044*a^8*b^4*d^2 + 7429*a
^8*b^4)*e^16 + 66*b^12*d^4 + 264*(3*a^7*b^5*d^10 + 225*a^7*b^5*d^8 + 2550*a
^7*b^5*d^6 + 9690*a^7*b^5*d^4 + 14535*a^7*b^5*d^2 + 7429*a^7*b^5)*e^14 + 12
*b^12*d^2 + 4*(231*a^6*b^6*d^12 + 18018*a^6*b^6*d^10 + 225225*a^6*b^6*d^8 +
 1021020*a^6*b^6*d^6 + 2078505*a^6*b^6*d^4 + 1939938*a^6*b^6*d^2 + 676039*a
^6*b^6)*e^12 + b^12 + 264*(3*a^5*b^7*d^14 + 231*a^5*b^7*d^12 + 3003*a^5*b^7
*d^10 + 15015*a^5*b^7*d^8 + 36465*a^5*b^7*d^6 + 46189*a^5*b^7*d^4 + 29393*a
^5*b^7*d^2 + 7429*a^5*b^7)*e^10 + 99*(5*a^4*b^8*d^16 + 360*a^4*b^8*d^14 + 4
620*a^4*b^8*d^12 + 24024*a^4*b^8*d^10 + 64350*a^4*b^8*d^8 + 97240*a^4*b^8*d
^6 + 83980*a^4*b^8*d^4 + 38760*a^4*b^8*d^2 + 7429*a^4*b^8)*e^8 + 44*(5*a^3*
b^9*d^18 + 315*a^3*b^9*d^16 + 3780*a^3*b^9*d^14 + 19404*a^3*b^9*d^12 + 5405
4*a^3*b^9*d^10 + 90090*a^3*b^9*d^8 + 92820*a^3*b^9*d^6 + 58140*a^3*b^9*d^4
+ 20349*a^3*b^9*d^2 + 3059*a^3*b^9)*e^6 + 66*(a^2*b^10*d^20 + 50*a^2*b^10*d
```

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^18 + 525*a^2*b^10*d^16 + 2520*a^2*b^10*d^14 + 6930*a^2*b^10*d^12 + 12012*a
^2*b^10*d^10 + 13650*a^2*b^10*d^8 + 10200*a^2*b^10*d^6 + 4845*a^2*b^10*d^4
+ 1330*a^2*b^10*d^2 + 161*a^2*b^10)*e^4 + 12*(a*b^11*d^22 + 33*a*b^11*d^20
+ 275*a*b^11*d^18 + 1155*a*b^11*d^16 + 2970*a*b^11*d^14 + 5082*a*b^11*d^12
+ 6006*a*b^11*d^10 + 4950*a*b^11*d^8 + 2805*a*b^11*d^6 + 1045*a*b^11*d^4 +
231*a*b^11*d^2 + 23*a*b^11)*e^2 - 8*(3*a^11*e^23 + 11*(3*a^10*b*d^2 + 23*a^
10*b)*e^21 + 33*(5*a^9*b^2*d^4 + 70*a^9*b^2*d^2 + 161*a^9*b^2)*e^19 + 99*(5
*a^8*b^3*d^6 + 95*a^8*b^3*d^4 + 399*a^8*b^3*d^2 + 437*a^8*b^3)*e^17 + 22*(4
5*a^7*b^4*d^8 + 1020*a^7*b^4*d^6 + 5814*a^7*b^4*d^4 + 11628*a^7*b^4*d^2 + 7
429*a^7*b^4)*e^15 + 6*(231*a^6*b^5*d^10 + 5775*a^6*b^5*d^8 + 39270*a^6*b^5*
d^6 + 106590*a^6*b^5*d^4 + 124355*a^6*b^5*d^2 + 52003*a^6*b^5)*e^13 + 6*(23
1*a^5*b^6*d^12 + 6006*a^5*b^6*d^10 + 45045*a^5*b^6*d^8 + 145860*a^5*b^6*d^6
 + 230945*a^5*b^6*d^4 + 176358*a^5*b^6*d^2 + 52003*a^5*b^6)*e^11 + 22*(45*a
^4*b^7*d^14 + 1155*a^4*b^7*d^12 + 9009*a^4*b^7*d^10 + 32175*a^4*b^7*d^8 + 6
0775*a^4*b^7*d^6 + 62985*a^4*b^7*d^4 + 33915*a^4*b^7*d^2 + 7429*a^4*b^7)*e^
9 + 99*(5*a^3*b^8*d^16 + 120*a^3*b^8*d^14 + 924*a^3*b^8*d^12 + 3432*a^3*b^8
*d^10 + 7150*a^3*b^8*d^8 + 8840*a^3*b^8*d^6 + 6460*a^3*b^8*d^4 + 2584*a^3*b
^8*d^2 + 437*a^3*b^8)*e^7 + 33*(5*a^2*b^9*d^18 + 105*a^2*b^9*d^16 + 756*a^2
*b^9*d^14 + 2772*a^2*b^9*d^12 + 6006*a^2*b^9*d^10 + 8190*a^2*b^9*d^8 + 7140
*a^2*b^9*d^6 + 3876*a^2*b^9*d^4 + 1197*a^2*b^9*d^2 + 161*a^2*b^9)*e^5 + 11*
(3*a*b^10*d^20 + 50*a*b^10*d^18 + 315*a*b^10*d^16 + 1080*a*b^10*d^14 + 2310
*a*b^10*d^12 + 3276*a*b^10*d^10 + 3150*a*b^10*d^8 + 2040*a*b^10*d^6 + 855*a
*b^10*d^4 + 210*a*b^10*d^2 + 23*a*b^10)*e^3 + 3*(b^11*d^22 + 11*b^11*d^20 +
 55*b^11*d^18 + 165*b^11*d^16 + 330*b^11*d^14 + 462*b^11*d^12 + 462*b^11*d^
10 + 330*b^11*d^8 + 165*b^11*d^6 + 55*b^11*d^4 + 11*b^11*d^2 + b^11)*e)*sqr
t(a)*sqrt(b))) + 2*dilog(-(a*e^2 + (b*d + I*b)*e*x + (I*e^2*x + (-I*d + 1)*
e)*sqrt(a)*sqrt(b))/(b*d^2 - 2*sqrt(a)*sqrt(b)*(-I*d + 1)*e - a*e^2 + 2*I*b
*d - b)) - 2*dilog(-(a*e^2 + (b*d + I*b)*e*x - (I*e^2*x + (-I*d + 1)*e)*sqr
t(a)*sqrt(b))/(b*d^2 + 2*sqrt(a)*sqrt(b)*(-I*d + 1)*e - a*e^2 + 2*I*b*d - b
)) - 2*dilog(-(a*e^2 + (b*d - I*b)*e*x + (I*e^2*x + (-I*d - 1)*e)*sqrt(a)*s
qrt(b))/(b*d^2 - 2*sqrt(a)*sqrt(b)*(-I*d - 1)*e - a*e^2 - 2*I*b*d - b)) + 2
*dilog(-(a*e^2 + (b*d - I*b)*e*x - (I*e^2*x + (-I*d - 1)*e)*sqrt(a)*sqrt(b)
)/(b*d^2 + 2*sqrt(a)*sqrt(b)*(-I*d - 1)*e - a*e^2 - 2*I*b*d - b)))/e)/sqrt(
a*b) + arctan(e*x + d)*arctan(b*x/sqrt(a*b))/sqrt(a*b) - arctan(b*x/sqrt(a*
b))*arctan((e^2*x + d*e)/e)/sqrt(a*b)
```

Giac [F]

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx=\int \frac{\arctan\left(ex+d\right)}{bx^2+a}\,dx$$

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \int \frac{\operatorname{atan}(d+e\,x)}{b\,x^2+a}\,dx$$

```
[In] int(atan(d + e*x)/(a + b*x^2),x)
```

```
[Out] int(atan(d + e*x)/(a + b*x^2), x)
```

3.62 $\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 367

$$\begin{aligned}\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx &= \frac{\arctan(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}}\\ &\quad-\frac{\arctan(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}}\\ &\quad-\frac{i\,\mathrm{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}}\\ &\quad+\frac{i\,\mathrm{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}}\end{aligned}$$

```
[Out] arctan(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+
e*(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arctan(e*x+d)*ln(2*e*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-
4*a*c+b^2)^(1/2)-1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(
1/2)))/(1-I*(e*x+d))/(2*I*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(
1/2)+1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(1-I*
(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {632, 212, 6860, 5155, 4966, 2449, 2352, 2497}

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx = \frac{\arctan(d+ex)\log\left(\frac{2e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(1-i(d+ex))\left(e\left(b-\sqrt{b^2-4ac}\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}} - \frac{\arctan(d+ex)\log\left(\frac{2e\left(\sqrt{b^2-4ac}+b+2cx\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}} - \frac{i\,\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2ic+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}} + \frac{i\,\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}$$

```
[In] Int[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcTan[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) +
(b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - (ArcTan
[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sq
rt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog
[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c -
 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c]
+ ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b
^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{2c\arctan(d+ex)}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}+2cx\right)}-\frac{2c\arctan(d+ex)}{\sqrt{b^2-4ac}\,\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx$$

$$=\frac{(2c)\int\frac{\arctan(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int\frac{\arctan(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{\arctan(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}e} - \frac{(2c)\text{Subst}\left(\int \frac{\arctan(x)}{\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}e}$$

$$= \frac{\arctan(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\arctan(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2\left(\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}\right)}{\left(\frac{2ic}{e}+\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}\right)(1-ix)}\right)}{1+x^2}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2\left(\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}\right)}{\left(\frac{2ic}{e}+\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}\right)(1-ix)}\right)}{1+x^2}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{\arctan(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\arctan(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{i\,\text{PolyLog}\left(2, 1-\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}} + \frac{i\,\text{PolyLog}\left(2, 1-\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx$$
$$= \frac{i\left(\log\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2c(i+d)+\left(-b+\sqrt{b^2-4ac}\right)e}\right)\log(1-i(d+ex))-\log\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2c(i+d)+\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(1-i(d+ex))-\log\left(\right.\right.}{}$$

```
[In] Integrate[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] ((I/2)*(Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(I + d) + (-b + Sqrt[
b^2 - 4*a*c])*e)]*Log[1 - I*(d + e*x)] - Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*(I + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 - I*(d + e*x)] - Lo
g[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(-I + d) + (-b + Sqrt[b^2 - 4*a
*c])*e)]*Log[1 + I*(d + e*x)] + Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2
*c*(-I + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 + I*(d + e*x)] - PolyLog[2,
 (2*c*(-I + d + e*x))/(2*c*(-I + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLo
g[2, (2*c*(-I + d + e*x))/(2*c*(-I + d) - (b + Sqrt[b^2 - 4*a*c])*e)] + Pol
yLog[2, (2*c*(I + d + e*x))/(2*c*(I + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] - P
olyLog[2, (2*c*(I + d + e*x))/(2*c*(I + d) - (b + Sqrt[b^2 - 4*a*c])*e)]))/
Sqrt[b^2 - 4*a*c]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(329) = 658.

Time = 1.36 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.43

method	result
risch	$\frac{e\ln(-iex-id+1)\ln\left(\frac{ibe-2icd-2(-iex-id+1)c+\sqrt{4ac\,e^2-b^2e^2}+2c}{ibe-2icd+2c+\sqrt{4ac\,e^2-b^2e^2}}\right)}{2\sqrt{4ac\,e^2-b^2e^2}}-\frac{e\ln(-iex-id+1)\ln\left(\frac{ibe-2icd-2(-iex-id+1)c-\sqrt{4a}}{ibe-2icd+2c-\sqrt{4ac\,e^2-}}\right.}{2\sqrt{4ac\,e^2-b^2e^2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arctan(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e*ln(1-I*d-I*e*x)/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d-2*(1-I*d-
I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c+(4*a*c*e^2-b^2*e
^2)^(1/2)))-1/2*e*ln(1-I*d-I*e*x)/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c
*d-2*(1-I*d-I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c-(4*a
*c*e^2-b^2*e^2)^(1/2)))+1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*
d-2*(1-I*d-I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c+(4*a*
```

```
c*e^2-b^2*e^2)^(1/2)))-1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d
-2*(1-I*d-I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c-(4*a*c
*e^2-b^2*e^2)^(1/2)))+1/2*e*ln(1+I*d+I*e*x)/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I
*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*
d-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))-1/2*e*ln(1+I*d+I*e*x)/(4*a*c*e^2-b^2*e^2)
^(1/2)*ln((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(
I*b*e-2*I*c*d+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))+1/2*e/(4*a*c*e^2-b^2*e^2)^(1/
2)*dilog((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I
*b*e-2*I*c*d-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))-1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2
)*dilog((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*
b*e-2*I*c*d+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))
```

Fricas [F]

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx=\int \frac{\arctan\left(ex+d\right)}{cx^{2}+bx+a}\,dx$$

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(e*x + d)/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx=\text{Timed out}$$

```
[In] integrate(atan(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx=\int \frac{\arctan\left(ex+d\right)}{cx^2+bx+a}\,dx$$

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx=\int \frac{\mathrm{atan}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

```
[In] int(atan(d + e*x)/(a + b*x + c*x^2),x)
```

```
[Out] int(atan(d + e*x)/(a + b*x + c*x^2), x)
```

3.63 $\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=-\frac{2i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

```
[Out] -2*I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+I*poly
log(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-I*polylog(2,I*(1+I*(b*x
+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5163, 5006}

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=-\frac{2i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

```
[In] Int[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((-2*I)*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]
)/b + (I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
- (I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5163

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\arctan(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}\\ &\quad+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.51

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{i\left(2\arctan\left(e^{i\arctan(a+bx)}\right)\arctan(a+bx)-\text{PolyLog}\left(2,-ie^{i\arctan(a+bx)}\right)+\text{PolyLog}\left(2,ie^{i\arctan(a+bx)}\right)\right)}{b}$$

```
[In] Integrate[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
[Out] ((-I)*(2*ArcTan[E^(I*ArcTan[a + b*x])]*ArcTan[a + b*x] - PolyLog[2, (-I)*E^
(I*ArcTan[a + b*x])] + PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/b
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

method	result
default	$\frac{-\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^{2}}}\right)+\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^{2}}}\right)+i\,\mathrm{dilog}\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^{2}}}\right)-i\,\mathrm{dilog}\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^{2}}}\right)}{b}$

```
[In] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+arctan(b*x+a)
*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+I*dilog(1+I*(1+I*(b*x+a))/(1+(b*
x+a)^2)^(1/2))-I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\arctan\left(bx+a\right)}{\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Sympy [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{atan}\left(a+bx\right)}{\sqrt{a^{2}+2abx+b^{2}x^{2}+1}}\,dx$$

```
[In] integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\arctan\left(bx+a\right)}{\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

```
[In] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
[Out] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.64 $\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{2i\sqrt{1+(a+bx)^2}\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}$$

```
[Out] -2*I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+
a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I
*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-I*polylog(2,I*
(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{5163, 5010, 5006\}$

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{2i\sqrt{(a+bx)^2+1}\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}} + \frac{i\sqrt{(a+bx)^2+1}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}} - \frac{i\sqrt{(a+bx)^2+1}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}$$

```
[In] Int[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt
[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
 + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2])
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5163

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\arctan(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b} \\
&= \frac{\sqrt{1+(a+bx)^2}\text{Subst}\left(\int \frac{\arctan(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b\sqrt{c+c(a+bx)^2}} \\
&= -\frac{2i\sqrt{1+(a+bx)^2}\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} \\
&\quad + \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} \\
&\quad - \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.44

$$
\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx =
-\frac{i\sqrt{1+(a+bx)^2}\left(2\arctan\left(e^{i\arctan(a+bx)}\right)\arctan(a+bx)-\text{PolyLog}\left(2,-ie^{i\arctan(a+bx)}\right)+\text{PolyLog}\left(2,i\right.\right.}{b\sqrt{c\,(1+(a+bx)^2)}}
$$

```
[In] Integrate[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] ((-I)*Sqrt[1 + (a + b*x)^2]*(2*ArcTan[E^(I*ArcTan[a + b*x])]*ArcTan[a + b*x
] - PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + PolyLog[2, I*E^(I*ArcTan[a + b
*x])]))/(b*Sqrt[c*(1 + (a + b*x)^2)])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.81

method	result
default	$-\frac{\left(\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-i\,\text{dilog}\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)+i\,\text{dilog}\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)\right.}{\sqrt{b^2x^2+2abx+a^2+1}\,bc}$

```
[In] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -(arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-arctan(b*x+a)*ln(
1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*dilog(1+I*(1+I*(b*x+a))/(1+(b*x+a)
^2)^(1/2))+I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(I
+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Fricas [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\text{Timed out}$$

```
[In] integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Giac [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

```
[In] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)
```

```
[Out] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)
```

3.65 $\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\operatorname{Int}\left(\frac{\arctan(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

```
[Out] Unintegrable(arctan(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

```
[In] Int[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcTan[x]/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral}=\frac{\operatorname{Subst}\left(\int \frac{\arctan(x)}{\sqrt[3]{1+x^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. $2(23) = 46$.

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.82

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(15+10(a+bx)\arctan(a+bx)+\frac{4(a+bx)\arctan(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{11}{6},\frac{}{1+(a}\right.}{1+(a+bx)^2}\right.}{20b\sqrt[3]{1+a^2+2abx+b^2x^2}\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)}$$

```
[In] Integrate[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(1
+ a^2 + 2*a*b*x + b^2*x^2)^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\arctan\left(bx+a\right)}{\left(b^2x^2+2abx+a^2+1\right)^{\frac{1}{3}}}dx$$

```
[In] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\arctan\left(bx+a\right)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas"
)
```

```
[Out] integral(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{atan}\left(a+bx\right)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

```
[In] integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
[Out] Integral(atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima"
)
[Out] integrate(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 56.78 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.11

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{1/3}} \,dx$$

```
[In] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
[Out] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.66 $\int \frac{\arctan(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx=\text{Int}\left(\frac{\arctan(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

```
[Out] Unintegrable(arctan(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx$$

```
[In] Int[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcTan[x]/(c + c*x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral}=\frac{\text{Subst}\left(\int \frac{\arctan(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 165 vs. $2(25) = 50$.

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$
$$= \frac{6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(15+10(a+bx)\arctan(a+bx)+\frac{4(a+bx)\arctan(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{11}{6},\frac{}{1+(a}\right.}{1+(a+bx)^2}\right.}{20b\sqrt[3]{c\,(1+a^2+2abx+b^2x^2)}\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)}$$

```
[In] Integrate[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(c*
(1 + a^2 + 2*a*b*x + b^2*x^2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\arctan\left(bx+a\right)}{\left(\left(a^2+1\right)c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

```
[In] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\arctan\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"fricas")
```

```
[Out] integral(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 6.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{atan}\,(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

```
[In] integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),x)
```

```
[Out] Integral(atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan\,(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"maxima")
```

```
[Out] integrate(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 59.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.09

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan\,(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

```
[In] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3),x)
[Out] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3), x)
```

3.67 $\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = &-\frac{\sqrt{1+(a+bx)^2}}{2b} \\ &+\frac{(a+bx)\sqrt{1+(a+bx)^2}\arctan(a+bx)}{2b} \\ &+\frac{i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} \\ &-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} \\ &+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}\end{aligned}$$

```
[Out] I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*I*pol
ylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*I*polylog(2,I*(1+I
*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a
)*arctan(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5165, 5072, 267, 5006}

$$\begin{aligned}\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = &\frac{i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\arctan(a+bx)}{b}\\ &+\frac{(a+bx)\sqrt{(a+bx)^2+1}\arctan(a+bx)}{2b}\\ &-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b}\\ &+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b}-\frac{\sqrt{(a+bx)^2+1}}{2b}\end{aligned}$$

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -1/2*Sqrt[1 + (a + b*x)^2]/b + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a +
b*x])/(2*b) + (I*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a
 + b*x)]])/b - ((I/2)*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a
 + b*x)]])/b + ((I/2)*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a +
b*x)]])/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
```

```
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5165

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2 \arctan(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\sqrt{1+(a+bx)^2}\arctan(a+bx)}{2b} \\
&\quad - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} - \frac{\text{Subst}\left(\int \frac{\arctan(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} \\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\arctan(a+bx)}{2b} \\
&\quad + \frac{i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} \\
&\quad - \frac{i\,\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} + \frac{i\,\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{-\sqrt{1+(a+bx)^2}+(a+bx)\sqrt{1+(a+bx)^2}\arctan(a+bx)-\arctan(a+bx)\log\left(1-ie^{i\arctan(a+bx)}\right)+\text{arc}}{2b}$$

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x
]
```

```
[Out] (-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x] -
 ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a + b*x])] + ArcTan[a + b*x]*Log[1 +
 I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + I*Po
lyLog[2, I*E^(I*ArcTan[a + b*x])])/(2*b)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

method	result
default	$\frac{(\arctan(bx+a)bx+a\arctan(bx+a)-1)\sqrt{b^2x^2+2abx+a^2+1}}{2b}+\frac{\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a}{\sqrt{1+(bx+a}}\right.}{2b}$

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/2*(arctan(b*x+a)*b*x+a*arctan(b*x+a)-1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b+1
/2*(arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-arctan(b*x+a)*l
n(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*dilog(1+I*(1+I*(b*x+a))/(1+(b*x+
a)^2)^(1/2))+I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))/b
```

Fricas [F]

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{(bx+a)^2\arctan\left(bx+a\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\text{Timed out}$$

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{{\left(bx+a\right)}^{2} \arctan\left(bx+a\right)}{\sqrt{b^{2} x^{2} + 2\,abx + a^{2} + 1}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Giac [F]

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{{\left(bx+a\right)}^{2} \arctan\left(bx+a\right)}{\sqrt{b^{2} x^{2} + 2\,abx + a^{2} + 1}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\mathrm{atan}\left(a+b\,x\right)\,{\left(a+b\,x\right)}^2}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.68 $\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 281

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\arctan(a+bx)}{2bc} + \frac{i\sqrt{1+(a+bx)^2}\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}}$$

```
[Out] I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^
2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-1/2*I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-
I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+1/2*I*polylog
(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b
*x+a)^2)^(1/2)-1/2*(c+c*(b*x+a)^2)^(1/2)/b/c+1/2*(b*x+a)*arctan(b*x+a)*(c+c
*(b*x+a)^2)^(1/2)/b/c
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5165, 5072, 267, 5010, 5006}

$$\begin{aligned}\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx &= \frac{i\sqrt{(a+bx)^2+1}\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\arctan(a+bx)}{b\sqrt{c(a+bx)^2+c}} \\ &\quad+\frac{(a+bx)\arctan(a+bx)\sqrt{c(a+bx)^2+c}}{2bc} \\ &\quad-\frac{i\sqrt{(a+bx)^2+1}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}} \\ &\quad+\frac{i\sqrt{(a+bx)^2+1}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}} \\ &\quad-\frac{\sqrt{c(a+bx)^2+c}}{2bc}\end{aligned}$$

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2]
,x]
```

```
[Out] -1/2*Sqrt[c + c*(a + b*x)^2]/(b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*Arc
Tan[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqr
t[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((
I/2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 -
 I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2)*Sqrt[1 + (a + b*x)^2]*
PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*
(a + b*x)^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5165

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2\arctan(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\sqrt{c+c(a+bx)^2}\arctan(a+bx)}{2bc}\\
&\quad-\frac{\text{Subst}\left(\int \frac{x}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{2b}-\frac{\text{Subst}\left(\int \frac{\arctan(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{2b}\\
&= -\frac{\sqrt{c+c(a+bx)^2}}{2bc}+\frac{(a+bx)\sqrt{c+c(a+bx)^2}\arctan(a+bx)}{2bc}\\
&\quad-\frac{\sqrt{1+(a+bx)^2}\text{Subst}\left(\int \frac{\arctan(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b\sqrt{c+c(a+bx)^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sqrt{c+c(a+bx)^2}}{2bc}+\frac{(a+bx)\sqrt{c+c(a+bx)^2}\arctan(a+bx)}{2bc} \\
&\quad+\frac{i\sqrt{1+(a+bx)^2}\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} \\
&\quad-\frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}} \\
&\quad+\frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.67

$$
\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx
$$

$$
=\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(-\sqrt{1+(a+bx)^2}+(a+bx)\sqrt{1+(a+bx)^2}\arctan(a+bx)-\arctan(a+bx)\log\right.}{2b\sqrt{c}}
$$

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*
c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt
[1 + (a + b*x)^2]*ArcTan[a + b*x] - ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a
 + b*x])] + ArcTan[a + b*x]*Log[1 + I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2,
 (-I)*E^(I*ArcTan[a + b*x])] + I*PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/(2*b
*Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

method	result
default	$\frac{(\arctan(bx+a)bx+a\arctan(bx+a)-1)\sqrt{c(bx+a-i)(bx+a+i)}}{2bc}+\frac{\left(\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx}{\sqrt{1+(bx}}\right.\right.}{2\surd}$

```
[In] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(b*x+a)*b*x+a*arctan(b*x+a)-1)*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/b/
c+1/2*(arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-arctan(b*x+a
```

```
)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*dilog(1+I*(1+I*(b*x+a))/(1+(b
*x+a)^2)^(1/2))+I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*
x)*(I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Fricas [F]

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{(bx+a)^2\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c
*x + (a^2 + 1)*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\text{Timed out}$$

```
[In] integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{(bx+a)^2\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1
)*c), x)
```

Giac [F]

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a+b\,x)\,\left(a+b\,x\right)^2}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
,x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
, x)
```

3.69 $\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \operatorname{Int}\left(\frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
[Out] Defer[Subst][Defer[Int][(x^2*ArcTan[x])/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2 \arctan(x)}{\sqrt[3]{1+x^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(30) = 60.

Time = 4.75 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.17

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx =$$

$$-\frac{3(1+(a+bx)^2)^{2/3}\Bigg(\frac{5\sqrt[3]{2}\sqrt{\pi}\,\mathrm{Gamma}\left(\frac{5}{3}\right)\,{}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+(a+bx)^2}\right)}{(1+(a+bx)^2)^2}+\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gamma}\left(\frac{7}{3}\right)\Bigg(15+\frac{90}{1+(a+bx)^2}}{140b\,\mathrm{Gamm}}$$

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)
,x]
```

```
[Out] (-3*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricP
FQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2
 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 + (a + b*x)^2) + (24*(a + b*x)*ArcTan
[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(1 + (a
+ b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a + b*x) + 6*Sin[2*ArcTan[a + b*x]])))
)/(140*b*Gamma[11/6]*Gamma[7/3])
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\left(bx+a\right)^{2}\arctan\left(bx+a\right)}{\left(b^{2}x^{2}+2abx+a^{2}+1\right)^{\frac{1}{3}}}dx$$

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\left(bx+a\right)^{2}\arctan\left(bx+a\right)}{\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(a+bx)^2 \operatorname{atan}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3)
, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

Giac [N/A]

Not integrable

Time = 172.49 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.09

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\mathrm{atan}(a+b\,x)\,\left(a+b\,x\right)^2}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{1/3}}\,dx$$

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
[Out] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.70 $\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \text{Int}\left(\frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/
3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcTan[x])/(c + c*x^2)^(1/3), x], x, a + b*x]/
b
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2 \arctan(x)}{\sqrt[3]{c+cx^2}}\,dx, x, a+bx\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 225 vs. $2(32) = 64$.

Time = 0.73 (sec) , antiderivative size = 225, normalized size of antiderivative = 5.62

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx =$$
$$-\frac{3\sqrt[3]{1+a^2+2abx+b^2x^2}(1+(a+bx)^2)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\,\mathrm{Gamma}\left(\frac{5}{3}\right)\,{}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+(a+bx)^2}\right)}{(1+(a+bx)^2)^2}+\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gam}\right.}{140b\sqrt[3]{c\,(1+}}$$

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^
2)^(1/3),x]
```

```
[Out] (-3*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)
*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a
+ b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 +
 (a + b*x)^2) + (24*(a + b*x)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/
6, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a
+ b*x) + 6*Sin[2*ArcTan[a + b*x]]))))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^
2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(bx+a)^2 \arctan(bx+a)}{\left((a^2+1)\,c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

```
[In] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x +
 (a^2 + 1)*c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 26.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{(a+bx)^2\,\operatorname{atan}\,(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

```
[In] integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),
x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(
1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{(bx+a)^2\arctan\,(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 173.56 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{(bx+a)^2\arctan\,(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)\,\left(a+b\,x\right)^2}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
,x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count i
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```